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Investigating the relation between complex mode shapes and local damage for structural assessment

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Abstract. Modal parameters define the inherent characteristics of real-world structures, being therefore employed as reference information for various purposes, including the assessment of structural damage, the evaluation of operational and environmental effects, and the calibration of realistic numerical models. Among frequencies, damping ratios and mode shapes, the latter have been proved far more effective in localizing structural damage given their spatial dependency on the nodal coordinates of vibrating systems. Most of modal analysis applications resort to the real part of these quantities for vibration-based damage identification of structural systems, assuming them as classically damped. However, the classical viscous damping assumption is often idealistic for real-world structures as the damping matrix cannot be considered as proportional to mass and stiffness matrices. It follows that the mode shapes of real systems are complex in nature, and their complexity level can vary with damage. Based on the above considerations, this work intends to shed light on the relationship between structural damage and modal complexity. Numerical investigations are carried out to track the variation of complex mode shapes in a multi-span bridge subjected to progressive damage scenarios and to infer about the generalization of a new index that relies on the variation of the imaginary content of complex eigenmodes to detect, locate and assess the structural damage.

1. Introduction

The Raileigh damping assumption, commonly adopted for modelling energy dissipation mechanisms in structural systems, is an approximation that turns simplistic and unrealistic in many cases because real-world structures cannot be always considered as classically damped. In case of nonproportional damping, the mode shapes of a system are generally characterized by real and imaginary components [1-2]; this complexity can arise from different factors, including non-uniform distributions of energy dissipation mechanisms caused by structural damage.

The above explains why, although real-valued modal analysis methods continue to be the standard approach for the dynamic characterization of structural systems, they might lead to inaccurate results when employed for damage identification purposes. In fact, structures tend to exhibit unsynchronized nodal movements in the presence of damage; thus, the real part of the mode shapes, which is related to the in-phase components of the structural response, can supply only limited or incorrect information about the damage location unlike the imaginary counterpart which is, conversely, linked to the out-of-phase components of the mode [3].

Based on the previous considerations, many researchers have focused their attention on the correlation between structural damage, modal damping and modal complexity with the aim of deriving useful and expeditive indicators to employ for structural health monitoring [4-5-6]. The resulting works have corroborated the relationship existing among these variables, yet most of the proposed indicators turned out to be unable to go beyond the simple detection of the damage occurrence. In other cases, validations have been performed using scaled experimental models and attributing the increase in mode shapes complexity exclusively to the energy dissipation associated with damage. But, when dealing with experimentally identified mode shapes, mode complexity can also increase due to measurements noise, identification errors, signal aliasing and leakage, mass loading effects, nonlinearities, and high levels of modal density [7]. Numerically, these aspects can be controlled, allowing to reduce the uncertainties in the analysis of the mode complexity and fostering an in-depth comprehension of the phenomenon.

In order to fill this gap, a new index has been recently formulated by the authors for attaining higher levels of damage identification in structural systems [8]. Hitherto, the proposed indicator has been only validated against numerical and experimental data obtained from the modal identification of elementary systems. The scope of the present research work is to go beyond and demonstrate the wide applicability of this new damage index to structures close to real-world physical systems. To this end, a well-known multi-span bridge is simulated and used as a numerical benchmark for the complex modal analysis.

2. Complex modal analysis

2.1. Theoretical background

The dynamic behaviour of real structures is commonly analysed by the Finite Element Method (FEM), assuming the hypothesis of a viscous damped multi degrees of freedom (MDOF) system, translated via the use of a damping matrix *C* proportional to the mass *M* and stiffness *K* matrices of the structure, in accordance with the Rayleigh assumption. In this case, the natural vibration modes of the system are real-valued and coincide with those of the undamped one; otherwise, the system results non-classically damped featuring complex-valued vibration modes.

In case of non-ordinary constructions consisting of many heterogeneous substructures and elaborated spatial geometries or even in case of common buildings but equipped with energy dissipation devices, the system is far from being classically damped, hence the second-order differential equations governing the motion are coupled and the free vibration problem is solved turning it into a complex polynomial eigenvalue problem [9]:

$$[s^2M + sC + K]\phi = 0 \tag{1}$$

where ϕ is a modal vector belonging to R^n and s is a complex number. This problem can be solved using a state-space model that describes the behaviour of the system as a set of first-order differential equations given by the expression:

$$A\dot{w}(t) + Bw(t) = 0 \tag{2}$$

in which the complex vector $w(t) = [u(t) \quad \dot{u}(t)]^T$, while A and B are $2n \times 2n$ symmetric matrices defined as:

$$A = \begin{bmatrix} C & M \\ M & 0 \end{bmatrix}, B = \begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix} \tag{3}$$

Assuming the free vibration solution of the damped structure as $u(t) = \phi e^{st}$, the complex vector $w(t)$ and its time derivative result equal to:

$$w(t) = \begin{bmatrix} \phi \\ s\phi \end{bmatrix} e^{st} = ze^{st}, \dot{w}(t) = \begin{bmatrix} s\phi \\ s^2\phi \end{bmatrix} e^{st} = sze^{st}, \text{with } se^{st} \neq 0 \quad (4)$$

Thus, the problem in Eq. (2) ultimately reads:

$$[sA + B]z = 0 \quad (5)$$

Being A and B real symmetric matrices, Eq. (5) yields to complex conjugate pairs of eigenvalues $s_j = -\zeta_j \omega_{n,j} \pm i\omega_{d,j}$ (function of the natural $\omega_{n,j}$ and $\omega_{d,j}$ damped circular frequencies of the system, the imaginary unit i and the damping ratio ζ_j) and associated complex conjugate pairs of eigenvectors [10].

2.2. Mode complexity and structural damage

Rooted in the assumption that non-uniform energy dissipation mechanisms induced by structural damage yield non-proportional damping distributions, ergo complex vibration modes, mode complexity has been widely accepted as a measure of structural damage. Therefore, as mentioned in the Introduction, several indicators based on mode shapes complexity have been proposed in the literature for damage identification purposes. Among the most relevant are the Modal Phase Collinearity (MPC) [11], the Modal Imaginary Ratio (MIR) [6], the Modal Dispersity (MD) and the Modal Polygon Area (MPA) [12]. MIR, MD, MPA lead to positive values ranging between zero - in case of null or proportional damping and real-valued vectors - and one - in case of non-proportional damping and fully complex modes; conversely MPC is equal to one for real mode shapes. The explicit expressions of the indices are reported in Table 1, where S_{xx}, S_{yy} and S_{xy} represent the variances and covariance of the real Re(·) and imaginary Im(·) parts of the mode shape; ‖·‖ indicates the Euclidean 2-norm operator and |·| means absolute value; ϕ_{jk} is the kth component of jth mode shape ϕ_j; n indicates the number of DOFs of the structure; A_j is the area enclosed by the polygon constituted by the components of ϕ_j in the complex plane, whilst $A_{j,max}$ indicates the maximum potential area of the modal polygons.

Table 1. MPC, MIR, MD, MPA expressions.

Index	Expression	
MPC	$\left(\frac{\alpha_1 - \alpha_2}{\alpha_1 + \alpha_2}\right)^2$	$\alpha_{1,2} = \frac{S_{xx} + S_{yy}}{2} \pm S_{xy}\sqrt{1 + \left(\frac{S_{yy} - S_{xx}}{2S_{xy}}\right)^2}$
MIR	$\frac{\lVert \mathrm{Im}(\phi_j) \rVert}{\lVert \lvert \phi_j \rvert \rVert}$	
MD	$\frac{\sum_{k=1}^{n} \lvert \mathrm{Im}(\phi_{jk}) \rvert}{n}$	
MPA	$\frac{A_j}{nA_{j,\max}}$	$A_{j,\max} = n\cos\left(\frac{\pi}{n}\right)\sin\left(\frac{\pi}{n}\right)$

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It is worth noting that the afore-mentioned state-of-the-art indicators enable the sole qualitative detection of the damage occurrence, being all coordinate-independent scalar quantities of complexity. As damage is a localized phenomenon that requires higher levels of identification beyond detection in order to plan targeted countermeasures in a timely fashion and prevent irreversible failures, the following damage index has been recently introduced by the authors to overcome the limitations of state-of-the-art mode complexity indicators:

$$\Delta I_{ij}(f_j) = \left| \frac{\mathrm{Im}({\phi_{ij}}^d) - \mathrm{Im}({\phi_{ij}}^u)}{\left| \frac{1}{n} \sum_{i=1}^{n} \mathrm{Im}\left({\phi_{ij}}^u\right) \right|} \right| \frac{{f_j}^u}{{f_j}^d} \tag{6}$$

where $\mathrm{Im}(\cdot)$ is the imaginary part of the mode shape; ϕ_{ij} is the ith component of the jth natural mode shape ϕ_j; n is the total number of imaginary elements contained in the mode shape; f_j is the eigenfrequency associated with the jth mode, and the upper scripts u and d stand for undamaged and damaged scenarios. The index in Eq. (6) is based on the weighted componentwise difference of the imaginary content of complex modes between different scenarios, thus enabling the detection, localization and quantification of structural damage. Like the other literature indices, it is estimated after a normalization and rotation process of the modal vectors aimed at removing the fictitious complexity typical of modes obtained by state-space modal analyses [13].

The proposed index has been already proved effective to attain higher levels of damage identification for simply supported beams and fixed-end arches [8]. Yet, no validation through data derived from realistic models calibrated on the basis of real-world physical systems has been performed. The next section of the paper focuses on this aspect in order to prove the wide applicability of the index and move towards the generalization of its formulation.

3. Analysis of modal complexity for damage identification

In order to validate the performance of the damage index proposed by the authors when applied to realistic non-proportionally damped systems as well as to better investigate the relationship between modal complexity and structural damage, a well-known benchmark structure was simulated and analysed under progressive damage scenarios. The simulations were carried out using NOSA-ITACA code (www.nosaitaca.it/software/), a non-commercial FE software developed in house by ISTI-CNR.

3.1. Numerical benchmark

The benchmark selected for the simulation is the Z24 bridge, a three-span post-tensioned concrete overpass built in Switzerland in the 1960s and subsequently demolished in favour of a new one with a larger side span. The bridge deck was centrally supported by two piers clamped into the box-girder, while the extremities rested on triple columns via hinges. The main dimensions of the bridge are reported in Fig. 1.

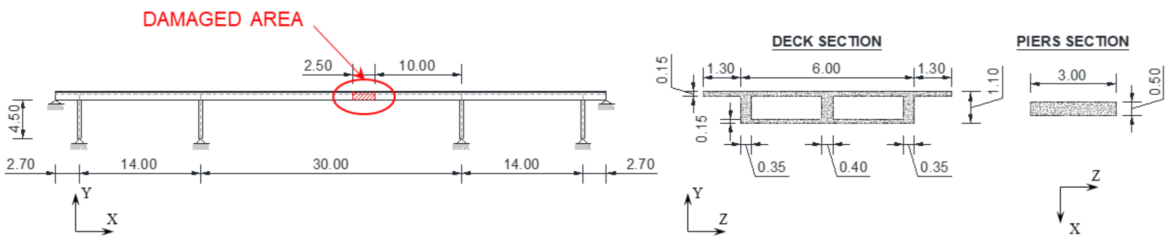

Figure 1. Geometry of the Z24 bridge used in the numerical simulations (length in meters) with identification of the damage location.

The physical system was numerically reproduced resorting to 176 Timoshenko beam elements (element n. 9 of NOSA-ITACA library) with cross sections as shown in Fig. 1, for a total of 177 nodes and 1062 degrees of freedom. As for the boundary conditions, fixed displacements along X, Y and Z directions are considered at the free ends of columns, piers, and deck of the bridge. A preliminary modal analysis was performed to calibrate the main frequencies and mode shapes of the bridge with respect to the corresponding modal parameters experimentally estimated for the reference undamaged scenario (RS). The analyses were conducted in the hypothesis of homogeneous material with Young's modulus E = 37.5 GPa, Poisson's ratio ν = 0.2 and mass density ρ = 2500 kg/m^3. Once a fine model updating of the bridge was achieved, the stiffness and mass matrices were extracted from the code and the damping matrix was computed under the Rayleigh assumption. The first four vibration modes of the system are shown in Fig. 2 along with their respective frequencies. For comparative purposes, the experimental frequencies are reported between parentheses [14]. As it can be observed, the first mode is a symmetric bending mode featuring a single-curvature vertical deflection of the main span, the second mode involves a lateral bending of the bridge deck, the third one is an asymmetric bending mode with a double-curvature vertical deflection of the main span and the fourth mode is a symmetric vertical bending mode with greater deflections at the side spans.

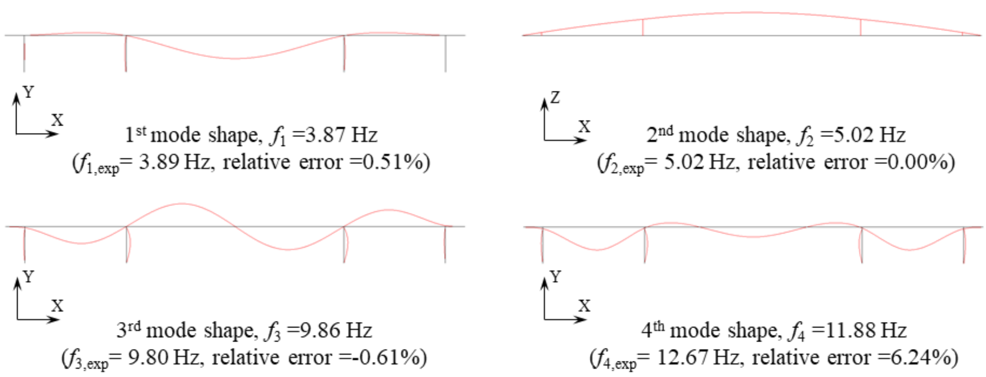

Figure 2. The first four vibration modes of the bridge in the reference scenario.

3.2. Modal complexity under progressive damage scenarios
After calibrating a realistic numerical model, four artificial damage scenarios (DS1-DS4) are induced in the bridge deck by applying an increasing penalty factor (0.1 to 0.6) to the elastic modulus of four elements of the girder (Fig. 1), whose position was selected in order to avoid null inflection points. For each scenario, the new stiffness matrix of the damaged system was evaluated through the code and employed to solve the complex eigenvalue problem according to the formulation described in Section 2. It is stressed that the damping non-proportionality of the bridge arises as an indirect effect of the stiffness loss.

The modal frequencies estimated for the bridge across the different scenarios are reported in Table 2 along with their percentage variation with respect to the initial undamaged condition. Indeed, the relative frequency decay and MAC decrement of the different modes with progressive damage are shown in Fig. 3. As expected, frequencies appear to be very sensitive to damage-induced stiffness variations, reading downshifts up to 4.8% (f_1), 2.3% (f_2) and 2.6% (f_3) for the lower modes, whereas MAC-weighted mode shape changes result more evident in the vertical bending modes, particularly those with a higher number of inflection points (modes 3 and 4). Similar considerations can be drawn by inspecting the trend of the complexity indices traced in Fig. 4, whose variation is consistent with the progressive severity of each damage scenario and turns to be greater for higher-order mode shapes in most cases, except for the MPA index, which shows an increase of modal complexity for all three vertical bending modes. Despite the meaningfulness of the results, the damage position cannot be inferred as the analysed indicators are scalar quantities and do not provide spatial information.

Table 2. Numerical frequencies of the bridge across the different scenarios.

Damage scenario	f_1 [Hz]	f_2 [Hz]	f_3 [Hz]	f_4 [Hz]
RS	3.87	5.02	9.86	11.88
DS1	3.85	5.00	9.83	11.88
DS2	3.81	4.98	9.78	11.87
DS3	3.76	4.95	9.72	11.86
Ds$	3.68	4.90	9.61	11.85

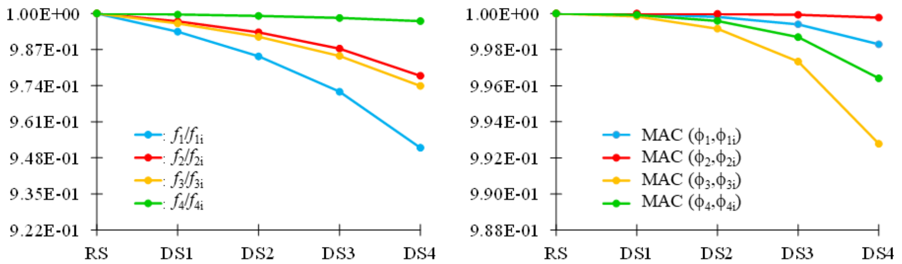

Figure 3. Frequency decay and MAC trend versus damage scenarios (DSs).

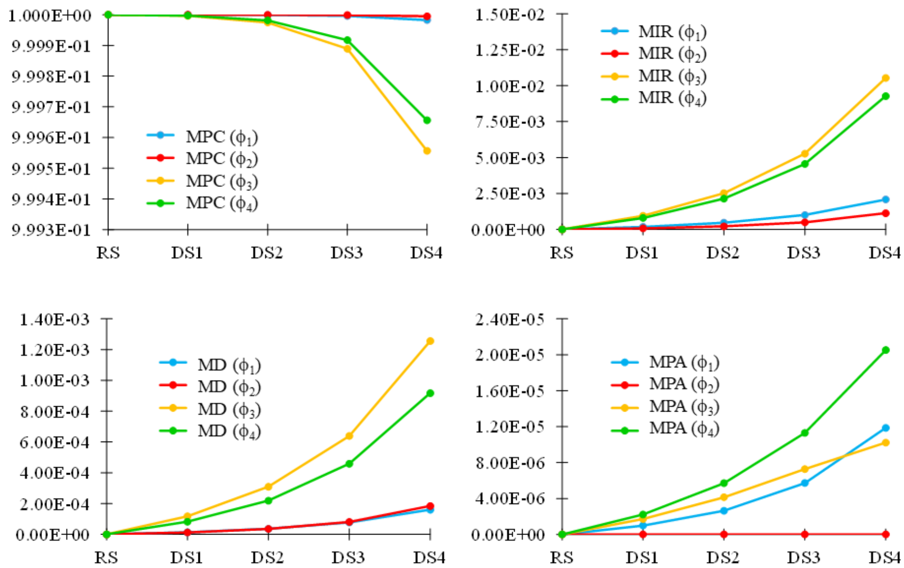

Figure 4. Complexity indices trend versus damage scenarios (DSs).

By analysing the pointwise variation of the imaginary part of the four predicted mode shapes with increasing damage, see Fig. 5, one can clearly notice the greater damage sensitivity of this component as compared to its real counterpart. Particularly, the relative percentage difference between imaginary components increases with progressive damage and, for the first two modes, it reaches its maximum value near the damage position. Though, for higher-order modes, the number of points of the structure, that do not pass through their undeflected position at the same instant in time, does increase thus leading to greater shifts of the imaginary components in multiple parts and to a higher number of candidate damage locations (false positives). This outcome corroborates the non-proportional correlation existing between the presence of structural damage and the amount of complexity in non-classically damped systems.

Moving from a global to a local damage identification, the damage indicator recently proposed by the authors [8] is computed by measuring for each DS the pointwise difference between reference and current magnitude of the imaginary part of each modal vector with respect to its initial average imaginary content, and weighing the resulting value by the ratio between the reference and current frequency of that mode, following the expression reported in Eq. (6). Accordingly, if damage occurs in the structure, ΔI provides a coordinate-dependent vector of scalar components different than zero and with higher values in correspondence of the damaged nodes. Indeed, each scalar is linked to a specific DOF of the system and the overall dimension of the vector equals the total number of measured DOFs (177 for the present case study).

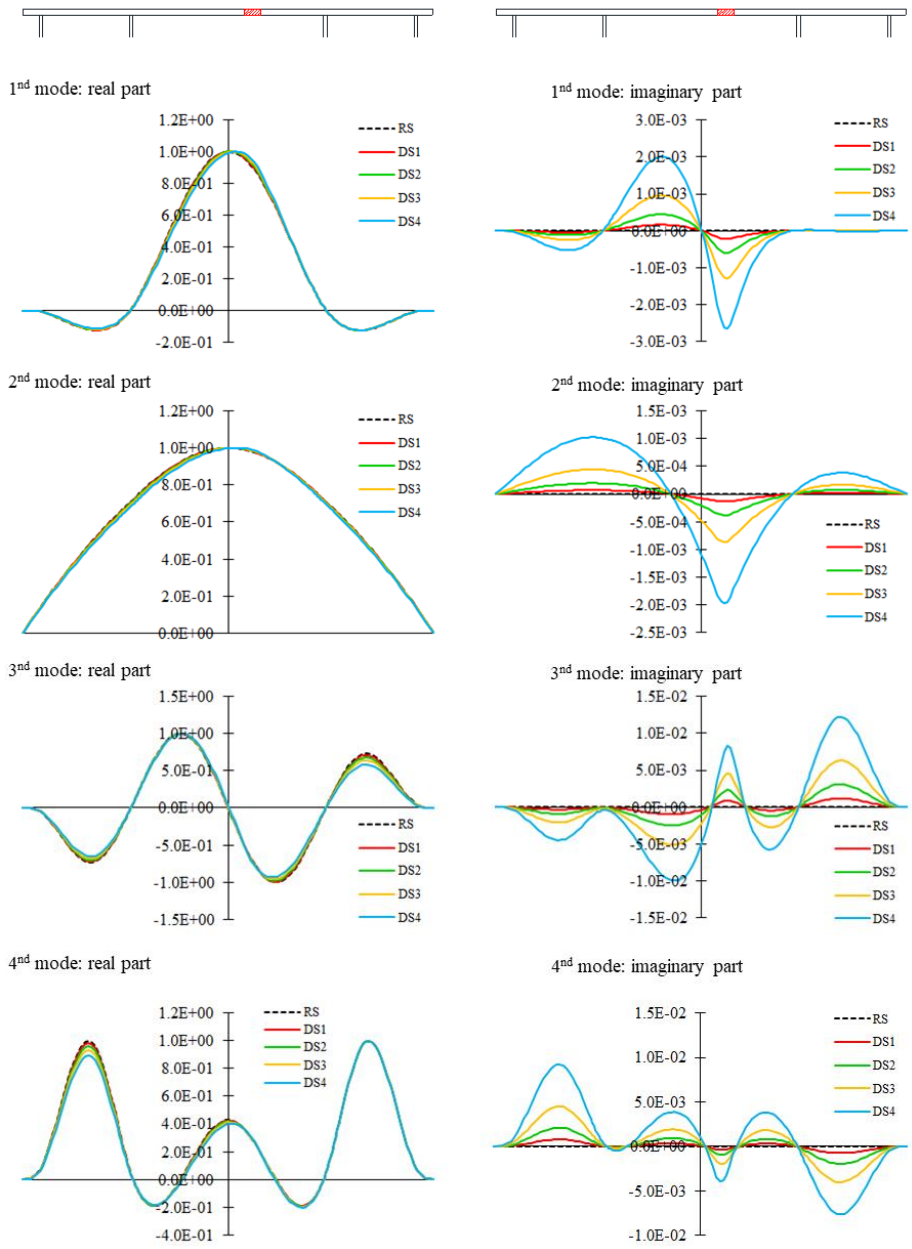

Figure 5. Variation of real and imaginary components of the identified mode shapes of the bridge over the four damage scenarios (DSs).

The results obtained by computing the proposed index based on single- as well as multi-mode shape contribution (the latter computed by summing the indices of single modes) are displayed in Fig. 6 and Fig. 7, respectively. It is found that, accounting for the relative imaginary content variation of all complex-valued eigenvectors between progressive scenarios, it is possible to localize the position of the structural damage, whereas single mode contributions might lead to false positives in case of higher-order modes.

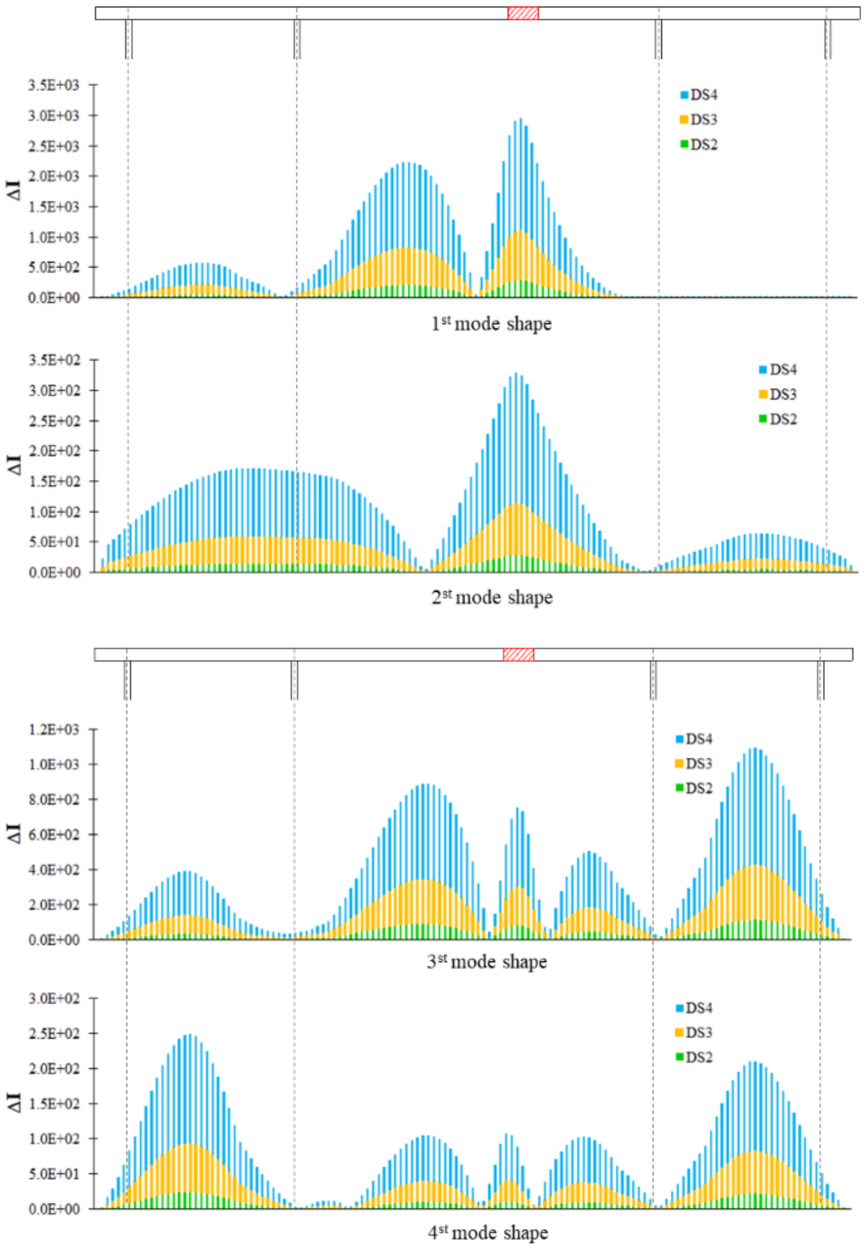

Figure 6. Pointwise damage index *ΔI* based on single-mode contribution.

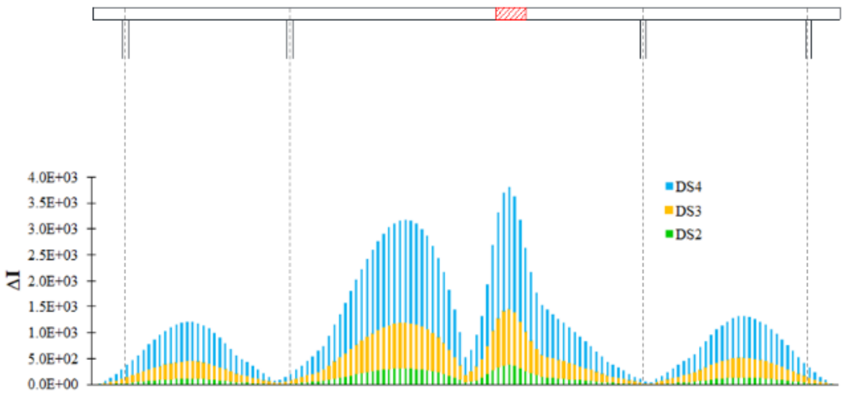

Figure 7. Pointwise damage index ΔI based on multi-mode contribution.

4. Conclusions

This paper explored the variation of complex vibration modes in a multi-span bridge model calibrated through experimental data and progressively damaged by simulating local stiffness reductions. The selection of the case study was driven by the necessity to validate the applicability of a new damage localization index to systems behaving as real-world physical structures. The underlying concept of this study is the profitability of spatial information that can be retrieved from the imaginary part of complex modal vectors when performing vibration-based assessments of the structural health.

It is shown that tracking the variation of complex mode shapes enables to capture modal deflection changes that would go otherwise undetected taking into consideration only the real components of the modal vectors. Moreover, in comparison with state-of-the-art indicators, the proposed index – which is based on the componentwise difference of the imaginary content of complex mode shapes between different scenarios, averaged with respect to the initial imaginary content of the modes and weighted by the ratio between initial and current frequencies – allowed not only to infer about the presence of damage in the structure but also to identify its location and provide a first estimate of its extent. Further analyses are being performed to thoroughly explore and quantify the correlation between amount of imaginary content variation and stiffness loss.

References

[1] D. De Domenico, G. Ricciardi (2019). Dynamic response of non-classically damped structures via reduced-order complex modal analysis: Two novel truncation measures, Journal of Sound and Vibration, 452 (2019), 169-190. https://doi.org/10.1016/j.jsv.2019.04.010

[2] E. Lofrano, A. Paolone, G. Ruta (2020). Dynamic damage identification using complex mode shapes, Structural Control and Health Monitoring, 27 (12), e2632. https://doi.org/10.1002/stc.2632

[3] M.G. Masciotta, L.F. Ramos, P.B. Lourenço, M. Vasta (2017). Spectral algorithm for non-destructive damage localisation: Application to an ancient masonry arch model, Mechanical Systems and Signal Processing, 84 (2017), 286–307. https://doi.org/10.1016/j.ymssp.2016.06.034

[4] G. Kawiecki (2001). Modal damping measurement for damage detection, Smart Mater Struct. 10(3), 466-471.

[5] F. Iezzi, D. Spina, C. Valente, 2015. Damage assessment through changes in mode shapes due to non-proportional damping, J Phys: Conf Ser. 628, 012019. https://doi:10.1088/1742-6596/628/1/012019

[6] F. Iezzi, C. Valente, Modal density influence on modal complexity quantification in dynamic systems, Procedia Eng. 199 (2017), 942–947. https://doi.org/10.1016/j.proeng.2017.09.245

[7] F. Deblauwe, R.J. Allemang, A possible origin of complex modal vectors, in: Proc. of the 11th International Seminar on Modal Analysis, paper A2-3, Katholic University of Leuven, 1986.
[8] M.G. Masciotta, D. Pellegrini (2022). Tracking the variation of complex mode shapes for damage quantification and localization in structural systems, Mechanical Systems and Signal Processing, 169 (2022), 108731. https://doi.org/10.1016/j.ymssp.2021.108731
[9] D. S. Mackey, N. Mackey, C. Mehl, V. Mehrmann, Vector spaces of linearizations for matrix polynomials. SIAM Journal on Matrix Analysis and Applications, 28:4 (2006) 971-1004. https://doi.org/10.1137/050628350
[10] A.M. Kabe, B.H. Sako, Structural Dynamics Fundamentals and Advanced Applications, Volume I, ISBN: 978-0-12-821614-9.
[11] R.S. Pappa, K.B. Elliott, A. Schenk, Consistent-mode indicator for the eigensystem realization algorithm, Journal of Guidance, Control, and Dynamics, 16:5 (1993) 852–858. https://doi.org/10.2514/3.21092
[12] G. Prater Jr, and R. Singh, Quantification of the extent of non-proportional viscous damping in discrete vibratory systems, Journal of Sound and Vibration. 104:1 (1986) 109-125. https://doi.org/10.1016/S0022-460X(86)80134-1
[13] K. Liu, M.R. Kujath, W. Zheng, Quantification of non-proportionality of damping in discrete vibratory systems, Computers and Structures. 77 (2000) 557-569. https://doi.org/10.1016/S0045-7949(99)00230-8
[14] A. Teughels & G. De Roeck (2004). Structural damage identification of the highway bridge Z24 by FE model updating. Journal of Sound and Vibration, 278 (3), 589-610. https://doi.org/10.1016/j.jsv.2003.10.041