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ScienceDirect

Procedia Engineering 158 (2016) 21 – 26

Procedia Engineering

www.elsevier.com/locate/procedia

VI ITALIAN CONFERENCE OF RESEARCHERS IN GEOTECHNICAL ENGINEERING – Geotechnical Engineering in Multidisciplinary Research: from Microscale to Regional Scale, CNRIG2016

A multiscale microstructural model of damage and permeability in fractured solids

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Abstract

Deterioration of mechanical and hydraulic properties of rock masses and subsequent problems are closely related to changes in the stress state, formation of new cracks, and increase of permeability in porous media saturated with freely moving fluids. We describe a coupled approach to model damage induced by hydro-mechanical processes in low permeability solids. We consider the solid as an anisotropic brittle material where deterioration is characterized by the formation of nested microstructures in the form of equi-distant parallel faults characterized by distinct orientation and spacing. The particular geometry of the faults allows for the analytical derivation of the porosity and of the anisotropic permeability of the solid. The approach can be used for a wide range of engineering problems, including the prevention of water or gas outburst in underground mines and the prediction of the integrity of reservoirs for underground CO_2 sequestration or hazardous waste storage.

Peer-review under the responsibility of the organizing and scientific committees of CNRIG2016

Keywords: damage; permeability; porous media; microstructure; faults; multiscale

1. Introduction

Fractures and discontinuities are among the most important features of geological structures. In natural rock formations, fractures and other types of discontinuities facilitate storage and movement of fluids, representing the

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Peer-review under the responsibility of the organizing and scientific committees of CNRIG2016
doi:10.1016/j.proeng.2016.08.399

most efficient conduit for fluid flow [1]. Despite the current availability of fault and fracture mappings in reservoirs, the understanding of the influence of these structures on fluid flow is nowadays not satisfactory, in particular when mechanical coupling is significant. The complexity of fault geometry and topology is an additional issue. In fact, each group or class of faults is characterized by different orientation, spacing, distribution and connectivity, affecting the entrapment of fluids, limiting or enhancing fluid flow in a particular environment [2]. In engineering technology fracture processes are often exploited, for example to improve the production and optimize well stimulation in low permeability reservoirs, to prevent water or gas outburst in underground mines, to predict the integrity of reservoirs for underground CO_2 sequestration or hazardous waste storage, and in various other areas of application [3]. The excavation of underground structures in rock masses induces cracking accompanied, in general, by significant changes in flow and permeability due to the deterioration of geotechnical and hydrogeological properties [4]. Damage induced by mechanical or hydraulic perturbations influences the permeability of the rock mass, with significant effects on the pore pressure distribution. Modifications in the pore pressure, in turn, affect the mechanical response of the material via poro-mechanical coupling.

In this contribution we present a recently developed model of brittle damage of confined rock masses [5], with particular emphasis on the influence of mechanical damage on the evolution of porosity and permeability. The model is based on an explicit micromechanical construction of connected patterns of parallel equi-spaced cracks, or faults. The dry multi-scale brittle damage model was first introduced in [6]. In contrast to the generic deterioration described by abstract damage mechanics, the fracture patterns that form the basis of the theory are explicit: the rock undergoes compatible deformations and remains in static equilibrium down to the micromechanical level. A relevant feature of the model is that the fracture patterns are not arbitrary, but their inception, orientation and spacing follow from energetic consideration. The constitutive model is derived within a thermodynamic framework, assuming the existence of an incremental work of deformation which accounts for reversible and dissipative behaviors of the material.

2. The brittle damage model

2.1. Constitutive equations

The brittle damage model is characterized by nested microstructures, with different length scale L_k and orientation $\mathbf{N}_k$, embedded in a homogeneous matrix. The number of levels of the nested microstructure is not limited in principle, as described in [5] where the finite kinematics version of the model is discussed. For the sake of simplicity, in the following we illustrate only the simplified model obtained through consistent linearization, and we limit the theoretical description to one single family of faults, characterized by the spacing L and the unit normal $\mathbf{N}$ to the plane of the faults.

It is assumed that the kinematics of the damaged material accounts for the deformations of the matrix and for fault opening. According to the small strain assumption, the additive decomposition of the total strain $\boldsymbol{\varepsilon}$ holds

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_m + \boldsymbol{\varepsilon}_f \tag{1}$$

where $\boldsymbol{\varepsilon}_m$ is the deformation of the homogeneous matrix and $\boldsymbol{\varepsilon}_f$ the deformation due to fault activity. By means of simple kinematic considerations (see [7] for the mathematical derivation), the deformation of the faults can be expressed through the fault displacement jump $\boldsymbol{\Delta}$ as (see Fig. 1a):

$$\boldsymbol{\varepsilon}_f = \frac{1}{2L}\left(\boldsymbol{\Delta}\otimes\mathbf{N}+\mathbf{N}\otimes\boldsymbol{\Delta}\right), \tag{2}$$

where $\otimes$ denotes the dyadic product between two vectors. Remarkably, equation (2) shows that, once L and $\mathbf{N}$ are known, there is a one-to-one correspondence between the deformation due to the fault $\boldsymbol{\varepsilon}_f$ and the displacement jump $\boldsymbol{\Delta}$. The behaviour of the matrix is assumed to be linear elastic (defined by Young modulus E and Poisson ratio ν).

The behavior of the faults follows standard cohesive theories. We assume that during the early stages of damage the separation of the fault planes is immediately resisted by cohesive forces $\mathbf{T}$. The dependence of cohesive forces on the displacement jump $\boldsymbol{\Delta}$ is expressed through a cohesive law $\mathbf{T} = \mathbf{T}(\boldsymbol{\Delta})$ that accounts for irreversibility of the fault opening. A simple class of cohesive laws can be derived in scalar terms by introducing an effective opening displacement, defined as the weighted average of the normal (Δ_N) and sliding (Δ_S) opening displacement of the fault (see Fig. 1a):

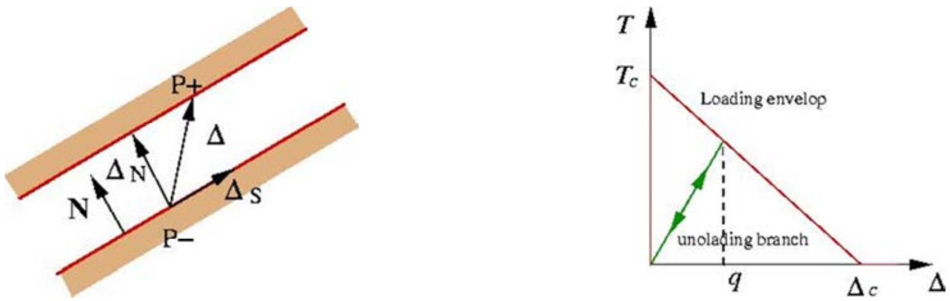
$$\Delta = \sqrt{\Delta_N^2 + \beta^2 \Delta_S^2}\ . \tag{3}$$

where β is the ratio between shear and tensile strengths of the material [6]. In line with standard cohesive theories, we rely on the existence of a cohesive energy per unit surface, $\phi(\Delta, q)$, dependent on the effective opening displacement and on some appropriate set of internal variables q describing the state of the faults. We assume a simple linear decreasing cohesive law defined by two parameters: the tensile strength T_c and the critical energy release rate G_c, see Figure 1b. The tensile strength T_c is the maximum effective traction while G_c is the area enclosed by the cohesive law. Once the critical opening displacement $\Delta_c = 2G_c/T_c$ is attained, faults completely loose cohesion and cohesive forces vanishes. In order to enforce irreversibility in the damage law, it is assumed that upon unloading faults follow a linear elastic path up to the origin: as a consequence, the only internal variable needed by the model is a scalar q, defining the maximum attained effective displacement of the faults.

Friction is included in the brittle damage model as an additional dissipative phenomenon acting concurrently with cohesion. When the material completely loses cohesion, friction remains the sole dissipation mechanism. Friction is included by means of a dual dissipation potential per unit fault area ψ^*, which for the classical Coulomb model reads

$$\psi^* = \mu \max\{0, -(\boldsymbol{\sigma}\mathbf{N})\cdot\mathbf{N}\}\left|\dot{\boldsymbol{\Delta}}\right| . \tag{4}$$

where μ the friction coefficient, $(\boldsymbol{\sigma}\mathbf{N})\cdot\mathbf{N}$ the normal component of the traction vector, and $\left|\dot{\boldsymbol{\Delta}}\right|$ the norm of the displacement jump rate. The dissipation potential is null when the displacement jump has a positive normal component $\Delta_N > 0$. If $\Delta_N = 0$ and the contact tractions are compressive, then ψ^* is convex and minimized for $\dot{\Delta} = 0$.



(a) (b)

Fig. 1. (a) Opening displacement of a single fault; (b) Linear decreasing cohesive law

2.2. Solution strategy

Let us consider a process of incremental deformation, and assume that the material state at time t_n is known. The goal of the solution strategy is to find the material state at time $t_{n+1} = t_n + \Delta t$ for a given total deformation $\boldsymbol{\varepsilon}_{n+1}$. According to [7], over the time interval Δt we define an incremental work of deformation $E_n(\boldsymbol{\varepsilon}_m, \Delta, q)$ as

$$E_n(\boldsymbol{\varepsilon}_m, \boldsymbol{\Delta}, q) = W^m(\boldsymbol{\varepsilon}_m) + \frac{1}{L}\phi(\boldsymbol{\Delta}, q) + \frac{\Delta t}{L}\psi^*\left(\frac{\boldsymbol{\Delta} - \boldsymbol{\Delta}_n}{\Delta t}, \boldsymbol{\varepsilon}, \boldsymbol{\Delta}\right). \tag{5}$$

where W^m is the elastic strain energy density per unit volume of the matrix, ϕ is the cohesive energy density per unit surface of the faults, and the term including ψ^* represents the frictional dissipation in Δt. The solution of the problem is obtained by minimizing the incremental work of deformation accounting for the constrained provided by the impenetrability of the closing faults ($\Delta_N \geq 0$) and the irreversibility of damage ($q \geq q_n$). The minimization process defines an effective potential for the stress of the material. With the inclusion of an extra energetic term, that describes the accommodation of the faults within the outermost fault or the external container through a length scale L_0, and of an additional constraint on the norm of the orientation $\boldsymbol{N}$, the minimization process provides the unknown spacing and orientation of the faults. The model accounts for material failure both in tension (according to the Galileo-Rankine criterion) and in shear (according to the Mohr-Coulomb criterion). In the latter case, the identity $\beta = \mu$ holds.

3. Permeability and porosity evolution

The explicit fracture pattern which characterizes the model provides the analytical expression of the porosity and the permeability owing to fault activity. The change in porosity due to the fault n^f can be easily obtained by equation (2) as the first invariant of the small strain tensor due to fault activity

$$n^f = -\varepsilon_{kk}^f = \frac{\Delta_N}{L}. \tag{6}$$

Fault porosity added to matrix porosity n^m provides the total porosity of the damaged medium:

$$n = n^f + n^m, \tag{7}$$

The matrix porosity is assumed to vary with the matrix volumetric strain as usual in soil mechanics. An additive decomposition holds also for the permeability tensor $\mathbf{k}$ of the fractured medium:

$$\mathbf{k} = \mathbf{k}^f + \mathbf{k}^m, \tag{8}$$

where $\mathbf{k}^m$ is the permeability of the matrix (assumed to be isotropic and dependent on matrix porosity via a Kozeny-Carman type relationship) and $\mathbf{k}^f$ is the permeability due to the faults:

$$\mathbf{k}^m = C_{KC}\frac{(n^m)^3}{(1-n^m)^2}\mathbf{I}; \qquad \mathbf{k}^f = \frac{\Delta_N^3}{12L}(\mathbf{I} - \mathbf{N}\otimes\mathbf{N}), \tag{9}$$

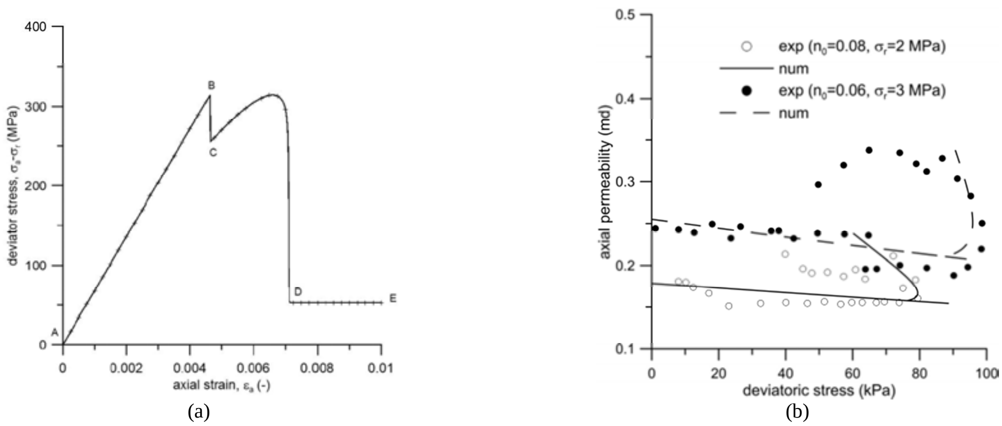
where $\mathbf{I}$ is the identity tensor and C_{KC} a material constant. The fault permeability $\mathbf{k}^f$ is derived from standard lubrication theories under the assumption of a laminar flux within the faults.

4. Model predictions

In order to understand the behavior of the model, we simulate an axial strain controlled triaxial test (confining pressure 10 MPa), see Figure 2a. The parameters used for the simulation are listed in Table 1. Model predictions are presented in terms of the evolution of axial stress with the axial strain ε_a. The response is characterized by the following stages are:

- Elastic compression (A-B), characterized by a linear relation between the deviatoric stress and the axial strain. The response is fully described by the elastic parameters and the opening displacement of the faults is null.
- Shear failure (B-C). The material fails in shear. The drop in the deviatoric stress, typical of brittle failure processes, is related to the new equilibrium condition attained by the damaged material with the presence of faults: the onset of the faults causes a reduction of the material stiffness, visualized by the vertical jump, evident also in the simulations presented in Figure 2a. Being the simulations performed at the material level, no structural redistribution effects are accounted for.
- Cohesive stage (C-D). The material response is dominated by the cohesive behavior. The deviatoric stress increases again and then experiences a softening stage. The path C-D is characterized by volumetric expansion due to fault opening.
- Residual stage (D-E). Faults have completely loose cohesion and the deviatoric stress remains constant for increasing axial strain. As long as the axial strain increases, the material goes on dilating at a constant rate.

The model has been validated with reference to experimental tests performed on rocks taken from the literature. The ability of the model in predicting the evolution of permeability in crystalline rocks and sandstones has been presented in [7]. Here we present, as a new example, the validation against some experimental results on Berea [8] and Flechtinger sandstone [9]. Figure 2b compares the numerical predictions and the experimental data in terms of permeability evolution versus deviatoric stress from two samples, characterized by different initial porosity n_0 and confining pressure σ_r, during the shear stage of a triaxial test. Remarkably, the model is able to simulate (the initial decrease in permeability for increasing deviatoric stress due to elastic compaction, followed by a permeability increase after fault formation and during the softening stage. Figure 3 reports the axial stress versus axial and radial strain for three triaxial tests on Berea sandstone samples, at the confining pressures of 0, 21 and 63 MPa, respectively. The model is able to reproduce the initial stiffness of the material as well as the non-linear mechanical response for increasing axial strain. Regrettably, experimental data do not include information on the post-peak behaviour of the tests, but report the volumetric response, in terms of porosity evolution, with reference to the shear stage of a triaxial test conducted at the confining pressure of 7 MPa. The model is able to capture the initial decrease in porosity occurring in the elastic stage, followed by a volume increase related to fault inception and dilatancy. The parameters used for the simulations are collected in Table 1.



(a) (b)

Fig. 2. (a) Model prediction of a strain control triaxial test; (b) Permeability evolution during triaxial tests on Flechtinger sandstone

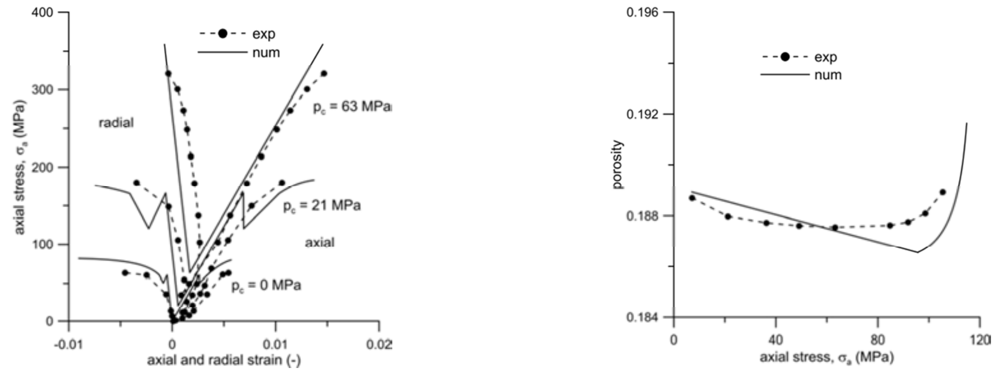


Fig. 3. Simulation of triaxial tests on Berea sandstone (experimental data from [8])

Table 1. Model parameters.

Simulation	E (MPa)	ν (-)	T_c (MPa)	G_c (N/mm)	μ (-)	L_0/Δ_c (-)	C_{CK} (mm^2)
Theoretical	68000	0.21	50	10	1.05	10	-
Flechtinger sandstone	11000	0.25	35	0.1	1.07	12.5	$3.6\cdot10^{-6}$
Berea sandstone	23000	0.19	18	0.01	0.90	1	$1.32\cdot10^{-5}$

4. Conclusions

This work has presented a brittle damage material model characterized by recursive cohesive/ frictional microstructures capable of describing both the hydraulic and mechanical behavior of brittle rocks in confined states. Microstructures assume the form of parallel faults, whose orientation and opening are obtained by optimizing the incremental work of deformation within a loading step. Faults are assumed to be cohesive until the attainment of full decohesion and to exhibit frictional behavior upon closure. Remarkably, the material model is characterized by a small number of mechanical parameters, i.e. two elastic constants, three cohesive parameters (two of them are the tensile resistance and the friction angle) and two parameters for the hydraulic behavior. The explicit geometry and connected topology of the faults enables the analytical characterization of the anisotropic permeability of the damaged rock. Model predictions in triaxial compression have been presented, highlighting the potentiality of the approach to simulate boundary value problems involving deterioration of the mechanical properties of rocks and the related evolution of hydraulic properties.

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