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An original approach to anomalies in intertemporal choices through functional data analysis: Theory and application for the study of Hikikomori syndrome

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ABSTRACT

The pattern of intertemporal preferences is related to critical behavioural aspects involving individuals' emotional and cognitive spheres. The characteristics of the discount function can provide a quantitative interpretation of pathological conditions, such as addiction, depression, and Attention Deficit Hyperactivity Disorder (ADHD). In the literature on intertemporal preferences, researchers commonly refer to parametric models, which are often characterized by structural limitations, mainly when behavioral anomalies manifest in the dynamics of choices. This paper proposes using Functional Data Analysis (FDA) as an innovative tool to analyze discount functions' characteristics and interpret intertemporal choices' anomalies, overcoming the limits of a completely parametric approach. To show the potential of FDA in studying intertemporal decisions, this research proposes a novel methodology for dealing with discount functions and a possible application to the so-called Hikikomori condition. The latter is a social phenomenon that has recently raised concerns and claimed the need for in-depth studies. Although the application focuses on an original implementation of the functional analysis of variance approach to capture possible differences between groups of curves, the main novelty of the paper lies in the methodology for treating discount functions via FDA in the context of intertemporal choices.

1. Introduction

Intertemporal choices are those types of decisions whose alternatives are distributed over multiple periods. The Discounted Utility (DU) model [1,2] is the primary reference for describing the decision maker's behaviour when facing an intertemporal choice. From a theoretical perspective, the utility of an outcome decreases over time concerning a factor expressed by the discount function. The greater the discount function, the less significant the indeterminacy of the future perceived by the individual. Among others, the discount function quantifies the psychological factors underlying the decrease in value, an essential element of preference trends [3]. Compared with the canons of classical rationality, the discount function is assumed to be exponential, and thus, the discount rate constant over time is a key feature to preserve the persistence of choices. Although Samuelson's model has been of considerable importance since its conceptualization in the study of intertemporal choices, its predictions deviate from individuals' empirical behaviour [4–7]. Over the years, research has attempted to describe preferences as "anomalous" [8,9] when not meeting the classical rationality standards assumed by the DU model. In effect, these anomalies in intertemporal choice appear when the instantaneous discount rate either decreases with respect to the delay (t) of the dated reward to be discounted (*delay effect*) or increases for smaller monetary amounts (x) (*magnitude effect*), or increases for gains ($x > 0$) compared to losses ($-x$) (*sign effect*). Consequently, the anomalies affect all parameters involved in a discount function, i.e., delay, reward amount, and sign outcome, in contradiction with the fact that the discount rate is constant and the discount function is separable (that is to say, it has the form $U(x)f(t)$,

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where $U(x)$ is a utility function and $f(t)$ is strictly decreasing with $f(0) = 1$). However, in Subsection 2.1, all these anomalies will be treated from the point of view of changes in impatience. The anomalies in intertemporal choices have been analyzed from modelling and behavioural perspectives. Concerning the latter, cognitive psychology has allowed the understanding that individuals are endowed with limited rationality [10] and systematic cognitive distortions [11], i.e., behavioural biases intervening during decision-making.

From a modelling perspective, however, the development of hyperbolic models for describing intertemporal preferences has reduced the gap between the empirical and normative models, achieving a greater adherence to empirical behaviour [12]. The main feature of hyperbolic models is the assumption of a discount rate that decreases over time; thus, the same value is discounted more in periods close to the present [13,14]. The behavioural significance behind the hyperbolic preference trend has prompted research to investigate the mathematical characteristics of the DU model. Next to the discount rate, decision-maker impatience [15] has been formalized and defined as the second key element for decision process. From a conceptual perspective, impatience expresses how much the decision-maker is willing to lose to receive a specific good immediately and, like the discount rate, it should be constant over time for the canons of classical rationality [13]. The degree to which impatience decreases has been discussed in the context of the DU's anomalies [16] and concerning the non-rational attitudes of individuals [13]. In addition, the distance between normative and empirical preferences has also been discussed in terms of temporal dis-perception [16–18].

The critical link between behaviour and intertemporal preferences has made discount rate and impatience trends tools for studying individuals' pathological attitudes and habits over time. Actually, research turns to intertemporal choice as a tool for interpreting and quantifying mechanisms such as obesity [19], attention deficit hyperactivity disorder [20,21], schizophrenia [22] and addiction [23]. Therefore, to accurately describe the non-rational aspects of intertemporal choices, it is essential to delve deeper into the analysis of the mathematical characteristics of the discount function. The Delay Discounted (DD) task is the most used as a behavioural measure of the individual behavioural and neurobiological index [24,25]. From a decisional perspective, DD is composed of selecting an intertemporal prospect from two available ones: the first proposes a lower figure available immediately (smaller sooner, SS) and the second a higher figure available after a certain period (larger later, LL). Moreover, the correlation between the behaviour manifested in the DD task and certain conditions of addiction [26] , depression [27] and ADHD [28–30] is well known in the literature.

Ventre et al. [31] and Ventre and Martino [18] recently proposed new methods for empirically defining the intertemporal discount function. Specifically, the authors created a questionnaire to interpolate the individual discount function and obtain a more detailed estimate than that offered by the DD task; certainly, FDA empowers us to achieve a heightened level of detail, unencumbered by the constraints of a parametric model.

Increasing the discount function's descriptive capacity makes it more challenging to capture subtle individual differences. Therefore, the fundamental problem of interpreting intertemporal preferences for behavioural analysis is understanding how to differentiate discount functions that express more significant degrees of non-rationality.

In the literature, it is common practice to focus on parametric models; however, the latter suffer from some limitations due to their parametric functional form. For this reason, in certain circumstances, classical models cannot always explain anomalies in intertemporal choices [8,32]. Since the cognitive mechanisms underlying decision-making can be expressed and represented as curves over time, functional data analysis (FDA) could support investigating discount function behaviour. Exploiting FDA for studying intertemporal choices can provide in-depth knowledge about variability and heterogeneity in behavioural profiles.

The first advantage is purely theoretical: FDA makes it possible to investigate intrinsic properties of the discount functions, such as the first derivatives, second derivatives, curvatures, radii of curvature, and arcs length (see, e.g., [33,34]). These elements could contribute to defining a model that is increasingly closer to empirical data by reducing the gap between expected and observed behaviours. The second benefit is directed at the possibility of being able to glimpse interconnections between individuals' characteristics, such as age, gender, academic level, and discount functions' shapes. These relationships, in fact, cannot always be hypothesized a priori, especially when considering new social phenomena or contexts in which multifactorial methods has an essential impact in describing decision-maker behaviour. In this regard, FDA also makes it possible to investigate high dimensional data (e.g., due to the presence of many time observations) (see, e.g., [35,36]).

This paper is the first original research proposing the use of FDA to study discount functions. To show the potential of the FDA approach in the context of intertemporal choices, this paper presents a novel methodology for dealing with discount functions and a possible application to the so-called Hikikomori syndrome, i.e., a condition of isolation that has become particularly alarming in recent decades. In fact, although it has been initially presented as a Japanese phenomenon, it has recently been discussed and spread to other countries [37–40]). The conceptualization of Hikikomori has been proposed through a bio-psycho-socio-cultural model [41] aimed at including personality dysfunction, social disorders, psychotic, bullying, anxiety and paranoid, obsessive-compulsive attitudes. The expansion of this phenomenon suggests the need for more investigation into its causes and characteristics to devise therapeutic and rehabilitative strategies. Given the relationship between Hikikomori and psychiatric disorders [42], depression [43], and Internet addiction [44], interest in the discount function behaviour of Hikikomori subjects is more than justified. Moreover, some of these issues have been exacerbated after the COVID-19 pandemic, as Di Filippo et al. [45] highlighted. This is why the proposed study aims to investigate the relationships between intertemporal preferences and individual factors characterizing the condition of Hikikomori by exploiting FDA. While the primary emphasis of this research is on the functional analysis of variance to detect potential variations among groups of curves, the genuine uniqueness of the paper resides in its approach to handling discount functions via FDA within the framework of intertemporal choices.

The paper is developed as follows: Section 2 discusses the methods for the experimental investigation; Section 3 provides a brief overview of the condition of Hikikomori and application; finally, Section 4 presents the discussion and conclusions.

2. Material and methods

2.1. Non-rationality in intertemporal choice

An intertemporal prospect is an n-tuple $(x_1, t_1; \dots; x_n, t_n)$ composed by n alternatives x_i available at time t_i. The DU model states that $U(x_1, t_1; \dots; x_n, t_n) = \sum_i^n U(x_i) f(t_i)$, i.e., the intertemporal utility is determined by the sum of the cardinal utility of the various alternatives x_i multiplied by the discount function evaluated at the points where the decision-maker will receive the outcome. The variation of the discount function is decisive for the pattern of preferences and encapsulates the psychological mechanisms behind the choice [3]. In particular, the discount rate $\rho(t) = -f'(t)/f(t)$ quantifies how the individual perceives the indeterminacy of the future: the higher the discount factor, the faster the cardinal utility value of the good x_i decreases. Another fundamental concept in the study of intertemporal choices is impatience, defined as "the value at 0 of a $1 reward available at instant t" [15]. The DD task consists of giving two different alternatives: the first is a smaller amount receivable earlier (SS), and the second is a larger amount later (LL). The normative model does not define the best alternative as its selection depends only on the individual's impatience and perception of the uncertainty of the future. Therefore, someone who prefers SS over LL is more impatient and discounts time more. Temporal inconsistency indicates that preferences are not constant over time, creating a gap between empirical evidence and the normative model. Over the years, the exponential discount function proposed by Samuelson was contrasted with the hyperbolic discount function [8,46,47], which justifies preference reversal and has greater explanatory power.

The present paper investigates inconsistency by using two different approaches. The first one determines the degree to which impatience decreases through the hyperbolic factor [48]. This approach is based on the study conducted by Ventre et al. [31], in which the hyperbolic factor is used to quantify the direct impact of the emotional factor on intertemporal choices. Individuals are administered the questions shown in Table 1, in which the hyperbolic factors for five different scenarios are indicated, each related to a particular anomaly in the DU: delay effect [49], magnitude effect [50], interval effect, and sign effect [9,51,52].

Table 1
Questionnaire used to derive the hyperbolic factor of individuals.

Anomaly	Step	Question
Delay Effect	**Step #1**: Given s, t and y_{DELAY}	You must receive a sum of y_{DELAY} in t months, not before the set date but, alternatively, there is the possibility to immediately collect a certain result by reducing the total to be collected. To accept the offer, how much at least (x) do you want to receive today? *** In mathematical terms: To choose x such that $(x,s) \sim (y_{\text{DELAY}},t)$.
	Step #2: Given s, t, τ, x and y_{DELAY}	You must receive a sum of y_{DELAY} in $t+\tau$ months, not before the set date but you are allowed to anticipate the application and collect x instead of y_{DELAY}. To accept the offer, how long (σ) do you want to receive the x-digit? *** In mathematical terms: To choose σ such that $(x,s+\sigma) \sim (y_{\text{DELAY}},t+\tau)$.
Magnitude Effect	**Step #1**: Given s, t and $y_{\text{MAGNITUDE}} < y_{\text{DELAY}}$	You must receive a sum of $y_{\text{MAGNITUDE}}$ in t months, not before the set date but, alternatively, there is the possibility to immediately collect a certain result by reducing the total to be collected. To accept the offer, how much at least (x) do you want to receive today? *** In mathematical terms: To choose x such that $(x,s) \sim (y_{\text{MAGNITUDE}},t)$.
	Step #2: Given s, t, τ, x and $y_{\text{MAGNITUDE}}$	You must receive a sum of $y_{\text{MAGNITUDE}}$ in $t+\tau$ months, not before the set date but you are allowed to anticipate the application and collect x instead of $y_{\text{MAGNITUDE}}$. To accept the offer, how long (σ) do you want to receive the x-digit? *** In mathematical terms: To choose σ such that $(x,s+\sigma) \sim (y_{\text{MAGNITUDE}},t+\tau)$.
Sign Effect	**Step #1**: Given s, t and $y_{\text{SIGN}} = -y_{\text{DELAY}}$	You must pay a sum of y_{SIGN} in t months, not before the set date but you are given the option to pay a lower result as long as the payment is paid on the same day. What is a figure (x) that you think you could offer to accept your offer? *** In mathematical terms: To choose x such that $(x,s) \sim (y_{\text{SIGN}},t)$.
	Step #2: Given s, t, τ, x and y_{SIGN}	You must pay a sum of y_{SIGN} in $t+\tau$ months, not before the set date but you are given the opportunity to pay x instead of y_{SIGN}. To accept the offer, how soon (σ) would you commit to making payment? *** In mathematical terms: To choose σ such that $(x,s+\sigma) \sim (y_{\text{SIGN}},t+\tau)$.

On the other hand, the second approach to inconsistency is directly related to individual time distortion, as proven and defined in Ventre and Martino [18]. Unlike the DD task, in which the alternatives are established a priori, and the discount function is constructed by using approximation techniques, the proposed approach creates its discount function for each individual by interpolating the values over the following time intervals: $\boldsymbol{t} = [0,2,4,7,10,14,20,30,45,60,90]$ ([31]; [18]). Fixed the initial amount, for $\boldsymbol{x} = 100€$ the discount function for everyone is calculated as in Table 2.

In other words, once the discount function has been calculated, the relationship between the time of the exponential discount function and the individual's perceived time associated with the hyperbolic trend will also be studied. Therefore, it is assumed that the perceived time $(\widetilde{t}^{-1}(t))$ is associated with the empirical curve $f(t)$. In effect, let k be the value such that $\widehat{f}(t) = e^{-kt}$ is the best exponential approximation of the empirical curve [18]. Thus, by identifying $f(t)$ and $\widehat{f}(t)$, the time expressed by the exponential function can be calculated as $\widetilde{t}^{-1}(t) = -\frac{\ln f(t)}{k}$.

2.2. *Using functional data analysis for the study of non-rationality in intertemporal preferences: the monotone smoothing procedure*

Adapting the available information to a well-known discounting model poses the challenge that it typically fails to detect changes in the mode of inconsistency or the potential existence of the so-called magnitude effect. As a possible solution, we propose using FDA [33] to fit the available data to a wide range of discount functions that reflect different degrees of inconsistency and non-separability. According to individuals' responses, each vector of indifference pairs can be seen as a function in the time domain rather than a finite-dimensional vector. Indeed, this approach allows us to represent the sequences of individual discrete observed data as functions and analyze them as single entities.

One usual solution to reconstruct the functional form of the data is to assume that sample paths belong to a finite-dimension space spanned by a basis. Thus, the function $f(t)$ could be represented by a basis expansion:

$$f_i(t) = \sum_{k=1}^{K} a_{ik}\varphi_k(t) \quad i = 1, \dots, n \tag{1}$$

where:

- $f_i(t)$ is the reconstructed function for the i-th unit;
- $\varphi_k(t)$ are linearly independent and known functions, called basis functions;
- a_{ik} is the coefficient that links each basis function together in the representation of $f_i(t)$.

However, to be considered a discount function, a B-spline approxi-

Table 2
Questionnaire used for interpolation of the individual discount function.

Anomaly	Question	Function
Delay Effect	You have to receive $U(x(t_i))$ euros today. How much do you want to receive in t_{i+1} days ($U(x(t_i))$) to consider the offer equivalent?	$f(t) = \begin{cases} f(0) = 1 \\ f(t_{i+1}) = \dfrac{f(t_i) \bullet U(x(t_i))}{U(x(t_{i+1}))} \end{cases}$

mation must also satisfy the following conditions:

- $f(0) = 1$;
- $f(t) > 0$;
- $f(t)$ is strictly decreasing.

Therefore, we refer to the monotone smoothing approach proposed by Ramsay and Silverman [33]. When functions must satisfy one or more constraints, e.g., positivity and monotony, linear combinations of basis functions are difficult to constrain for this purpose. The authors suggested transforming the problem to one where the estimated curves are unconstrained [33]. Some applications require a fitting function $f(t)$ that is monotonically increasing or decreasing, even though the observations may not exhibit perfect monotonicity:

$$y_j = b_0 + b_1 f(t_j) + e_j. \tag{2}$$

To solve this problem, we can express *Dx* as the exponential of an unconstrained function *W* to obtain:

$$Df(t) = \exp[w(t)]dt. \tag{3}$$

By integrating both sides of this equation, we get:

$$f(t) = \int_{t_0}^{t} \exp[w(u)]du, \tag{4}$$

where t_0 is the fixed origin for the range of t-values for which the data are being fit. The intercept term b_0 is the value of the approximating function at t_0 [33] (in this context $f(0) = 1$). To allow for monotonically decreasing functions, the authors suggested to keep b_1 separate and select normalize $w(u)$ for numerical stability. Substituting, we get:

$$y_j = b_0 + b_1 \int_{t_0}^{t_j} \exp[w(u)]du + e_j \tag{5}$$

The function $w(u)$ is now the logarithm of the data-fitting function $f(u) = \exp[w(u)]$.

2.3. Using the functional analysis of variance (fANOVA) to detect differences in discount functions' means

In many application contexts, it is necessary to test whether groups of functions are statistically significantly different. In the FDA context, to assess the impact exerted on the functional observation by various factors operating at multiple levels, one can employ the functional analysis of variance (fANOVA). In particular, when examining the between-groups fANOVA, the objective is to ascertain if a statistically significant distinction exists among sets of independent curves.

Various statistical methods have been suggested for assessing the null hypothesis of equal functional means. For the remainder of this article, we will concentrate on the Ramsay and Silverman technique [33, 53]. We work under the assumption of a single factor involving V distinct groups ($v = 1, 2, \ldots, V$) and $N = \sum_{i=1}^{V} n_v$ observations, with n_v observations within group v. Thus, the model for the i-th functional observation ($i = 1, 2, \ldots, N$) in the v-th group can be expressed by:

$$y_{iv}(t) = \mu(t) + \gamma_v(t) + u_{iv}(t), \tag{6}$$

where $y_{iv}(t)$ is the functional response of the i-th curve in the v-th group, $\mu(t)$ is the grand mean function, $\gamma_v(t)$ is the functional effect of being in a specific group v, and $u_{iv}(t)$ is the residual function, i.e., the unexplained variation for the i-th observation within the v-th group.

The null hypothesis posits that the groups share identical functional means, whilst the alternative one suggests the existence of a difference among them as follows:

$$H_0 : \mu_1(t) = \mu_2(t) = \cdots = \mu_V(t)$$

$$H_1 : \mu_{v'}(t) \neq \mu_{v''}(t) \quad \text{for at least one couple } (v', v'') \text{ with } v' \neq v''$$

Given that *SSA(t)* represents the functional variance between groups and *SSE(t)* is the functional variance within groups, the pointwise functional F-statistics is given by:

$$F(t) = \frac{\frac{SSA(t)}{V-1}}{\frac{SSE(t)}{N-V}} \tag{7}$$

where *V-1* and *N–V* are the degrees of freedom of the two components.

Like classical ANOVA, a high *F(t)* value indicates that the variance accounted for by the model surpasses the unexplained variance. The primary difference between this approach and conventional univariate or multivariate ANOVA is that the *F(t)* value is not fixed but varies across the entire range. The classical significance level was originally designed for a single hypothesis rather than a continuous spectrum. Consequently, we must guard against making unwarranted claims within this range. To address this issue, a practical solution is to employ the permutation test, which is functionally equivalent to the univariate *F*-test statistic. This test enables us to identify any significant distinctions between the groups. The concept revolves around computing the Fisher test statistic as a function derived from a series of point estimates for each domain point. However, a unique test statistic is needed to formally examine the null hypothesis of no-relationship. By adopting the maximum value from the observed *F*-statistics function, we can derive the null hypothesis distribution by repeatedly estimating the test statistic while permuting the curves randomly. Specifically, the objective is to evaluate the *F*-statistic function and obtain the observed functional *F*, whose maximum is crucial for estimating the test's p-value. In the second step, the curves are randomly re-labeled with different curve numbers, keeping the grouping structure unchanged. For the set of re-labeled curves, we compute the *F*-statistics at each time domain point and determine the maximum of these functions. This re-labeling process is repeated numerous times, and for each iteration, we estimate the pointwise *F*-statistic function along with its maximum. In the third step, we identify the pointwise 0.05 critical value within the null distribution at each domain point and calculate the 95th percentile of the *F*-statistic values corresponding to that point. This final step furnishes us with the maximum 0.05 critical value from the null distribution, obtained by computing the 95th percentile of the distribution generated through the permutations in the second step. Taking the 95th percentile, we ascertain the critical threshold value without needing conventional statistical tables for the *F* distribution [33,54,55].

3. An application to a real dataset

3.1. The Hikikomori condition dataset

The term Hikikomori denotes a condition of isolation and was coined by psychiatrist Tamaki Saito [56] to describe individuals who choose to escape from social life physically. The phenomenon usually affects young people between 19 and 30 [57]. The main symptom of Hikikomori is withdrawal from society, which can present with varying degrees of severity, ranging from isolated behaviour to not leaving one's room entirely for months [58]. The school or job rejection consequently accompanies these extreme levels of confinement. In general, it is difficult to focus on the causes of this phenomenon, but cultural values and social pressures are crucial to the voluntary imprisonment of individuals [59]. Factors related to the social and school environment and family and character elements are possible determinants of this condition: a susceptible temperament, difficulty in establishing stable and rewarding relationships, fatigue in overcoming life's inevitable disappointments, excessive attachment to the mother and absence of the father can all be determining factors in the development of the phenomenon.

The original Hikikomori questionnaire was developed and validated in Japan and can identify the degree of voluntary social isolation. In

2018, Teo et al. [60] administered the test on the Japanese population by defining three groups: the Hikikomori, the non-Hikikomori, and the pre-Hikikomori. The results prove a significant difference between the Hikikomori and the remaining two groups. The questions consist of 25 responses ranging from "0: Strongly disagree" to "4: Strongly agree", and it investigates the intensity of the individual's Hikikomori condition in the past six months. The version used in the present paper is the one proposed by Amendola et al. [61], which is the first to validate the Italian version of the HQ-25 for a population of adults (aged between 18 and 50). The model is based on three factors: socialization, isolation, and emotional support, correlated positively with psychoticism, personality dysfunction and Internet use. The total score is the sum of the values attributed to each question, giving 100 points. The authors showed that with a sensitivity of 94% and a specificity of 61%, a cut-off score of 42 could discriminate between those at risk of Hikikomori and those who were not [61]. Therefore, the present article will be based on dividing the sample into two categories, whose division threshold is 42.

3.2. The questionnaire and data collection

The test was administered by using the *vercel.app* platform, from which the data was downloaded and converted into an Excel file. The users were anonymous; the only information was gender and age (18 years and older). The questionnaire consists of two parts preceded by an introduction section explaining how to answer. The first part addresses the calculation of the individual discount function, and the second part aims to define the Hikikomori score. The link for the questionnaire was shared on digital platforms and social channels and addressed to all Italian individuals willing to voluntarily complete the questionnaire (clearly, the results can be distorted by self-selection bias, but in any case, the objective of this research is to introduce a new methodology and not to delve into medical problems). After opening the link, the individual is asked to accept the informed consent and participate in the study.

The questionnaire in the first section consists of two alternating questions: the first is used to collect the values needed to construct the empirical discount function using the interpolation method and according to the indications given in Table 2; and the second question is used to elicit the sense of confusion [18]. Therefore, the individual must answer the following questions:

- "*You have to receive* $U(x(t_i))$ *euros in* t_i *days*; how *much do you want to receive in* t_{i+1} *days to consider the equivalent offer?"*
- "*You have to receive* €100 *today*; how *much do you want to receive in* t_i *days to consider the equivalent offer*?"

It is important to note that the second question, used to distract, is the same as the previous one but keeps the value of the outcome constant over time. This experimental structure makes it more difficult for respondents to keep track of the replies given in previous questions. As shown in Table 2, the initial result was set at $U(x(0)) = 100$, as it appears to be a moderate value that allows people of all ages to realistically realize the hypothesis and the times $t = [0, 2, 4, 7, 10, 14, 20, 30, 45, 60, 90]$ are used to define the indifference pairs. In the instructions of the first part, it is also explained to the respondents that, for each question, the maximum time to answer is 20 s. The timer was constantly visible in a box below the question so that each respondent constantly perceived the passage of time. This experimental dynamic was designed to induce a sense of hurry and excitement in each respondent. At the end of the timer, if the individual had not yet entered the answer to the question, the following sentence would appear to prompt them to enter the digit:

- "*Time's up! Time to answer the question has expired. Please answer the question IMMEDIATELY, without further thought*".

Before the start of the test, individuals were informed through the instructions that if the time limit were exceeded more than ten times, the test would be invalidated as unreliable. Fortunately, all participants complied with the limit of 10 countdown timers.

On the other hand, the second section of the questionnaire consists of 25 questions aimed at defining the Hikikomori score. All 25 questions are shown in Table A2 of the Appendix, linked to an initial one asking the following: "*Over the past six months, how accurately do the following statements describe you*?". The instructions in the second section explain that individuals must indicate a score from 0 to 4 for each question, where 0 stands for *strongly disagree,* and 4 stands for *strongly agree*. Thus, "one" shows ***a little bit of disagreement***, "two" ***neither agree nor disagree***, and "three" ***a little bit of agreement***. To prevent individuals from confusing the reading of the scores, this indication was given in small print in all 25 questions. The sum of three sub-scores provides the Hikikomori with score: the socialization score, defined by the sum of the answers to questions 1, 4, 6, 8, 11, 13, 15, 18, 20, 23, and 25; the isolation score, defined by the sum of the answers to questions 2, 5, 9, 12, 16, 19, 22, and 24; the emotional support score, represented by the sum of the answers to questions 7, 10, 14, 17, and 21. Concerning the summation operation, questions 4, 7, 10, 15, 21, 22, and 23 are characterized by reverse scoring, i.e. they are read by inverting the scale: if "0" is indicated then "4" is added; if "1" is shown then "3" is added; if "2" is indicated then "1" is added; and finally, if "4" is indicated then "0" is added.

The sample consists of 271 individuals, of whom 74 are men (27.31%), 195 are women (71.96%), 2 are "other" (0.74%), the mean age is 21.56, and the mean Hikikomori score is 35.09. In total, given the score threshold of 42 as indicated in Amendola et *al.* (2022), there are 169 individuals classified as non-Hikikomori/non-at risk of Hikikomori (62.36%, NH) compared to those who scored greater than or equal to 42, which are 102 individuals (37.64%), classified as Hikikomori/at risk of Hikikomori (H).

3.3. Descriptive analysis

Fig. 1 highlights that the median of the distributions assessed at each time instant decreases over time, with a hyperbolic trend confirming data collection's efficiency (this is what we expect from this type of data). Fig. 1 highlights how, as time progresses, the asymmetry of the distributions increases, concentrating on smaller values of the discount function (this is also what we suppose when dealing with intertemporal choices).

Fig. 2 compares the distributions between the Hikikomory (H) and Non-Hikikomory (NH) categories. Specifically, it is possible to capture how the median of H differs from that of NH starting from D7. The different lengths of the boxes also indicate that H subjects apply a gentler discount rate than the NH subjects. The lower presence of outliers also confirms this attitude.

The first analysis is conducted by reconstructing the discount function of both categories, interpolating the median values of the $f(t_i)$ for each category. Fig. 3 shows the discount functions of the values shown in Table 3.

The possibility of deriving the value of the discount function over several time instants shows a slight difference between the performance of Hikikomori and non-Hikikomori. Even plotting inconsistency, as in Ventre and Martino [18], the difference between the behaviour of the discount function of H and NH is not clear-cut but only shows that the most significant degree of inconsistency is determined by the NH class, as shown in Fig. 4. However, when plotting the difference between perceived time and objective time, the H class achieves the most remarkable inconsistency, as shown in Fig. 5.

However, when plotting the difference between perceived and objective time, the H class achieves the most remarkable inconsistency, as shown in Fig. 5. In particular, for H class $\widetilde{t}^{-1}(t) = -\frac{\ln f(t)}{0.042}$ and for NH $\widetilde{t}^{-1}(t) = -\frac{\ln f(t)}{0.045}$.

Turning to the analysis of the hyperbolic factors obtained with the

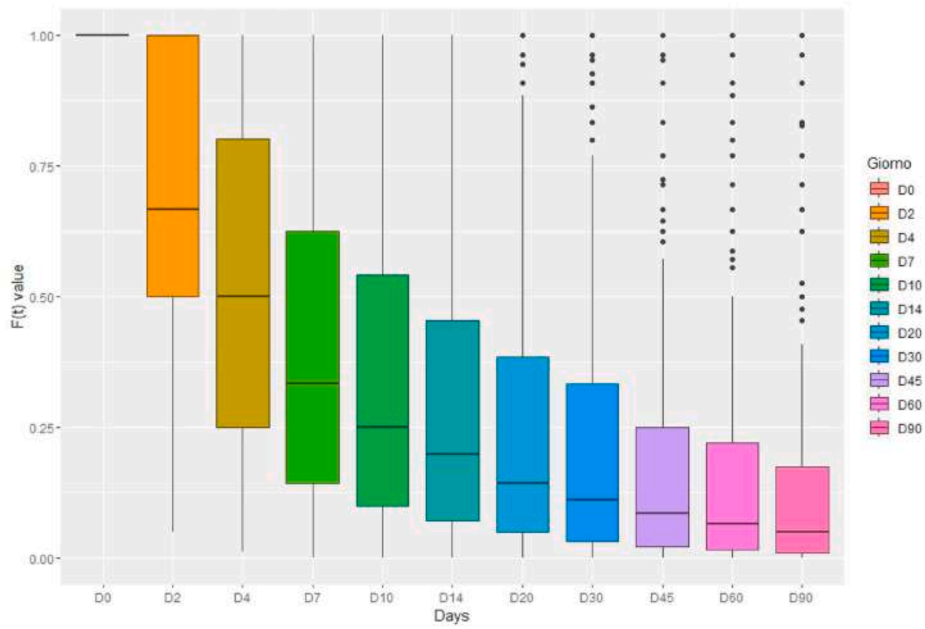

Fig. 1. Boxplot of *f*(*t*) values on different days.

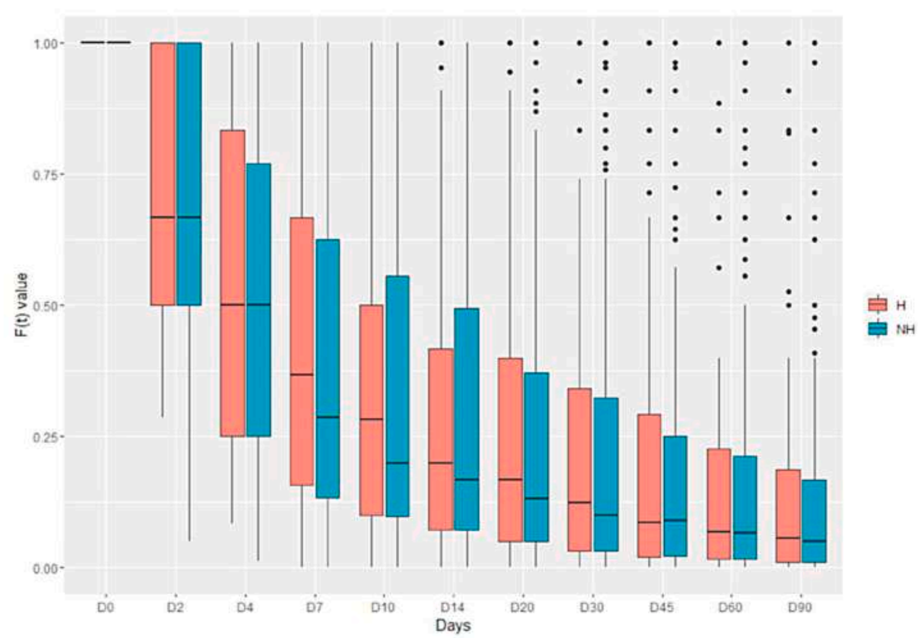

Fig. 2. Boxplot of *f*(*t*) values at different days for Hikikomori and non-Hikikomori classes.

questionnaire in Table 1, it is possible to observe from both Table 4 and the box-plots shown in Figs. 6 and 7 that it is still not possible to determine a significant distinction between the two classes, except for H (0,6,50,12) whereby the median of the NH is more than twice that of the H.

Concerning intertemporal choice, it is only possible to state that the H individuals are less sensitive to the magnitude effect, an anomaly linked to associating lower outcomes with less distant consumption [31].

3.4. Results of the functional data analysis approach

Section 3.3 remarks that the survey conducted without FDA does not allow for a comprehensive understanding and interpretation. To gain a deeper insight into the phenomenon, the data is interpreted using FDA. The discount functions were analyzed by dividing the individual score into several severity classes suggested in the literature, varying the number of classes from 1 to 5, as shown in Figs. 8–12.

Fig. 8 presents the discount functions for each individual treated as functional data. The latter are obtained using the monotone smoothing procedure described in Section 2.2. Also, Fig. 8 shows the presence of high functional variability within both Hikikomori (blue functional mean) and non-Hikikomory subjects (red functional mean). The functional averages seem very similar, but a statistical test is needed to understand if there are significant differences (i.e., fANOVA).

Fig. 9 proposes the insertion of a new subclass [42,60], whereas

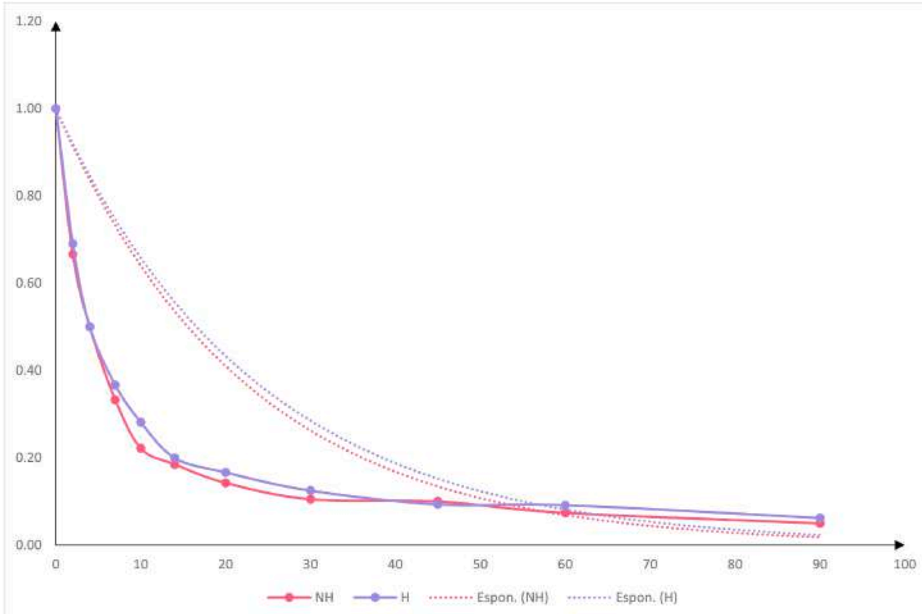

Fig. 3. Discount function obtained with interpolating median values of *f*(*t*) at different days for H and NH class. The dotted line represents the exponential approximation of the empirical curves.

Table 3
Median values of *f*(*t*) at different days for H and NH class.

T	NH	H
0	1.00	1.00
2	0.67	0.69
4	0.50	0.50
7	0.33	0.37
10	0.22	0.28
14	0.19	0.20
20	0.14	0.17
30	0.11	0.13
45	0.10	0.09
60	0.07	0.09
90	0.05	0.06

Fig. 10 illustrates a four-class subdivision. The goal of the analysis with three or four classes, depending on the severity of the Hikikomori condition (instead of the use of a binary outcome), is dictated by the fact that we can inspect for differences between different intermediate states. Indeed, Fig. 8 displays a discount mean function with a more hyperbolic trend observed for the green class. Finally, inserting a final class [0, 20] in Fig. 11 investigates how the functional means' trends change, concentrating on the group with a lower Hikikomory score.

Including the last class confirms that *High F. Mean* and *Very High F. Mean* have the smallest and most significant decreases in the discount function, respectively, and all other categories have a decrease between these. It is interesting to observe the development of the first and second derivatives of the discount function shown in Figs. 12 and 13, respectively, for the five classes identified above. Concerning the first derivative, it is possible to distinguish the class *Very High F. Mean,* which, compared to the others, has a much steeper growth that occurs mainly between D0 and D2. Still showing the opposite trend of classes *High F. Mean* and *Very High F. Mean*: the former reaches smaller modulus values of the first derivative before all the others; the latter, on the other hand, reaches the smallest modulus values when the variations of the others are already low.

With the second derivatives, on the other hand, it is interesting to

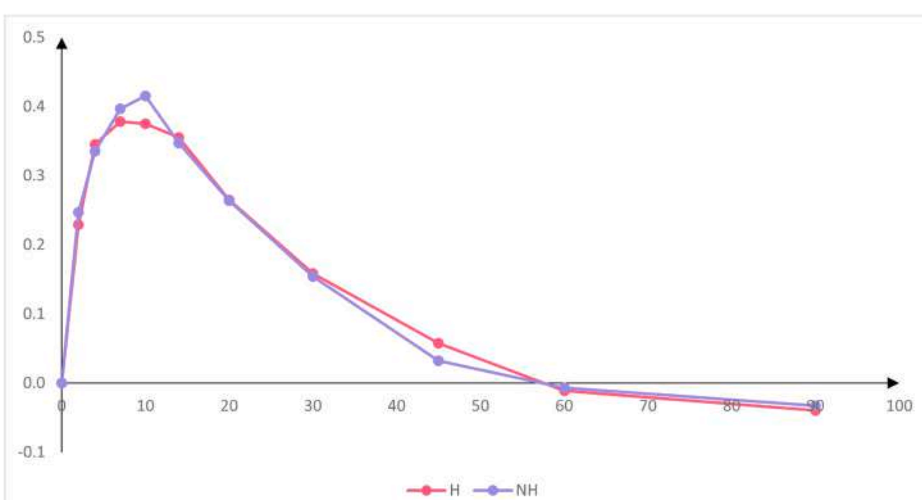

Fig. 4. Difference between the empirical discount function and the exponential discount function for each category, H and NH.

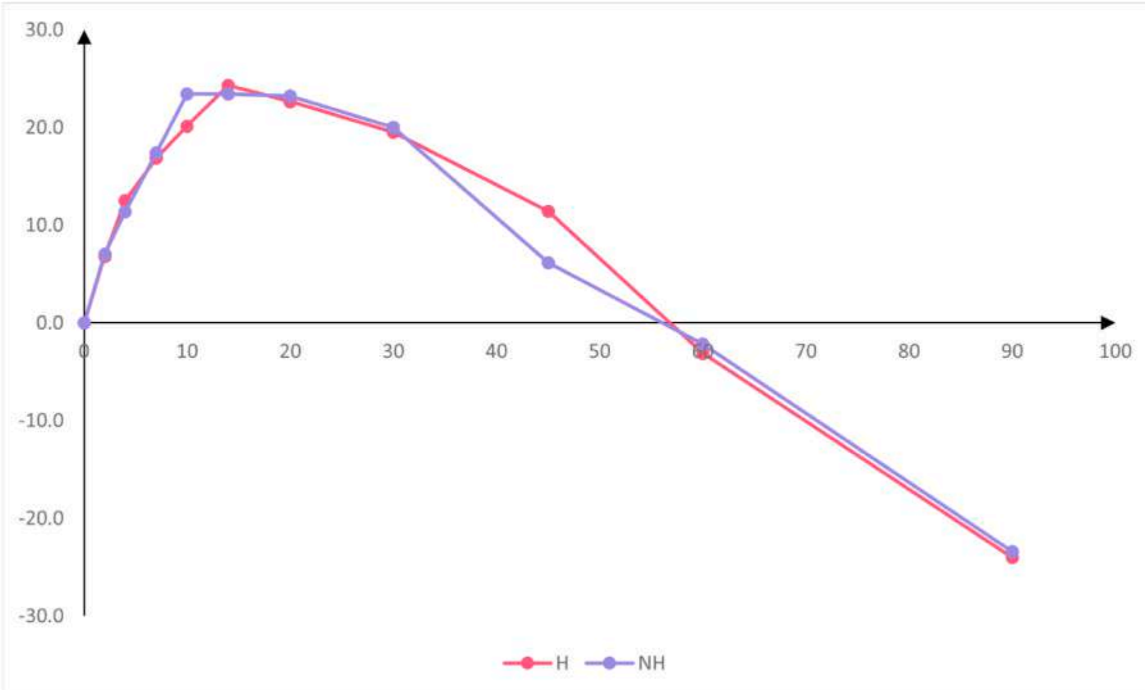

Fig. 5. Difference between the objective and perceived time for each category.

Table 4
The median of the hyperbolic factors for each category.

	H (0,6,500,12)	H (0,6,50,12)	H (0,1,50,1)	H(0,6,-500,12)	H(0,6,-500,12) gain
NH	0.7	3.8	4.1	1.8	0.8
H	0.5	1.8	3.1	1.8	0.8

observe how all classes reach the point of maximum in a neighbour of the second time instant, the value of which increases as the classes' scores increase. Finally, the trend of the discount rate in Fig. 14 shows that when the *High F. Mean* class presents low values, the *Very High F. Mean* class presents high values and vice versa. In addition, the *Very High Mean* class stands out as the only one with an initial peak, diversifying significantly from other classes, and growth from D30 onwards.

At this point, we aim to understand whether the functional differences observed by dividing the groups of functions based on the Hikikomori score classes (the factor) have a statistically significant effect on the functional form of the discount functions. From a statistical point of view, performing a fANOVA test on the group means will be necessary. Below are the results of three different fANOVA analyses.

The first analysis considers the case where we consider three groups based on Hikikomori scores (see Fig. 9). Consequently, the test is based on the following assumptions:

$$H_0 : \mu_1(t) = \mu_2(t) = \mu_3(t)$$

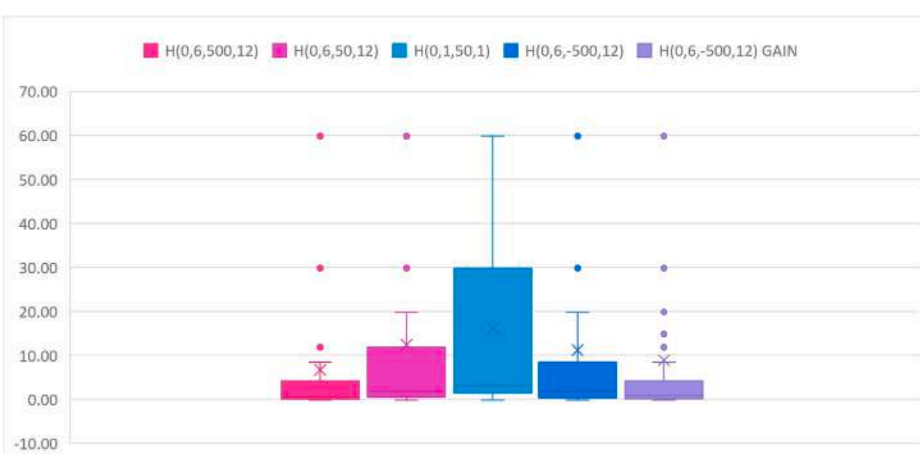

Fig. 6. Box plot of hyperbolic factors by H category.

H_1 : *At least one functional mean of the discount function groups is different from the others*

The second analysis considers the case in which we consider four groups (see Fig. 10). Consequently, the test is based on the hypotheses:

$$H_0 : \mu_1(t) = \mu_2(t) = \cdots = \mu_V(t)$$

H_1 : *At least one functional mean of the discount function groups is different from the others*

The last analysis contemplates the case in which we consider five groups (see Fig. 11). The test is based on the following hypotheses:

$$H_0 : \mu_1(t) = \mu_2(t) = \cdots = \mu_V(t)$$

H_1 : *At least one functional mean of the discount function groups is different from the others*

The results of the three distinct applications are shown in Figs. 15–17, respectively. All analyses highlight that the observed functional *F* statistic lies below the pointwise critical level and the maximum. In all three cases, we have enough evidence in favour of the null hypothesis. Therefore, we can state that there is no statistically significant difference between the functional means of the discount functions of different groups based on the severity of the Hikikomori condition. In other words, it seems that the Hikikomori condition does not really affect intertemporal choices, as one might hypothesize from a theoretical point of view.

4. Discussion and conclusions

The present study addressed the problem of temporal inconsistency in intertemporal choices through FDA. In particular, the latter was used to interpret the data more comprehensively for two primary purposes: understanding relations not known a priori and distinguishing the behaviour of groups of individuals with similar intertemporal choices. The motivation behind integrating FDA with the study of the discount function stems from the fact that more detailed methods for obtaining the individual discount function have been defined. In particular, the fundamental difference between the intertemporal choice task proposed in the present work and other intertemporal choice tasks lies in the fact that, whilst in the former, the indifference pairs are determined on an individual-by-individual basis, in the latter, the individual must express preferences on pairs constructed a priori by the experimental design. Therefore, the data are potentially much more detailed than the behavioural and cognitive aspects of the individual.

The application of the study turned towards the phenomenon of Hikikomori, a complex mechanism that has attracted the interest of many research areas. Specifically, to the best of our knowledge, this is the first study analyzing the intertemporal choice behaviour of individuals according to the degree of Hikikomori status. However, our analyses were limited to describing the procedure and usefulness of data analysis in a context that is still new and difficult to study.

Concerning the data collected, it was possible to see in Figs. 3 and 8 and Table 4 that the division into two categories does not allow any particular difference to be observed between the two classes defined as NH, i.e., individuals not Hikikomori/not at risk of Hikikomori, and H, i.e., individuals Hikikomori/at risk of Hikikomori for which the threshold score is 42. Functional analysis made it possible to investigate the characteristics of the discount function with smaller and smaller range groups.

FDA yielded an unexpected result for the characteristics of the discount function. Indeed, although from Fig. 11, there appeared to be an essential difference between the five classes of individuals, the contribution of the fANOVA, presented in Figs. 15–17, indicates that this difference is not statistically significant. In this regard, it is essential to emphasize that the novelty and importance of the present work do not lie in determining whether or not there is a difference concerning an individual behavioural condition, the discussion of which is also beyond us, but in demonstrating how FDA and decision theory can contribute, in combination, to investigations of individual behaviour by determining statistically significant results. However, although there is no significant difference between groups, the results show curious differences between individuals scoring between 61 and 70 and above 70. In particular, the transition from the discount function to FDA, and then back to the study of the discount function and degree of impatience, reported in the APPENDIX, shows that the highest and lowest degrees of temporal inconsistency are displayed by individuals with scores above 70 and between 61 and 70, respectively. The trend of the first derivative also confirmed this result, while the second derivative and the degree of temporal misperception have a behaviour that varies consistently with the degree of Hikikomori.

The latter considerations motivate interest in possible future studies, as even if there is no difference between groups in any of the three cases with 3, 4 and 5 classes, a development of the present research could aim to understand whether there are significant differences at the level of derivatives or discount rates. Indeed, a lack of significant differences between discount functions does not mean that intertemporal choice

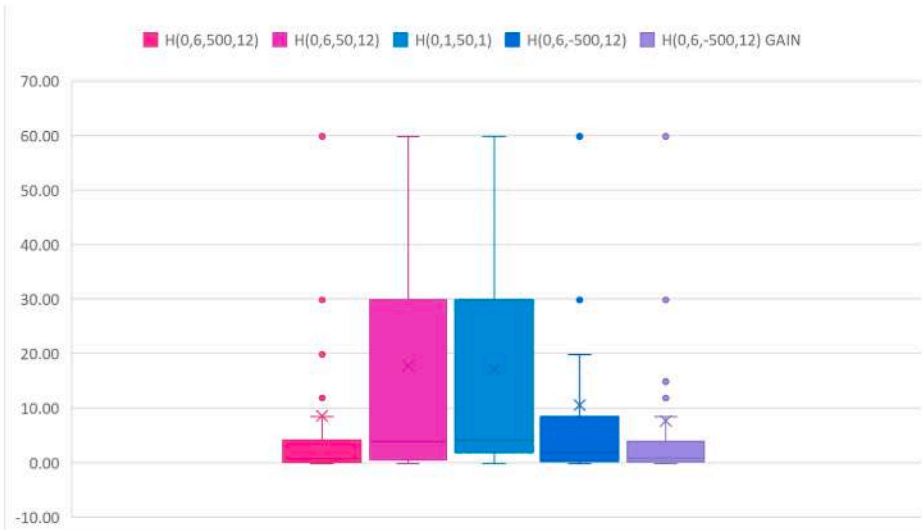

Fig. 7. Box plot of hyperbolic factors by NH category.

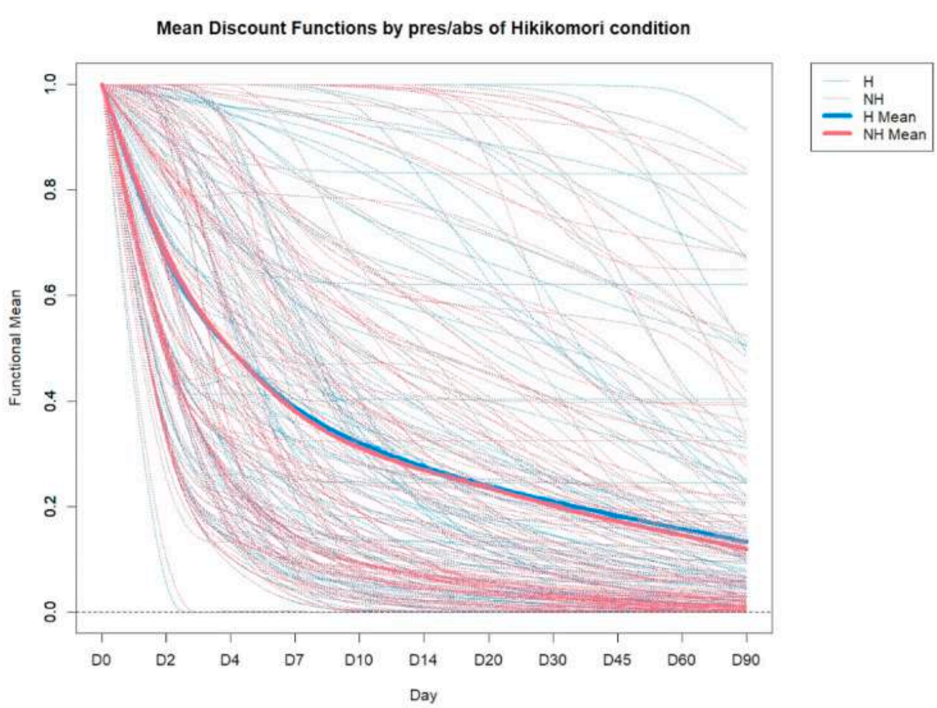

Fig. 8. Discount function with two classes based on the Hikikomori scores.

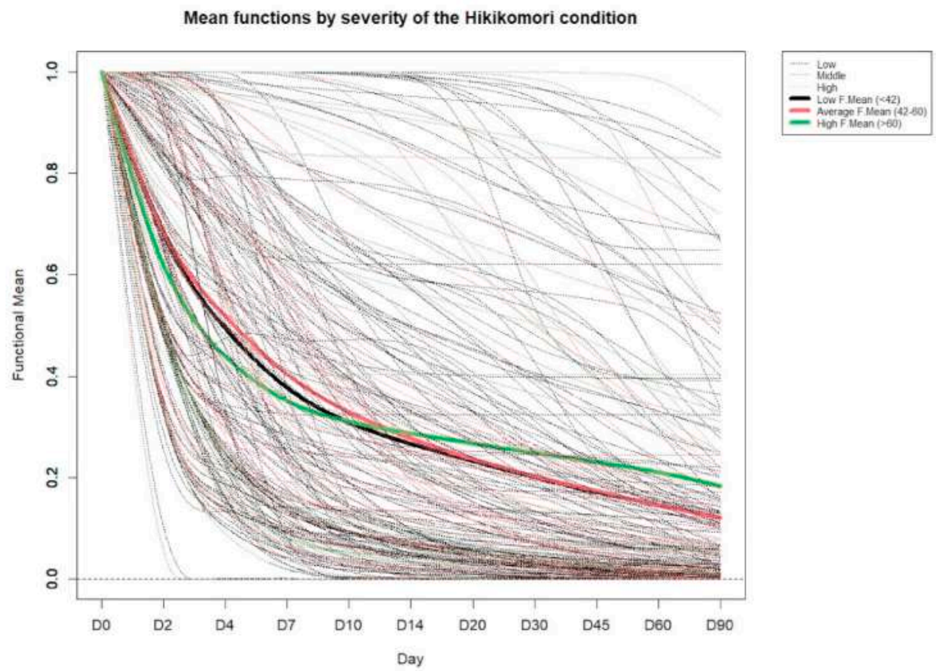

Fig. 9. Discount function with three classes based on the Hikikomori scores.

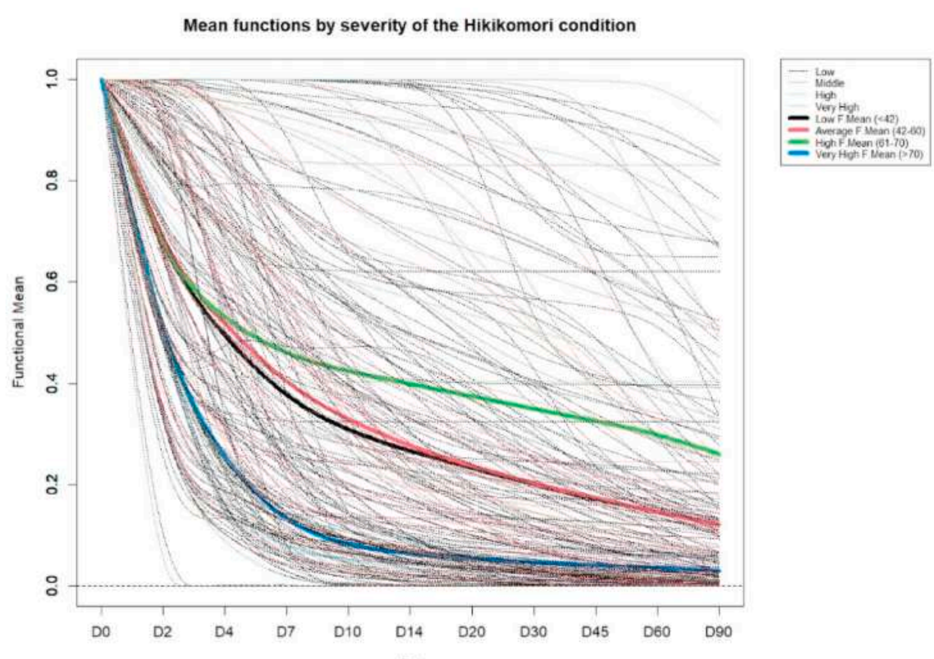

Fig. 10. Discount function with four classes based on the Hikikomori scores.

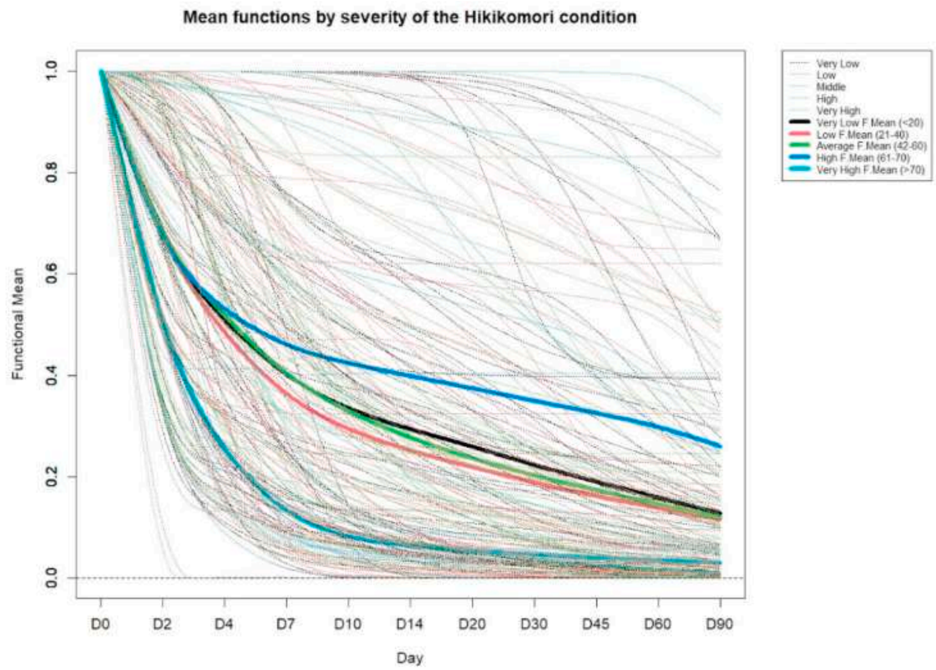

Fig. 11. Discount functions with five classes based on the Hikikomori scores.

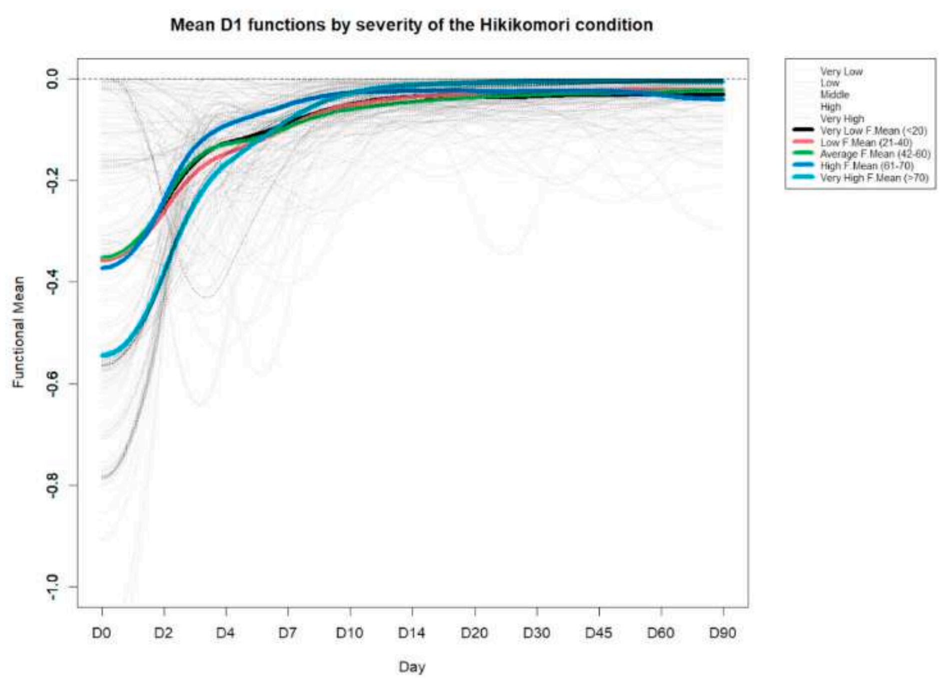

Fig. 12. The first derivatives of the discount functions with five classes based on the Hikikomori scores.

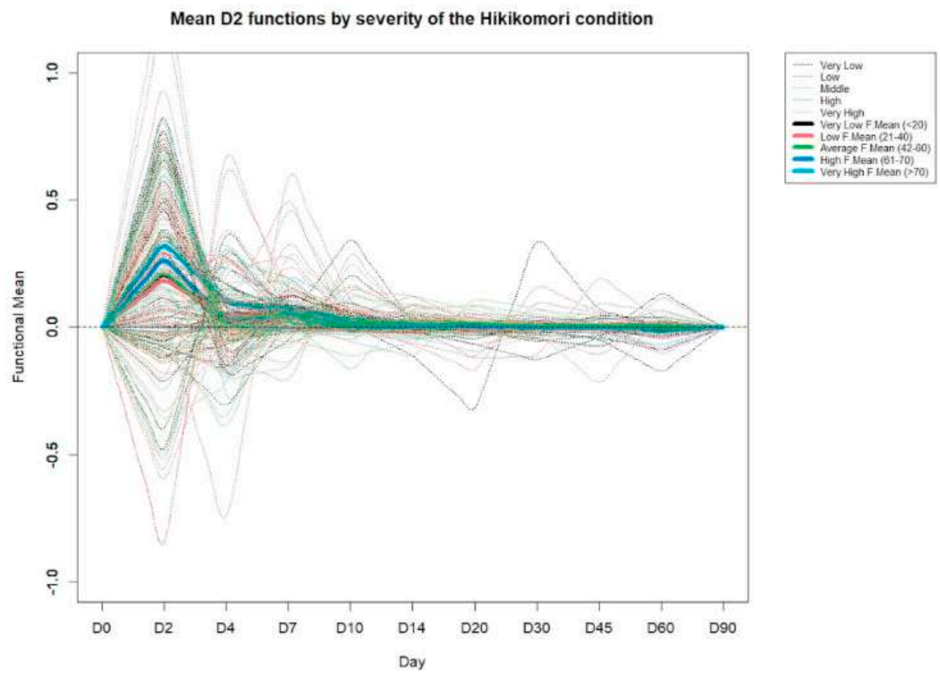

Fig. 13. The second derivative of the discount functions with five classes based on the Hikikomori scores.

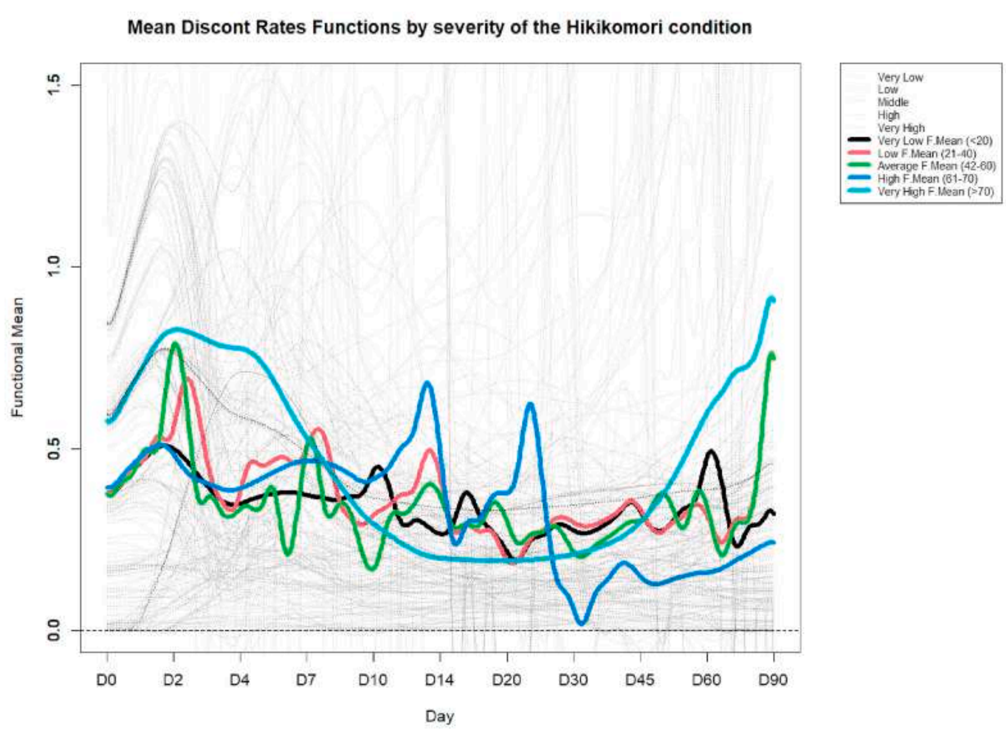

Fig. 14. The discount rates with five classes based on the Hikikomori scores.

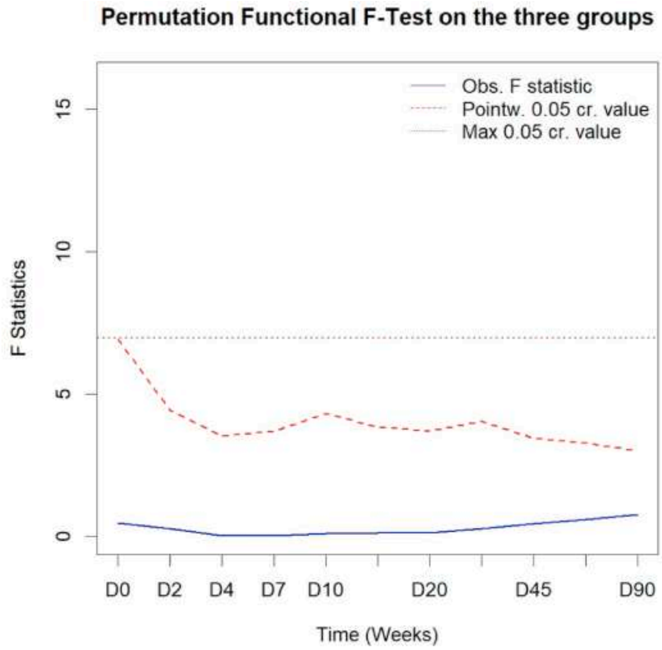

Fig. 15. Permutation F-test using three groups based on the Hikikomori scores.

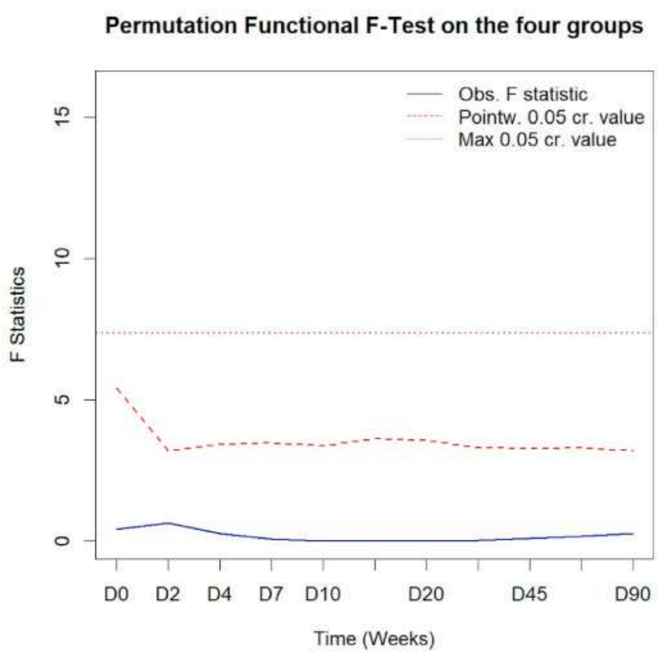

Fig. 16. Permutation F-test using four groups based on the Hikikomori scores.

behaviour cannot differ significantly on other dimensions.

Future studies may also clarify other exciting observations. For example, in Fig. 9 (three categories of Hikikomori syndrome), there is a change of convexity in the graph of the *High F. Mean* (in green). Specifically, the curve goes from convex to concave behaviour, as confirmed by Fig. 10 (four categories) and 11 (five categories). Unfortunately, only f concave (second derivative negative) implies $\ln f$ concave. Therefore, it

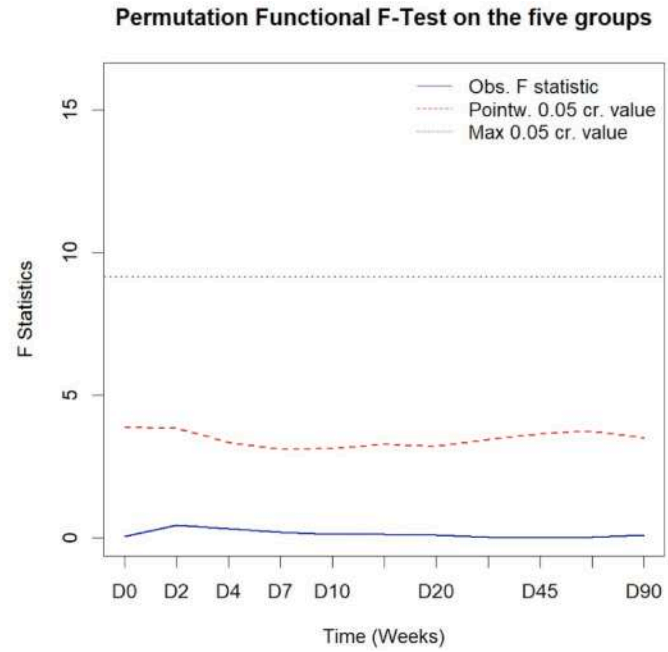

Fig. 17. Permutation F-test using five groups based on the Hikikomori scores.

could be interesting to apply the FDA methodology to $\ln f$ instead of f to analyze the increase or decrease of the instantaneous discount rate. This analysis could be complemented by Fig. 13, where it is unclear if the second derivative cut the t-axis: plotting the second derivative of $\ln f$ instead of f could determine the extraction of new features and relations. Finally, Fig. 14 exhibits changes in the increase/decrease of the discount rate. These changes are less frequent when the Hikikomori score is high or very high (curves in light and dark blue). From a decision-making point of view, one could investigate the phenomenon whereby, in these two cases, the discount rates are much higher than the corresponding to the rest of the cases (average, *low* and *very low F. Mean*). This would demonstrate that people exhibiting high or very high Hikikomori scores are more impatient but possibly less inconsistent as their convexity is more stable (i.e., exhibit less number of changes). From this point of view, therefore, impatience and impulsiveness might no longer be considered synonymous in the context of intertemporal choices.

As we conclude, it is essential to acknowledge a potential limitation in this research: the presence of self-selection or voluntary bias inherent in the data collection process. The participants in the study were volunteers who actively chose to take part in it, which may have introduced a bias into the sample. Therefore, caution should be exercised from an inferential standpoint when interpreting the application results.

CRediT authorship contribution statement

Viviana Ventre: Data curation, Formal analysis, Methodology, Supervision, Writing – original draft, Writing – review & editing. **Roberta Martino:** Conceptualization, Data curation, Investigation, Methodology, Writing – original draft, Writing – review & editing, Visualization. **Salvador Cruz Rambaud:** Formal analysis, Methodology, Supervision, Writing – original draft, Writing – review & editing. **Fabrizio Maturo:** Methodology, Software, Supervision, Visualization, Writing – original draft, Writing – review & editing. **Annamaria Porreca:** Methodology, Software, Visualization, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data that has been used is confidential.

APPENDIX

Although no significant differences were found between the five classes concerning the performance of the discount function, individual behaviours are characterized by interesting different shapes (also on other dimensions). In fact, in addition to the observations discussed in Section 4 concerning the development of the discount rate and the derivatives of the discount function, the measures of time inconsistency and decreasing impatience concerning the five classes also motivate the need to investigate further via FDA the possible relationships. In particular, through the analysis of the inconsistency of the discount functions, shown in Table A1, Figure A1 and Figure A2, it is now possible to see that the fifth class has both the highest degree of inconsistency and dispersion; the first, second, and third classes have a similar trend indicating a similar attitude in the intertemporal choice task; finally, the fourth class has the lowest degree of inconsistency but not the lowest degree of temporal dispersion. The degree of inconsistency does not increase as the severity of the hikikomori score rises. The highest degree is determined by the fourth class, which has a value of 0.52, compared to the third class, which has the lowest degree, which has a value of 0.37. Furthermore, the third class has the lowest median values, indicating a lower propensity for the degree of non-zero impatience to decrease, except for $H(0,6,500,12)$. This result is consistent with the degree of inconsistency, which is remembered to be determined by the maximum distance between the empirical function obtained by interpolation and its best exponential approximation. Since these characteristics are related to magnitudes different from the discount function alone, future studies may be interested in understanding whether or not these differences are also significant in the dynamics of intertemporal choice.

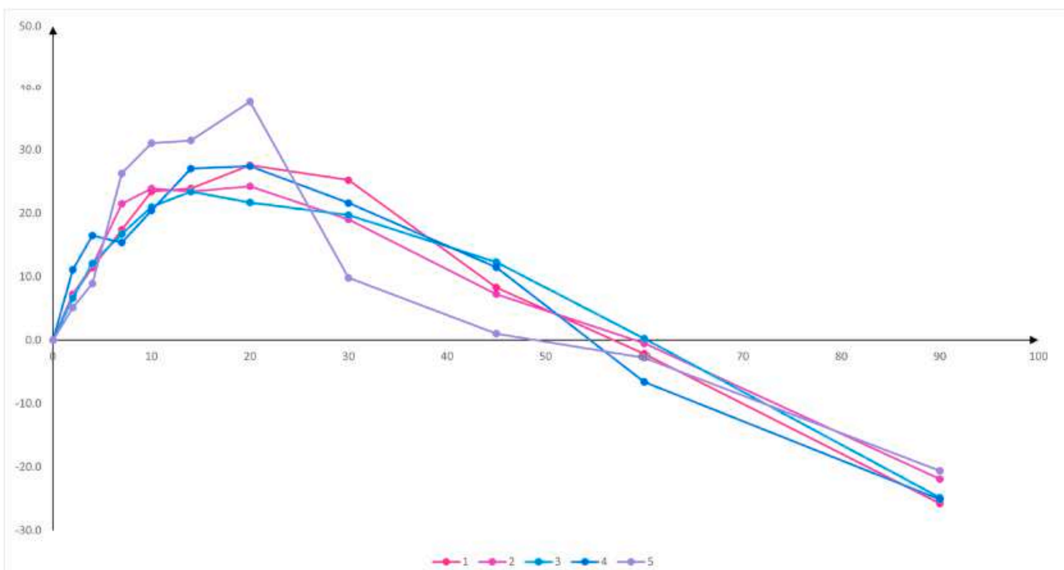

Fig. A1. Difference between the empirical discount function and the exponential discount function for each class.

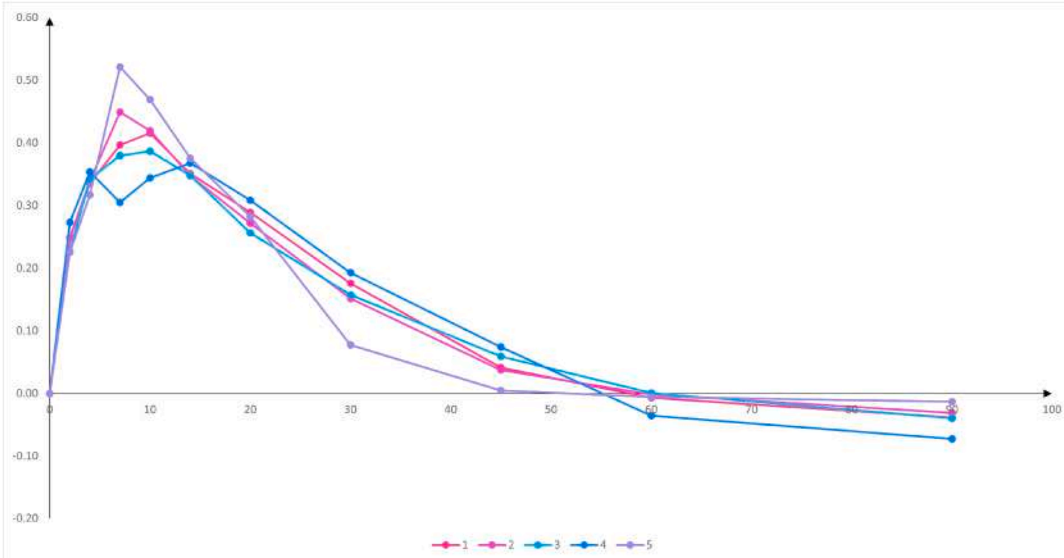

Fig. A2. Difference between the objective and perceived time for each class.

Table A1
Characteristics of the four classes concerning attitude in intertemporal choices

	1	2	3	4	5
Max Degree of inconsistency	0.42	0.42	0.39	0.37	0.52
Max Degree of time misperception	27.6	23.4	23.4	27.4	37.6
Median H(0,6,500,12)	1.41	0.61	0.50	1.00	0.24
Median H(0,6,50,12)	1.83	3.26	1.83	1.83	4.12
Median H(0,1,50,1)	4.12	4.12	4.12	1.83	2.98
Median H(0,6,-500,12)	1.45	1.83	1.83	1.05	0.86
Median H(0,6,-500,12) gains	0.83	0.83	0.83	0.67	1.94

Table A2
The Italian version of the 25-item Hikikomori questionnaire [61]. "R" means the score has been reversed for data analysis purposes.

Over the past six months, how accurately do the following statements describe to you?

		Strongly in disagree	A little bit in disagree	Nor agree nor disagree	A little bit in agree	Strongly in agree
1.	*I stay away from other people.*	0	1	2	3	4
2.	*I spend most of my time at home.*	0	1	2	3	4
3.	*There's really no one with whom I can discuss important things.*	0	1	2	3	4
4. R	*I like meeting new people.*	0	1	2	3	4
5.	*I locked myself in my room.*	0	1	2	3	4
6.	*People annoy me.*	0	1	2	3	4
7. R	*There are people in my life who try to understand me.*	0	1	2	3	4
8.	*I don't feel comfortable with people around.*	0	1	2	3	4
9.	*I spend most of my time alone.*	0	1	2	3	4
10. R	*I can share my thoughts with different people.*	0	1	2	3	4
11.	*I don't like to be seen by others.*	0	1	2	3	4
12.	*I rarely meet people in person (face-to-face).*	0	1	2	3	4
13.	*It's hard for me to join (or participate) to groups.*	0	1	2	3	4
14.	*There are few people with whom I can discuss important topics.*	0	1	2	3	4
15. R	*I like to be in social situations.*	0	1	2	3	4
16.	*I do not live by the rules and values of society.*	0	1	2	3	4
17.	*In my life, there is really anyone very important.*	0	1	2	3	4
18.	*I avoid talking to other people.*	0	1	2	3	4
19.	*I have little contact with others, such as talking, writing and similar things.*	0	1	2	3	4
20.	*I much prefer to be alone than with others.*	0	1	2	3	4
21. R	*I have someone I can confide my problems.*	0	1	2	3	4
22. R	*I rarely spend time alone.*	0	1	2	3	4
23.	*I don't like social situations.*	0	1	2	3	4
24.	*I spend very little time interacting with other people.*	0	1	2	3	4
25. R	*I strongly prefer to be with other people.*	0	1	2	3	4

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