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A DISCONTINUUM FINITE ELEMENT MODELLING APPROACH FOR REPRODUCING THE STRUCTURAL BEHAVIOUR OF MASONRY WALLS

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Abstract. This paper defines an innovative approach for modelling masonry walls when the structural behaviour of new or existing buildings, subjected to vertical and lateral load, has to be evaluated. Such an approach aims to provide a calculation tool that allows to model the non-linear behaviour of masonry structures with a reduced numerical effort, but, nonetheless, without jeopardizing the accuracy of obtained results.

The proposed model is a typical D-FEM (Discontinuum - Finite Element Model) that, differently by the most common methodologies, is composed by deformable blocks separated by interface elements arranged along pre-established surfaces of potential cracks. To this aim, the "Combined Cracking-Shearing-Crushing" model, proposed by Lourenco for the FEM analysis with the so called simplified micro-models, is used.

Some experimental tests taken by litterature are described. Such tests are used as reference for setting up a non-linear model with the "simplified micro-modelling" approach, which considers the presence of blocks, of the same geometry of the stone units, separated by interface elements. Once that this modelling approach is validated, it is used to obtain the non-linear response of 65 masonry panels which differ in terms of geometry, vertical loads, as well as in terms of the most significant mechanical parameters.

The obtained responses for the 65 panels are taken into account for the calibration of the here proposed model. In detail, a proper variation of the coefficients contributing to the "Combined Cracking-Shearing-Crushing" formulation is implemented through a trial and error procedure, which ends when a satisfying comparison between the results provided by the two different methods of modelling is achieved; this for taking into consideration the constraints imposed on the development of cracking surfaces.

The outcomes obtained with the here proposed modelling approach are then elaborated in order to develop suitable closed form equations, which provide the necessary coefficients that have to be used for implementing the modified "Combined Cracking-Shearing-Crushing" when a generic masonry panel has to be modelled.

1 INTRODUCTION

Among the most used modeling techniques there are numerous methods that can be more or less appropriate, depending on the purpose for which they are to be used. Therefore, the purpose combined with the computational costs drive the choice between one model rather than another.

By considering only numerical analysis, it is possible to distinguish two main types: (i) the Finite Element models (FEM - Finite Element Model) and (ii) the Discrete or Distinct Element models (DEM - Distinct Element Model).

The most used FEM models are divided into Continuous models (Macromodels) and Discontinuous models (Micromodels). Discontinuous approaches are based on a micro-modeling of the wall, with mortar joints and bricks considered as distinct units; for this reason, their implementation requires a considerable knowledge of the mechanical characteristics of the blocks, the joints and the interface.

In continuous approaches with macro-modeling, the masonry material is considered as an *anisotropic continuum* whose mechanical behavior is deduced from phenomenological observations, or through homogenization techniques.

A further fragmentation of models present in the literature, subdivides micromodels into Detailed and Simplified which, as their names suggest, capture or not those mechanisms that are established at the microscopic level. In fact, the detailed micromodels, mainly used for research purposes on small samples as they have the peculiarity of reproducing even the smallest collapse or re-adjustment mechanisms, consider the presence of units, mortar joints and interfaces. On the contrary, the simplified micro-models focus the modeling of the mortar's properties in the modeling of the interface elements, resulting in lower computational demand and significantly reducing the analysis times. However, despite the simplifications they still allow the identification of the global collapse mechanism that leads to structural collapse.

With specific reference to masonry, since it is a material with multiple different mechanical properties, due to the mortar joints that form "weak surfaces", the discretization must be implemented according to precise criteria also as a function of the phenomena and failure modes that are the aims of the simulation.

In general, the approach to be followed may include the micro-modeling of individual components, units (brick, block, etc.), mortar and interface, or the macro-modeling of masonry as a composite [1]. Different modeling strategies can be developed, as shown in (Figure 1):

- "detailed micro-modeling": units and mortar joints are discretized with continuous elements, while the interface is modeled through discontinuous elements;
- "simplified micro-modeling": the units are modeled as continuous elements that incorporate the mortar joints, while the interface is represented by discontinuous elements;
- "macro-modeling": units and mortar joints are coupled together (masonry) and treated as a continuum.

In the first approach, the Young's modulus, the Poisson's coefficient and eventually the inelastic properties of the single elements are taken into consideration, and the interface represents a potential plane of breakage/sliding with fictitious stiffness to avoid compenetrations of the continuous.

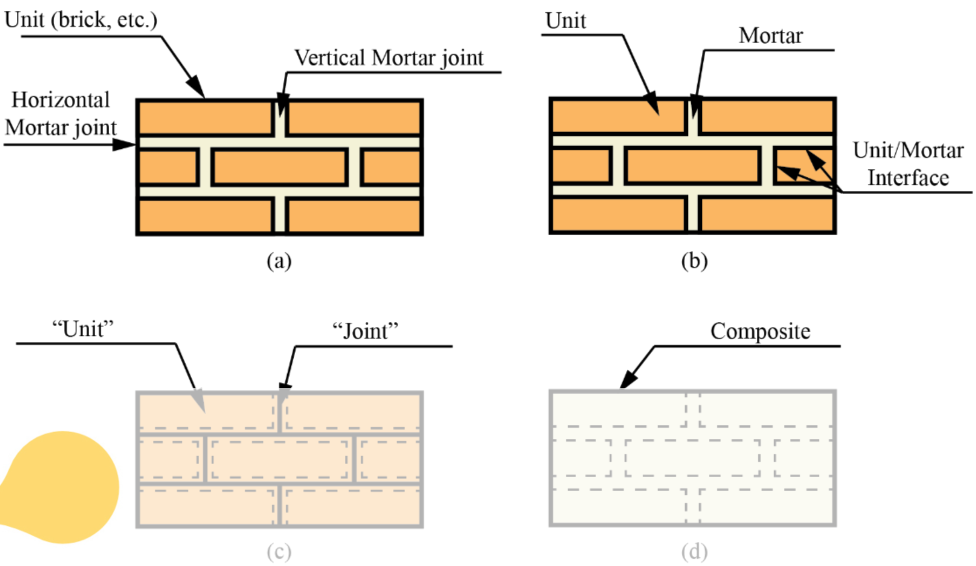


Figure 1: Modeling strategies for masonry structures: (a) real panel; (b) detailed micro-modeling; (c) simplified micro-modeling; (d) macro-modeling

In the second approach, the mortar joints are incorporated into the units. It should be noted that, by assigning a zero thickness to the interface, the geometry of the units is greater and the stiffness assigned to the interface must be calculated taking into account the mechanical properties of all the components (blocks and mortar) that make part of the masonry.

The first two modeling techniques are part of the discontinuous/discrete approach, in which cracks are concentrated in predefined paths, such as mortar joints or units. If the detailed or simplified approach of micro-modeling is computationally burdensome for the analysis of large masonry structures, it represents an important research tool for the reproduction of experimental laboratory tests.

In the third approach, we do not distinguish between unity, mortar and interface, but treat the wall as an anisotropic composite. This modeling approach returns satisfactory results when there are evenly distributed cracks throughout the wall. Furthermore, it is widely used when a compromise between accuracy and efficiency (professional applications) is required.

In this article, the proposed model is calibrated through the a simplified micro-modelling approach.

2 THE PROPOSED MODEL

2.1 General

The basic idea of the modeling proposed in this article is that a periodic masonry wall can be divided into a limited number of modules having the same mechanical characteristics. If each of them is able to reproduce the potential rupture of the portion of masonry considered in different loading conditions, together they can simulate the global non-linear behavior of the entire element. On the other hand, this idea implicitly characterizes all those methods that in recent decades have been proposed for masonry with periodic layouts, even if the size of the module considered was always too small (usually two units separated by a mortar joint). Therefore, the application to complex structures is inadequate for numerical problems.

The test model is characterized by the use of interface elements whose formulation is obtained by joining the Coulomb resistance criterion with a longitudinal vertical part for the tensile strength of the mortar (Tension mode) and an elliptical part for the compression resistance (Cap mode).

An accurate model must include all types of failure mechanisms that characterize the masonry: (a) tensile failure of the joints, (b) sliding along the bed or head joints, in particular for low values of normal stress, (c) rupture of the units by pure traction, (d) rupture by diagonal traction of the units with normal stress sufficient to develop the necessary friction and (e) "masonry crushing", a consequence of the crushing of the mortar joint for high values of stress normal which, by dilating outwards, leads to breakage of the units, as shown in (Table 1).

It is evident that mechanisms (a) and (b) concern mortar joints, (c) is a mechanism of unity, (d) and (e) are combined mechanisms involving units and joints.

2.2 The module

The methodology proposed here is based on a square module with sides of 250 mm. This dimension seems sufficient to represent the periodicity of all masonry. However, sensitivity studies are also carried out to understand if it is possible to adopt even larger modules.

The module is represented through a D-FEM discretization, as shown in (Figure 2a). Specifically, it is subdivided into four elastic prismatic blocks, two trapezoidal and two pentagon shaped, both having linear behavior and the same elastic modulus and coefficient of Poisson assumed for the units in the simplified micro-modeling proposed by Lourenco. The four elastic blocks are separated by zero-thickness interface elements characterized by the Combined Cracking-Shearing-Crushing formulation introduced in [2].

The geometry of these elements has been defined on the basis of considerations inherent to the collapse mechanisms, frequently found in masonry structures and which in the modeling are simulated through preferential slides which well describe the actual behavior of the element.

The assembly of the elements described above allows the modelling of a specific structural element, as shown in (Figure 2b).

It should be noted that in the assembly process, the interface elements are also diffused along the free borders of each elastic unit, in order to model the interaction between two adjacent modules, but an internal rigid constraint is imposed along the vertical surfaces to avoid possible slides in that direction. This mechanism, in fact, is not usually emplyed for real masonry panels.

It is evident that the proposed modeling approach does not allow to identify the precise location of cracks that can be found on a real masonry panel, for the simple reason that their possible formation along with sliding and crushing mechanisms are bound to the layout and position of the elements of interface. However, all possible collapse mechanisms for different loading conditions can be identified, as shown schematically in (Table 1), through the formation of cracks in predetermined points of the model which, moreover, very well reproduce the observable phenomenon in reality.

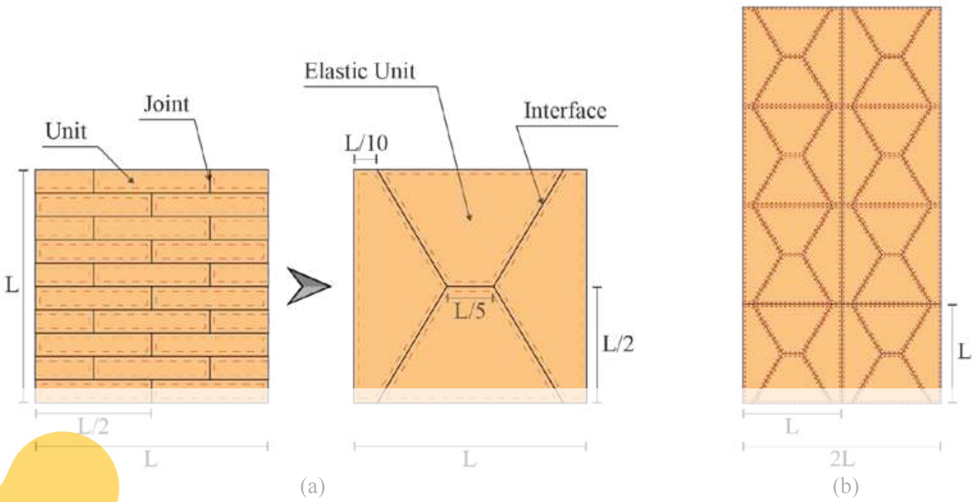


Figure 2: (a) elementary module used for the proposed approach, (b) module assembly technique

For this reason, we may think that, if a reliable model for the interface elements is adopted, the calibration applied to the mechanical parameters in order to consider the simplifications made in the shape may be of modest.

As in the case of simplified micro-modeling, the proposed D-FE model is based on the hypothesis that the inelastic characteristics of the mortar joints can be diffused along the interfaces between the blocks with zero thickness, and they are governed by the "Combined Cracking-Shearing-Crushing" multi-surface yield model, which is developed by the union of the three different constitutive bonds listed below:

- tension cut-off criterion;
- coulomb friction criterion;
- compressive cap criterion.

The introduced interface elements allow the modeling of the discontinuities in the displacement field, and their behavior is described in terms of the relationship between the tractions t and the relative displacements Δu in the interface. The linear elastic relationship between these generalized tensions and the deformations can be written in the equation shown in (1):

$$\sigma = D\varepsilon \tag{1}$$

where, for a 2D configuration, $\sigma = \{\sigma, \tau\}^T$, $D = \mathrm{diag}\{kn, ks\}$ and $\varepsilon = \{\Delta un, \Delta us\}^T$, with n and s indicating the normal and shear components, respectively.

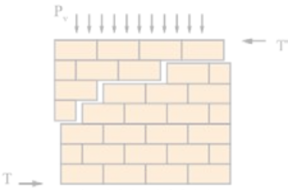
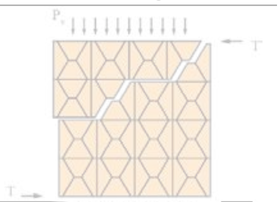
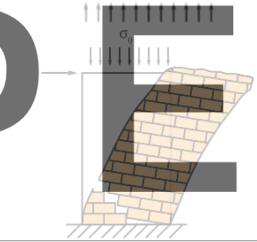
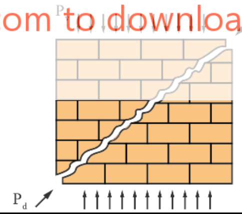
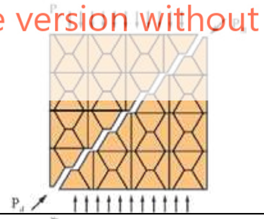
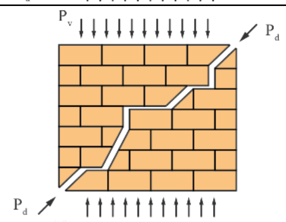
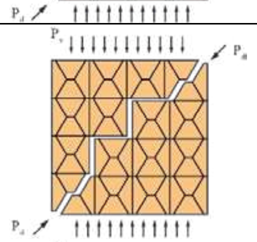
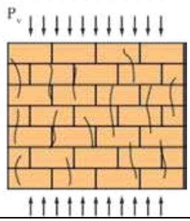
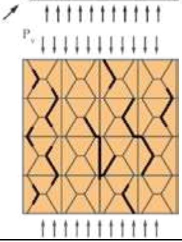
The elastic stiffness matrix D is obtained from the properties of the two components of the masonry (unit and mortar) and from the thickness of the joint. By assigning a zero thickness to the interfaces, the size of the units must be expanded by $h_m/2$ on each side. It follows that the elastic properties of the "expanded" unit and the "interface joint" must be correctly calibrated to obtain accurate results. One possibility is to reduce the elastic modulus of the unit and to use interface elements with high fictitious stiffness to avoid interpenetration of the continuum. Instead, in the following approach it is assumed that the elastic properties of the unit do not change and through the assumption of a “stack bond”, such as a series connection between the components, and a uniform stress distribution in both the unity and the mortar, the normal and tangent stiffness of the interface are determined as shown in (2), see [3] for more details:

$$k_n = \frac{E_u E_m}{h_m(E_u - E_m)} \qquad\qquad k_s = \frac{G_u G_m}{h_m(G_u - G_m)} \tag{2}$$

where, E_u and E_m are the Young modules and G_u and G_m the shear modules of the unit and the mortar, respectively, and h_m is the actual thickness of the joint.

It is clear that the interpenetration will be greater as the stiffness of the interface decreases; moreover, the interface model includes the compression behavior through the "Cap mode".

Table 1: Real collapse mechanisms reproduced by the D-FE model

MASONRY TYPE	COLLAPSE TYPE	MECHANISM OF GLOBAL COLLAPSE	GLOBAL COLLAPSE FOR D-FEM MODEL
WEAK CONNECTIONS	Sliding Joints		
	Axial-Flexure		
WEAK BRICKS	Diagonal Units Cracking		
MIXED	Diagonal Units Cracking and Sliding Joints		
	Compression		

3 THE PROPOSED FORMULATIONS

3.1 General

The form equations proposed in this paper are introduced to modify the fundamental characteristics of the Lourenco formulation, in order to take into account the approximations proposed by the D-FEM modeling with reference to the development of the cracks.

In order to determine these form equations, the technique of the Evolutionary Polynomial Regression (EPR) will be applied. This is a data mining technique based on evolutionary computing developed by Giustolisi and Savic (2006) [4]. This technique combines the power of a genetic algorithm with numerical regression to develop symbolic models.

The EPR is a two-step technique in which, in the first phase, the exponents of the symbolic structures are searched using a genetic algorithm (GA), which represents the EPR key, and in the second phase the parameters of the symbolic structures are determined by solving a linear least squares (LS) problem.

The case study starts with the monitoring of input data used for the numerical simulation of masonry panels. The Evolutionary Polynomial Regression technique is applied to extract in a symbolic way the correlations, between parametric data and input data of the new structural modeling proposed as a result of the simplifications made.

3.2 EPR-MOGA research objectives

The database includes the results of 65 numerical analyses conducted through the proposed D-FEM modeling.

In this particular case, the following parameters have been considered as explanatory variables within the EPR-MOGA procedure in order to provide the 3 fundamental mechanical parameters for the realization of the proposed numerical D-FE model: tensile strength of the mortar σ_t, cohesion c and friction angle Φ.

Therefore, the data are subdivided into a calibration set and a validation set according to the procedure described in [5], in such way that the two subsets have similar statistical properties. This step of numerical analysis has often been overlooked in previous studies, but it is essential to evaluate the effectiveness of adopted equations in predicting results. By disregarding this aspect, any conclusion on the predictive ability of the proposed solutions will be strongly distorted.

The calibration set and the validation set consist respectively of 65 samples. The calibration set will be used only to execute the Pareto front of equations that are not controlled by EPR-MOGA. The structure of the basic model is adopted, without the function f selected. It is assumed that each monomial term added is a combination of input variables. The MOGA starts with an initial population of individuals, created randomly by resorting to a Latin Hypercube sampling of set. The GA encoding is used to determine the tentative values from the specified set. During the search, the mathematical structures are created by assigning the tentative values described above to the relevant inputs. The parameters are then estimated (for example using LS or a non-negative LS minimization) in order to determine the complete mathematical expression. All expressions are classified in terms of data suitability and model complexity.

3.3 EPR-MOGA search results

The formulas selected by the Pareto front obtained with the EPR-MOGA, for different CoD levels, are shown below:

$$\sigma_{tra} = +1.059\sigma_t \qquad \text{CoD} = 98.38\% \tag{3}$$

$$\sigma_{tra} = +0.113p^{0.5} + 0.985\sigma_t \qquad \text{CoD} = 99.93\% \tag{4}$$

$$\sigma_{tra} = +0.035\phi^{0.33}p^{0.5} + 0.986\sigma_t \qquad \text{CoD} = 99.93\% \tag{5}$$

$$\sigma_{tra} = +0.228\sigma_t^{0.33}p^{0.5} + 0.906\sigma_t + 0.127\frac{\sigma_t^3}{\phi^{0.5}p^{0.5}b^{0.33}} \qquad \text{CoD} = 99.96\% \tag{6}$$

$$\sigma_{tra} = +0.004\frac{\sigma_c^2\phi^3}{\sigma_t \cdot b^3} + 0.209\sigma_t^{0.33}p^{0.5} + 0.92\sigma_t + 0.003\frac{\sigma_t^3}{p} \qquad \text{CoD} = 99.97\% \tag{7}$$

$$\sigma_{tra} = 0.004\frac{\sigma_c^2\phi^3}{\sigma_t \cdot b^3\, {B/H}^{0.33}} + 0.209\sigma_t^{0.33}p^{0.5} + 0.92\sigma_t + 0.003\frac{\sigma_t^3}{p} \qquad \text{CoD} = 99.97\% \tag{8}$$

$$c = +1.483\sigma_t \qquad \text{CoD} = 98.41\% \tag{9}$$

$$c = +0.157p^{0.5} + 1.379\sigma_t \qquad \text{CoD} = 99.93\% \tag{10}$$

$$c = +0.049\phi^{0.33}p^{0.5} + 1.38\sigma_t \qquad \text{CoD} = 99.93\% \tag{11}$$

$$c = +0.008\sigma_c^{0.33}p^{0.33} + 0.292\sigma_t^{0.33}p^{0.5} + 1.256\sigma_t + 0.066\frac{\sigma_t^2}{\phi^{0.33}p^{0.5}} \qquad \text{CoD} = 99.97\% \tag{12}$$

$$c = +0.126\frac{\sigma_c^{0.33}p^{0.33}}{b^{0.5}} + 0.292\sigma_t^{0.33}p^{0.5} + 1.257\sigma_t + 0.064\frac{\sigma_t^2}{\phi^{0.33}p^{0.5}} \qquad \text{CoD} = 99.97\% \tag{13}$$

$$c = 0.112\frac{\sigma_c \cdot \phi^3}{{\sigma_t}^{0.5}b^3\, {B/H}^{0.33}} + 0.304\sigma_t^{0.33}p^{0.5} + 1.259\sigma_t + 0.113\frac{\sigma_t^2}{\phi^{0.5}p^{0.5}} \qquad \text{CoD} = 99.97\% \tag{14}$$

$$\Phi = +0.814\phi \qquad \text{CoD} = 46.29\% \tag{15}$$

$$\Phi = +6.32\frac{1}{p^{0.33}} + 0.015\phi^2 \qquad \text{CoD} = 96.71\% \tag{16}$$

$$\Phi = +6.206\frac{1}{p^{0.33}} + 0.428\frac{1}{{B/H}^{0.33}} + 0.015\phi^2 \qquad \text{CoD} = 96.80\% \tag{17}$$

$$\Phi = +0.189\frac{\phi}{p^{0.33}} + 0.345\phi + 0.005\phi^2 + 9.466e-07\sigma_c^3 b \qquad \text{CoD} = 97.35\% \tag{18}$$

$$\Phi = +0.03\frac{\phi \cdot b^{0.33}}{p^{0.33}} + 855848.77\frac{\phi}{b^3} + 0.256\phi + 0.001\phi^2 b^{0.5} + 1.992e-06\frac{\sigma_c^2\phi^{0.5}b}{p^{0.33}} \qquad \text{CoD} = 97.62\% \tag{19}$$

The obtained formulas are examined considering the correlation coefficient R^2, the mean and the coefficient of variation (COV). The correlation coefficient is defined here as [6]:

$$R^2 = \frac{\sum_{i=1}^{N}(y_i - \bar{y})(\hat{y}_i - \bar{\hat{y}})}{\sqrt{\sum_{i=1}^{N}(\hat{y}_i - \bar{\hat{y}})^2 \sum_{i=1}^{N}(y_i - \bar{y})^2}} \tag{20}$$

where $\bar{\hat{y}}$ is the average of the estimates.

4 EXPERIMENTAL TESTS VS. PROPOSED MODEL

4.1 Experimental tests

The first four specimens considered are those treated during the experimental campaign carried out by Vermeltfoort and Raijmakers [7,8].

The one-head masonry panels are characterized by a width of 990 mm and a height of 1000 mm. All of them are made by eighteen layers of clay bricks (brick size: 210x52x100 mm^3) separated by 10 mm thick joints.

Panels were subjected to three different vertical loading conditions (30 kN for two panels, 120 kN and 210 kN for the other two) and pushed laterally, through a displacement control procedure, until collapse was reached. Both the vertical and horizontal loads were transmitted to the structure through a rigid steel beam placed on the upper part of the wall.

The collapse mechanisms of the four panels examined are mainly due to diagonal cracking by shear forces, a predictable behavior given the dimensional ratio (near 1:1).

The last panel considered for the calibration, is the one tested by Magenes and Calvi [9]. It is characterized by a width of 1000 mm and a height of 2000 mm, and made by twenty-seven layers of bricks. The units have a size of 242.5x60x120 mm^3, and are separated by 10 mm thick mortar joints.

The wall was subjected to a vertical load of 60 kN and pushed laterally, through a displacement control procedure, until collapse was reached. The collapse mechanism found was due to pressure/bending, presenting horizontal cracks in the areas subject to traction and vertical cracks in the areas stressed by compression.

4.2 Reproduction of experimental tests

To reproduce the results of the experimental tests, the basic parameters of the "Combined Cracking-Shearing-Crushing" formulation were calibrated using the trial and error procedure, performing a sequence of analyses by changing the parameter’s values until a suitable fitting was reached.

In this iterative procedure, the initial values of the aforementioned parameters are those selected according to the indications of Lourenco [2], which have proved to be quite suitable to simulate the behavior of masonry panels through a simplified micro-modeling. Values of the panels for which experimental tests were also available are reported in (Table 2), along with the values obtained after the trial and error procedure.

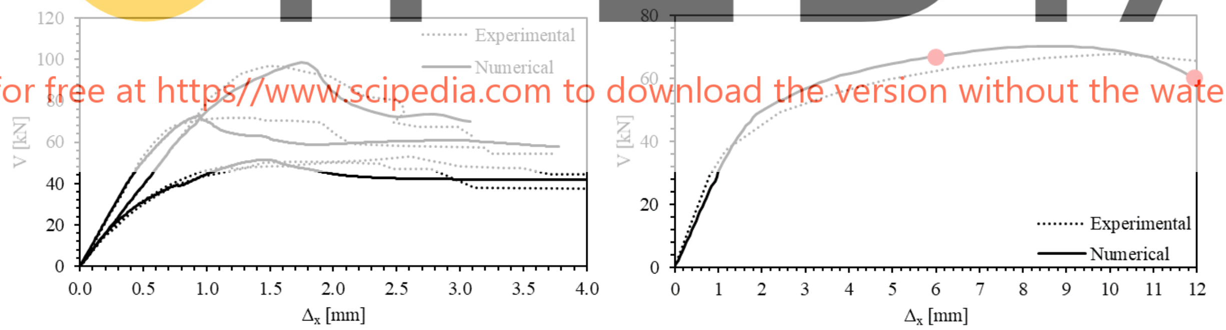
The comparison between the results of Vermeltfoort and Raijmakers experimental tests and those of D-FEM, obtained with values listed in (Table 2), is shown in (Figure 3a). As we can see, the experimental shear strength of the panels is very well reproduced by the D-FEM estimate, even in the post-peak phase.

Table 2: Inelastic properties of the joints for the Simplified Micro Model and for the D-FE model

		MORTAR JOINTS		TENSION CUT-OFF		COULOMB FRICTION			COMPRESSIVE CAP			
TEST	MODEL	k_n	k_t	σ_t	G^I_f	c	ϕ	G^{II}_f	σ_c	C_s	G^{III}_f	k_p
Vermeltfoort and Raijmaker (p =30 kN)	Lourenco Model	82.0	36.0	0.20	0.016	0.28	36.87	0.125	11.0	9	6	0.09
	Assumed D-FE Model	41.0	18.0	0.26	0.600	0.36	30.80	0.125	11.0	9	120	0.09
Vermeltfoort and Raijmaker (p = 120 kN)	Lourenco Model	110.0	50.0	0.16	0.012	0.22	36.87	0.050	11.5	9	6	0.09
	Assumed D-FE Model	61.1	27.8	0.31	0.600	0.43	25.00	0.125	11.5	9	120	0.09
Vermeltfoort and Raijmaker (p = 210 kN)	Lourenco Model	82.0	36.0	0.16	0.012	0.22	36.87	0.050	11.5	9	6	0.09
	Assumed D-FE Model	41.0	18.0	0.31	0.600	0.43	25.00	0.125	11.5	9	120	0.09
Magenes and Calvi (p = 60 kN)	Lourenco Model	30.1	13.1	0.04	0.150	0.06	24.30	0.085	10.0	9	50	0.09
	Assumed D-FE Model	16.7	7.2	0.10	0.600	0.14	15.00	0.125	10.0	9	120	0.09

The sudden loss of strength, visible in some points of the curves, is due to local failures in the single integration points, in the units or in correspondence of the cracks formation in the mortar joints.

The same comparison is shown in (Figure 3b) for the panel tested by Magenes and Calvi. Also in this case, the proposed D-FEM model, set according to the parameters of (Table 2), works very well.



(a) (b)

Figure 3: Experimental results vs. D-FEM: (a) panels tested by Vermeltfoort and Raijmakers, (b) panel tested by Magenes and Calvi

4.3 Computational efficiency of the D-FE model

In order to highlight the potential of the proposed model with respect to what is available in literature, some information is provided on the computational demand connected to the various analyses performed. In (Figure 4), a direct comparison was made between the calculation times needed for different analyses with FEM and with D-FEM models.

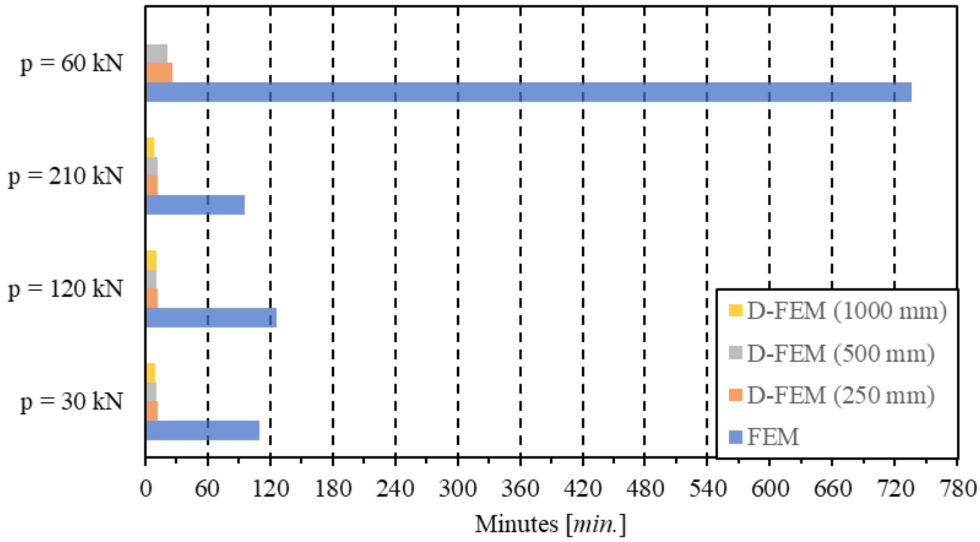


Figure 4: Computational charges of the models used

In fact, one of the aims of the paper is to provide a new calculation tool for masonry structures capable of reducing the computational demand of the analyses, allowing to broaden the horizons of detailed numerical analyses to complex structures.

As we can see, the D-FEM model has much lower computational demands than the traditional model. The analysis times are drastically reduced, maintaining however an adequate quality of the results. The use of a larger module does not bring great benefits, as the analysis times are significantly reduced but the results also undergo a non-negligible variation. Moreover, an excessively large module leads to significant numerical problems, a topic of interest for future studies. On the contrary, a smaller module should guarantee more accurate results and a slight increase in calculation times but further studies will have to confirm these assumptions.

The results are undoubtedly astonishing: in fact, the analyses performed have shown an efficiency that varies between 85% and 95% of less analysis time. The D-FEM model has more than halved the analysis time while maintaining a remarkable accuracy in the results; this gives high expectations for future applications by opening up new scenarios in the current panorama of detailed numerical analyses for new and existing masonry structures.

5 CONCLUSIONS

This work introduced a new modelling approach for masonry panels under axial and shear forces. In particular, a new type of “Discontinuum Finite Element (D-FEM)” modelling was presented, with elastic blocks separated by interface surfaces along predefined potential cracks, characterized by the "Combined Cracking-Shearing-Crushing" model proposed by Lourenco.

In conclusion, the results achieved and the proposals for future developments can be summarized the following bullet points:

- Numerical modeling techniques considered to be the most reliable in the scientific literature have been described in depth. The FEM Models (Micro and Macro

Modeling), have been described individually to understand both their strengths and weaknesses and to establish which one may be more suitable for the purpose of the analysis.

- Form functions have been provided, through EPR application, capable of correcting the input data used for the interfaces in the proposed model, obtained following the trial and error procedure, and the analogous values of the formulation proposed by Lourenco.
- The comparison between the numerical results of the proposed D-FEM modeling and those of the parametric analyses conducted previously showed the reliability of the proposed approach, after the fundamental parameters of the Lourenco model were correctly calibrated through a trial and error procedure.
- As we can deduce from the obtained results, the D-FEM model has a much lower computational demand compared to the traditional model. The analysis time is drastically reduced, maintaining an adequate quality of the results.

The obtained results are only a starting point for the proposal of a new reliable tool for the analysis of masonry structures. In the short term, further sensitivity analyzes will be carried out to guarantee an increasingly reliable procedure in the definition of the module and to provide users with a closed solution, free from any critical issue that could compromise their use.

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