

Supplementary Information.

Standard error calculation:

We adopted the “Delta Method” to evaluate approximated values of the standard error of a ratio between two independent random variables (whose standard errors are known). For estimators, the delta method is important because while the theoretical standard error of a mean of an SRS is known, there is no general theoretical expression for standard errors of most functions of a mean, such as the inverse of a mean, or the ratio of two means.

Suppose we have random variables K_1, K_2 . A Taylor series expansion of $f(K_1, K_2)$ about the values (\bar{K}_1, \bar{K}_2) is given by:

$$f(K_1, K_2) = f(\bar{K}_1, \bar{K}_2) + \left. \frac{\partial f(K_1, K_2)}{\partial K_1} \right|_{(\bar{K}_1, \bar{K}_2)} (K_1 - \bar{K}_1) + \left. \frac{\partial f(K_1, K_2)}{\partial K_2} \right|_{(\bar{K}_1, \bar{K}_2)} (K_2 - \bar{K}_2) + \dots$$

If we suppose $f(K_1, K_2) = K_2 / K_1$ then $\frac{\partial f(K_1, K_2)}{\partial K_1} = -\frac{K_2}{K_1^2}$, $\frac{\partial f(K_1, K_2)}{\partial K_2} = \frac{1}{K_1}$

$$\Rightarrow f(K_1, K_2) = \frac{K_2}{K_1} \approx \frac{\bar{K}_2}{\bar{K}_1} + \frac{-\bar{K}_2}{\bar{K}_1^2} (K_1 - \bar{K}_1) + \frac{1}{\bar{K}_1} (K_2 - \bar{K}_2)$$

Hence if we want to evaluate the standard error of the latter ratio:

$$(SD(K_2 / K_1) \approx \sqrt{\frac{\bar{K}_2^2}{\bar{K}_1^4} SD^2(K_1) + \frac{1}{\bar{K}_1^2} SD^2(K_2) - 2 \frac{\bar{K}_2}{\bar{K}_1^3} Cov(K_1 K_2)})$$

Since the random variables K_1, K_2 are independent:

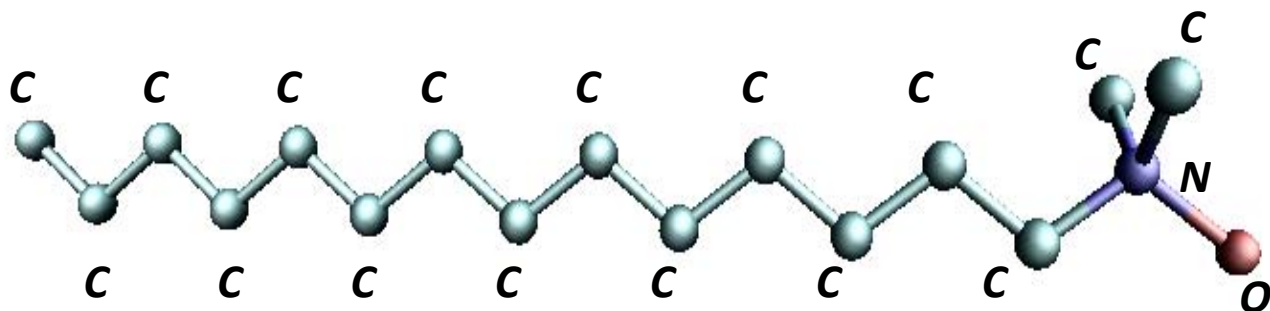
$$SD(K_2 / K_1) \approx \sqrt{\frac{\bar{K}_2^2}{\bar{K}_1^4} SD^2(K_1) + \frac{1}{\bar{K}_1^2} SD^2(K_2)}$$

Furthermore if $SD(K_1) \approx SD(K_2)$:

$$SD(K_2 / K_1) \approx SD(K_1) \sqrt{\frac{\bar{K}_2^2}{\bar{K}_1^4} + \frac{1}{\bar{K}_1^2}} = \frac{SD(K_1)}{\bar{K}_1^2} \sqrt{\bar{K}_1^2 + \bar{K}_2^2}$$

which represents the final formula for our calculations.

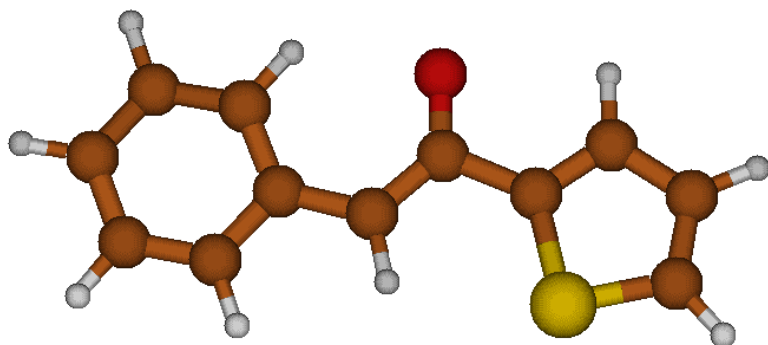
TDDNO monomer Gromacs topology:



```
[ moleculetype ]
; Name          nrexcl
tddno           3

[ atoms ]
; nr           type  resnr residue  atom  cgnr      charge      mass  typeB  chargeB
massB
    1          CH3    1     TDD    C     1         0.         15.035 ; qtot 0.129
    2          CH2    1     TDD    C     2         0.         14.027 ; qtot 0.377
    3          CH2    1     TDD    C     3         0.         14.027 ; qtot 0.625
    4          CH2    1     TDD    C     4         0.         14.027 ; qtot 0.873
    5          CH2    1     TDD    C     5         0.         14.027 ; qtot 1
    6          CH2    1     TDD    C     6         0.         14.027 ; qtot 1
    7          CH2    1     TDD    C     7         0.         14.027 ; qtot 1
    8          CH2    1     TDD    C     8         0.         14.027 ; qtot 1
    9          CH2    1     TDD    C     9         0.         14.027 ; qtot -0.8
   10          CH2    1     TDD    C    10         0.         14.027 ; qtot 1
   11          CH2    1     TDD    C    11         0.         14.027 ;
   12          CH2    1     TDD    C    12         0.         14.027 ; qtot=-1
   13          CH2    1     TDD    C    13         0.         14.027 ;
   14          CH2    1     TDD    C    14         0.0268    14.027 ;
   15           N     1     TDD    N    14         0.6164    14.0067 ;
   16          CH3    1     TDD    C    14         0.0316    15.035 ;
   17          CH3    1     TDD    C    14         0.0314    15.035 ;
   18           O     1     TDD    O    14        -0.7062    15.9994 ;
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Enolate topology:



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[ moleculetype ]
; Name          nrexcl
Protein_A      3
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```
[ atoms ]
; nr          type  resnr residue  atom  cgnr    charge      mass  typeB    chargeB
massB
  1           S     1     FNT     S     1     -0.1     32.065   ; qtot -0.83
  2          CR1    1     FNT     C     1     -0.2     12.011   ; qtot -0.415
  3          CR1    1     FNT     C     1     -0.2     12.011   ; qtot -0.415
  4          CR1    1     FNT     C     1     -0.1     12.011   ; qtot -0.415
  5          CR1    1     FNT     C     1     -0.1     12.011   ; qtot -0.415
  6          HC     1     FNT     H     1      0.2      1.008   ; qtot 0
  7          HC     1     FNT     H     1      0.1      1.008   ; qtot -0.28
  8          HC     1     FNT     H     1      0.1      1.008   ; qtot 0
  9           C     1     FNT     C     1      0.7     12.011   ; qtot 0
 10          O     1     FNT     O     1     -0.8    15.9994   ; qtot 0.38
 11          C     1     FNT     C     1     -1.0     12.011   ; qtot -0.28
 12          H     1     FNT     H     1      0.2      1.008   ; qtot 0
 13          CR1    1     FNT     C     1      0.8     12.011   ; qtot 0
 14          CR1    1     FNT     C     1     -0.3     12.011   ; qtot 0
 15          CR1    1     FNT     C     1      0.00     12.011   ; qtot 0
 16          CR1    1     FNT     C     1     -0.3     12.011   ; qtot 0
 17          CR1    1     FNT     C     1     -0.1     12.011   ; qtot 0
 18          CR1    1     FNT     C     1     -0.4     12.011   ; qtot -0.83
 19          HC     1     FNT     H     1      0.1      1.008   ; qtot -0.415
 20          HC     1     FNT     H     1      0.1      1.008   ; qtot 0
 21          HC     1     FNT     H     1      0.1      1.008   ; qtot 0.38
 22          HC     1     FNT     H     1      0.1      1.008
 23          HC     1     FNT     H     1      0.1      1.008
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