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Modal Density Influence on Modal Complexity Quantification in Dynamic Systems

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Abstract

From an engineering point of view, it is important to recognize the proportional or non-proportional nature of the damping in dynamic systems. In the first case, the mode shapes are real, whereas in the second case they are complex. Typically, the entity of non-proportional damping is estimated through appropriate indices that measure the relative weight of the imaginary part of the mode shapes. A serious problem for the practical applicability of these indices is their dependence on the accuracy of the mode shapes identification. Above all, the modal density affects the identification quality by injecting fictitious complexity in the mode shapes regardless the identification technique used. It is the purpose of the paper to contribute to evaluate the effectiveness of the non-proportionality indices in the presence of modal density. The work is based on the comparison between theoretical (exact) solutions and experimental solutions. The results show that the error is almost constant up to a certain value of the modal density beyond which it markedly diverges. The results show also that the indices share the same trend even if differently scaled. The selection of the best index is also addressed.

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Keywords: Non-proportional damping; modal density; complex mode shapes

1. Introduction

The viscous damping in dynamic systems can be proportional or non-proportional, depending whether the damping matrix is a combination of the mass and stiffness matrices or not [1]. In the first case, the mode shapes are

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real and the degrees of freedom (dofs) of the system oscillate in phase whereas, in the second case, the mode shapes are complex and the dofs oscillate out of phase [2].

Therefore, the imaginary content of the mode shapes can be used for the quantification of the non-proportional damping. Several indices exist on purpose to provide estimates of the modal complexity and then of the damping non-proportionality [3]. These indices are zero for real mode shapes and increase along with the imaginary content of the mode shapes.

Sometimes the dynamic systems have close frequencies and this circumstance affects the experimental identification of the mode shapes regardless the accuracy of the method used for the identification. As consequence, depending on the modal density, a fictitious over-content of the imaginary part of the mode shapes is detected [4], and analytically showed [5].

The paper is aimed at providing a contribution to evaluate the effectiveness of the non-proportionality indices in the presence of significant modal densities. In order to assess the errors due to modal density, the indices obtained through the experimental identification are compared against the theoretical solution, obtained via standard modal analysis capable to provide the "exact" real or complex mode shapes regardless the modal density. The indices are first classified and selected according to their actual experimental applicability; then their trend against an appropriate scaling factor devised to control the modal density is analysed.

2. Mode shapes identification

2.1. Theoretical solution

The theoretical mode shapes of a discrete linear N-dofs system are identified using the standard modal analysis based on the well known state space formulation capable to deal with general viscous damping [6] and will not followed further hereafter.

2.2. Experimental solution

Several techniques exist for the experimental identification of the mode shapes [7]. Among these, one commonly employed makes use of the Frequency Response Function (FRF) of the system [8]. The FRF of a viscously damped discrete linear N-dofs system, at the resonance frequency ω_k, is given by the following two expressions valid respectively for proportional (P) and non-proportional (NP) damping where the response H_{mn} is measured at dof m for a harmonic excitation applied at dof n:

$$H_{mn}^{P} = \frac{\Psi_{mk}\Psi_{nk}}{\omega_k \nu_k + i\left(\omega_k - \omega_k\sqrt{1-\nu_k^2}\right)} + \sum_{r\neq k}^{N}\left[\frac{\Psi_{mr}\Psi_{nr}}{\omega_r \nu_r + i\left(\omega_k - \omega_r\sqrt{1-\nu_r^2}\right)}\right] \tag{1}$$

$$H_{mn}^{NP} = \frac{\Psi_{mk}\Psi_{nk} + \Psi_{mk}^{*}\Psi_{nk}^{*}}{\omega_k \nu_k + i\left(\omega_k - \omega_k\sqrt{1-\nu_k^2}\right)} + \sum_{r\neq k}^{N}\left[\frac{\Psi_{mr}\Psi_{nr} + \Psi_{mr}^{*}\Psi_{nr}^{*}}{\omega_r \nu_r + i\left(\omega_k - \omega_r\sqrt{1-\nu_r^2}\right)}\right] \tag{2}$$

In (1) and (2) ω_k and ν_k are the frequency and the modal damping ratio of the mode shape k; whereas ω_r and ν_r are the frequency and the modal damping ratio of the mode shapes $r \neq k$; Ψ_{mk} and Ψ_{nk} are, respectively, the modal components of the k-th mode shape at dofs m and n; Ψ_{mr} and Ψ_{nr} have the same meaning but they refer to the mode shapes $r \neq k$; i is the imaginary unit. In the case of non-proportional damping (2) the mode shapes oscillate out of phase and the complex conjugate pairs Ψ_{mk}^*, Ψ_{nk}^* and Ψ_{mr}^*, Ψ_{nr}^* respectively of Ψ_{mk}, Ψ_{nk} and Ψ_{mr}, Ψ_{nr} add to the system response.

In (1) and (2) the contribution to the FRF of the interested k-th mode shape has been separated with respect to the others N-1 modal residues that pollute the pure response in k. Each modal residue depends on ν_r and ω_r and will be progressively smaller as ω_r departs from ω_k, i.e. as the modal density decreases. Each mode shape $r \neq k$ will add a different contribution to FRF; this contribution is weighted, for each dof, by $\Psi_{mr}\Psi_{nr}$ in case of proportional damping (1), and by the addition of its complex conjugate in the case of non-proportional damping (2). It is important to note that when the damping is proportional (1), the *k*-th mode shape should be real; on the contrary, when the damping is non-proportional (2), the same *k*-th mode shape should be complex.

3. Modal complexity quantification

Several indices exist to quantify the mode shapes complexity, and then the damping non-proportionality, of dynamic systems. As shown in [3], the indices can be divided in two groups depending on their identifiability: unmeasurable indices and measurable indices. The unmeasurable indices require the knowledge of the damping matrix of the system, which is not identifiable directly from experimental tests. On the contrary, the measurable indices require the knowledge of the modes shapes of the system, which can be identified directly from experimental tests. Consequently, from a practical point of view, only the measurable indices are experimentally useful even if their identification suffers for the modal density problem.

3.1. Unmeasurable indices

The unmeasurable indices have essentially theoretical value since they cannot be experimentally derived. They are therefore beyond the purpose of this work and are not further considered. The interested reader can refer to [3] for details.

3.2. Measurable indices

A number of measurable indices has been used to try to relate the structural damage to the damping non-proportionality regardless their sensitivity to the modal density problem [9,10].

The effectiveness of these indices is here reconsidered in the light of their dependency on the fictitious complexity originating in the mode shapes because of closed spaced frequencies. The measurable indices considered are: I_1 the modal imaginary ratio; I_2 the modal collinearity; I_3 the modal dispersity and I_4 the modal phase difference. In particular, I_1 weighs the imaginary part with respect to the overall length of the mode shape; I_2 considers the degree of correlation between the real and imaginary parts of the mode shape; I_3 provides the average amplitude of the imaginary part of the mode shape and I_4 is related to the maximum out of phase of the mode shape. The explicit expressions of the indices are reported below:

$$I_1 = \frac{\|\mathrm{Im}(\boldsymbol{\Psi}_k)\|}{\|\boldsymbol{\Psi}_k\|} \quad I_2 = 1 - \frac{\left|\mathrm{Re}(\boldsymbol{\Psi}_k)^T \,\mathrm{Im}(\boldsymbol{\Psi}_k)\right|}{\sqrt{\left(\mathrm{Re}(\boldsymbol{\Psi}_k)^T \,\mathrm{Re}(\boldsymbol{\Psi}_k)\right)\left(\mathrm{Im}(\boldsymbol{\Psi}_k)^T \,\mathrm{Im}(\boldsymbol{\Psi}_k)\right)}} \quad I_3 = \frac{\sum_{j=1}^{N}\left|\mathrm{Im}(\Psi_{kj})\right|}{N} \quad I_4 = \frac{|\varphi_{k,\max}| - |\varphi_{k,\min}|}{\pi} \tag{3}$$

where $\boldsymbol{\Psi}_k$ is the *k*-th mode shape; Ψ_{kj} is the *j*-th component of $\boldsymbol{\Psi}_k$; $\varphi_{k,max}$ and $\varphi_{k,min}$ are respectively the maximum and minimum phase angle of $\boldsymbol{\Psi}_k$; $\|\cdot\|$ is the Euclidean 2-norm operator and $|\cdot|$ is intended as the componentwise absolute value; N is the number of system dofs. The mode shapes used to compute the indices are treated according to [11] to ensure the minimization of the errors in the identification of the imaginary part of the mode shapes.

4. Mechanical system

In order to discuss the influence of the modal density on the measurable indices, as a consequence of the fictitious complexity of the identified mode shapes, a simple mechanical system capable to highlight the main features of the problem is used, Fig. 1.

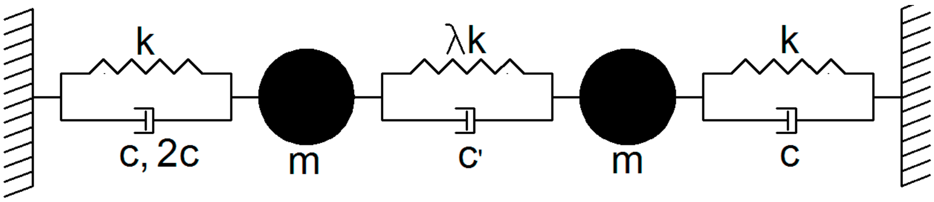

Fig. 1. Mechanical system.

The system is a 2-dofs viscously damped discrete linear system that allows to control the modal density in both cases of proportional or non-proportional damping. The mass matrix **M** of the system (m = 20 x 10^3 Kg) is diagonal, the stiffness matrix **K** (k = 56880 kN/m) is banded and the damping matrix **C** is adjusted as necessary to provide proportionality or not with respect to **M** and **K**, i.e.: $\mathbf{C}_P = \alpha\mathbf{M} + \beta\mathbf{K}$ or $\mathbf{C}_{NP} \neq \alpha\mathbf{M} + \beta\mathbf{K}$.

In order to control the modal density, a parametric stiffness is attributed to the central spring via the scaling factor λ = (0, 1] that makes the central spring variable in (0, k] and preserves the system symmetry. Changes in λ leave the frequency ω_1 of the first mode shape unchanged and modify only the frequency ω_2 of the second mode shape. More precisely, when λ decreases, ω_2 gets close to ω_1, which is invariant with λ. It is initially assumed $\lambda = 1$, in this case the two frequencies ω_1 and ω_2 are very well spaced; on the contrary, when λ is let approaching zero, ω_1 and ω_2 become undistinguishable. Consequently, it is possible to study the modal density influence by analyzing the effects only on the first mode shape.

In the case of proportional damping $\mathbf{C}_P(\lambda) = \alpha\mathbf{M} + \beta\mathbf{K}(\lambda)$, the constants α and β are selected, respectively, equal to 3,38 and 6,87 x 10^{-4}, in order to have the modal damping ratio equal to 5% both for the first and the second mode shape whatever λ. The three conditions $\mathbf{K}\,\mathbf{M}^{-1}\mathbf{C} = \mathbf{C}\,\mathbf{M}^{-1}\mathbf{K}$, $\mathbf{M}\,\mathbf{K}^{-1}\mathbf{C} = \mathbf{C}\mathbf{K}^{-1}\mathbf{M}$, $\mathbf{M}\mathbf{C}^{-1}\mathbf{K} = \mathbf{K}\mathbf{C}^{-1}\mathbf{M}$ posed in [12] to ensure the proportional damping existence are satisfied. Consequently, the mode shapes are real and the measurable indices in (3) must be zero for any λ.

The case of non-proportional damping is introduced by doubling the damping constant c of the left damper, Fig. 1. Because of that, the modal damping ratio is now equal to 7,5%, both for the first and for the second mode shape, regardless of λ; in fact, as in the proportional damping case, the value of the central spring stiffness does not affect the modal damping ratio. More precisely, the non-proportional damping case is realized by keeping $\mathbf{C} = \mathbf{C}(\lambda = 1) = const.$ and let varying only **K** along with λ. Furthermore, in the present case, the above three conditions for the proportional damping existence are not satisfied, therefore the mode shapes are complex and the measurable indices in (3) must be different than zero.

In conclusion, the constancy of the damping in both cases (proportional or not) uncouples the problem and let it possible to study the changes of the mode shapes as a consequence of the modal density (frequency spacing) effect alone. Moreover, due to the analytical nature of the problem, the theoretical (exact) solution is available along with the experimental one and the errors embodied in the measurable indices can be easily estimated by comparing the two solutions. In particular, the theoretical solution is derived from the state space formulation and the experimental solution is derived through conventional identification using FRFs (1) and (2).

5. Results

The comparison between the results of the theoretical and experimental solutions is given in Fig. 2, where the measurable indices I_1, I_2, I_3 and I_4 (th = theoretical, exp = experimental) are plotted against the parameter λ which controls the modal density.

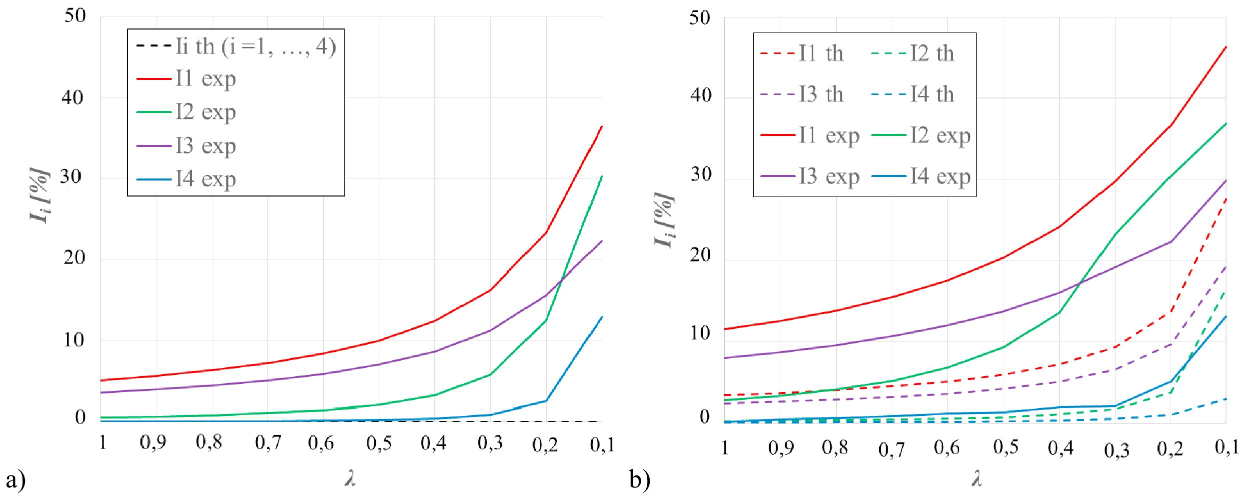

Fig. 2. Influence of modal density (λ) on the modal complexity indices - first mode shape: a) proportional damping; b) non-proportional damping. I_i *th* = theoretical solution, I_i *exp* = experimental solution.

Theoretical case (exact solution). In the case of proportional damping (Fig. 2a), the mode shapes are real and, therefore, the theoretical indices I_i *th* are zero regardless the λ value. On the contrary, in case of non-proportional damping (Fig. 2b), the mode shapes are complex and the I_i *th* are not zero. They increase rightly along with the λ reduction due to the progressive loss of damping proportionality. In particular, it is noted that the four indices I_i *th* share the same trend against λ, even if they are differently scaled in intensity. This means that the different indices show a different sensitivity and do not provide for a unique solution.

Experimental case. In either cases of proportional (Fig. 2a) and non-proportional (Fig. 2b) damping, the experimental indices I_i *exp* are not zero. They show a moderate increase along with the modal density until approximately $\lambda > 0{,}3$ beyond which a sharply slope is observed. Furthermore, the I_i *exp* are not zero also for $\lambda \cong 1$; this means that they are influenced also in the case of well spaced frequencies. However, in the first case (Fig. 2a) the measured modal complexity by I_i *exp* is entirely fictitious. In fact, this complexity is a pure consequence of the modal residues ($\Psi_{mr}\Psi_{nr}$). Instead, in the second case (Fig. 2b), the complexity of the mode shapes estimated by I_i *exp* is the consequence of two combined contributions: one actual and one fictitious. The first is related to the entity of non-proportional damping ($\Psi^*_{mk}\Psi^*_{nk}$); the second is related to the modal density ($\Psi_{mr}\Psi_{nr}$, $\Psi^*_{mr}\Psi^*_{nr}$). Therefore and rightly, the I_i *exp* values of the proportional damping case (Fig. 2a) are lower than those of the non-proportional damping case (Fig. 2b). The difference between the two is precisely the amount of the error due to the modal density. The comparison between I_i *th* and I_i *exp* shows that the best index is I_4 and the worst is I_1. In fact, I_4 is the less sensitive to the modal density since its experimental values depart less from the theoretical ones in either case proportional (Fig. 2a) or not (Fig. 2b).

In conclusion, it appears that the modal density problem affects the correct identification of the damping nature (proportional or non-proportional) and entity. This is apparent by the graphs of Fig. 2a where it is observed I_i *exp* $\neq 0$ for $\lambda = 1$ and by the graphs of Fig. 2b where the I_i *exp* curves are systematically higher than the corresponding I_i *th* curves.

6. Conclusions

The paper deals with the problem of the modal density influence on the complexity of the identified mode shapes in dynamic systems. This problem arises when the mode shapes or their appropriate functionals (indices) are used to detect the system damping nature (proportional or non-proportional) and its entity. In fact, errors in the measure of the modal complexity induce errors in the damping non-proportionality quantification.

The purpose of the work is to contribute to evaluate these errors through the comparison between theoretical (exact) and experimental (actual) solutions. The comparison is carried out using selected literature indices that provide an estimate of the modal complexity. Theoretically, the indices must be zero for proportional damping and different than zero for non-proportional damping. The indices are first classified according to their experimental application and then to their sensitivity to the modal density.

The results indicate that the indices show the same trend with respect to the modal density entity (in this sense they are equivalent), but they are differently scaled in intensity (in this sense they are endowed with different sensitiveness to the frequency spacing). In particular, the indices obtained via the experimental solution increase along with the increase of the modal density. The trend is almost constant up to very high modal density beyond which the error diverge with sharply slope. It is also found that in the case of proportional damping and well spaced frequencies the indices are not zero and then fail to address the right nature of the damping. Finally, the indices tested do not provide for a unique solution, but show different sensitivity to the modal density. The less sensitive index to the frequency spacing is the one formulated in terms of modal phase difference and it is the one suggested to use for problems of detection of non-proportional damping.

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