Note added to
"Information Geometry of a regime-switching model with time-varying parameters"

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In the paper "Information Geometry of a regime-switching model with time-varying parameters", [1], we discussed the possibility to use tools derived from Information Geometry in order to analyze dynamical systems in which the motion randomly switches between two regimes using Markovian transitions. We proved that, in order to understand when an change can appear, the network properties of the system are important because they accelerate the diffusion of the information. In this note we recall the most important results obtained announcing some possible applications to detect the regime-switching in real phenomena.

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I. INTRODUCTION

Regime changes are very frequent in social and natural phenomena. As an example, Figure 1 shows on the left the historical dynamics of electricity prices observed at the Californian electricity market. Note the fast recovery of the electricity price normal regime after only a short time in the turbulent regime with spikes and jumps due to outages and consequent blackouts. On the right we have the historical behavior of the total seismic activity, observed in the Californian area, which shows the restabilization of the normal situation of the earth after a period of seismic activity, [9, 10].

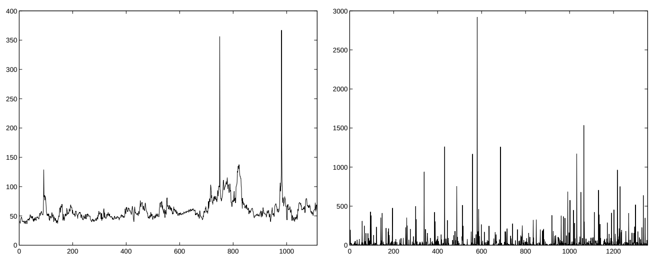

FIG. 1: Left side: California electricity prices from October 22, 2002 until January 27, 2007. Right side: historical behavior of the Californian total seismic activity.

To describe all those situations Hamilton [12, 13] proposed the regime-switching models. The assumption is that some imperfectly predictable forces produce the switches, therefore the motion is described by a larger model where a hidden random variable changes according to an assumed probability distribution. The simplest specification is that the switching mechanism between the regime is governed by an unobservable Markov process.

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II. A REGIME-SWITCHING MODEL WITH TIME-VARYING PARAMETERS

In particular we consider the model of the price behavior proposed by Hamilton, Gray, Kim and Nelson and Ning [6, 12, 18–20]. If y_t is the logarithm of price, the model for y_t depends on an explanatory variable x_t. For the regime $S_t = i$, with $i = 1, 2$, the form of the dynamics is the following:

$$y_t = \mu_{it} + \varepsilon_{it} = \alpha_i + \phi_i y_{t-1} + \gamma_i x_t + \varepsilon_{it} \quad (1)$$

where α_i, ϕ_i, and γ_i are unknown and ε_{it} is an unobservable residual that is $N(0, \sigma_i^2)$. Assuming markovian transitions, the model is mean reversing and the conditional mean of y_t

$$E[y_t \mid x_t, y_{t-1}, S_t = i] = \mu_{it} \quad (2)$$

varies in time. The four transition probabilities for the switching from one regime to another are:

$$\begin{array}{l} Pr[S_t = 1 \mid S_{t-1} = 1] = P_{1t} \\ Pr[S_t = 2 \mid S_{t-1} = 1] = 1 - P_{1t} \\ Pr[S_t = 2 \mid S_{t-1} = 2] = P_{2t} \\ Pr[S_t = 1 \mid S_{t-1} = 2] = 1 - P_{2t} \end{array} \quad (3)$$

With regard to the example of an electricity market, the two regimes are referred to the low price and high price. The first is usually the marginal cost, while the second is fixed by fundamental power markets at a maximum price which is sometimes reached. The log-price is represented by the variable y_t which is driven by an unknown variable x_t summarizing unpredictable situations like particular weather conditions or outages in the transmission system.

We denote with $\Phi_t = [y_1, y_2, ..., y_{t-1}, x_1, x_2, ..., x_t]$ the information available to make a one-step ahead forecast of y_t, in both regimes, and with $\rho_{it} = Pr[S_t = i \mid \Phi_t]$ the conditional probability of y_t being in state i. After specifying a startup value for the probability process, the whole series of regime probabilities for period t, given information at period $t-1$, can be derived recursively.

More specifically, the probability of being in regime i given the information in the previous period, is

$$\rho_{it|t-1} = Pr(S_t = i \mid \Phi_{t-1}) \quad (4)$$

for $i = 1, 2$ (Note: $\rho_{1t|t-1} = 1 - \rho_{2t|t-1}$). When new information from the current period is available, these probabilities can be updated to:

$$\rho_{it|t} = Pr(S_t = i \mid \Phi_t) \quad (5)$$

for $i = 1, 2$ (Note: $\rho_{1t} = 1 - \rho_{2t}$). Hamilton and Gray have shown [6, 12, 18] that the updated conditional regime probabilities are weighted averages of the updated regime probabilities using the transition probabilities for regime-switching:

$$\begin{aligned} \rho_{1t|t-1} &= \rho_{1t-1}P_{1t} + \rho_{2t-1}(1 - P_{2t}) \\ \rho_{2t|t-1} &= \rho_{1t-1}(1 - P_{1t}) + \rho_{2t-1}P_{2t} \end{aligned} \quad (6)$$

The likelihood value for an observed value of y_t in a given regime can be written:

$$f_{it} = f(y_t \mid S_t = i, y_{t-1}, x_t; \theta_i) \quad (7)$$

where $\theta_i = \{\alpha_i, \phi_i, \gamma_i, \sigma_i\}$, for $i = 1, 2$. Under the assumption of normality, the expression of f_{it} is:

$$f_{it} = \frac{1}{\sqrt{2\pi}\sigma_i} \exp[-\frac{1}{2}\frac{(y_t - \mu_{it})^2}{\sigma_i^2}] \quad (8)$$

for $i = 1, 2$, where μ_{it} is a function of θ_i. The conditional likelihood value for an individual observation can be written as a weighted average of the likelihood for the two regimes as follows:

$$g(y_t \mid y_{t-1}, x_t, w_t; \phi_i) = f_{1t}\rho_{1t|t-1} + f_{2t}\rho_{2t|t-1} \quad (9)$$

where $\phi_i = \{\theta_i, c_i, d_i\}$, for $i = 1, 2$. In any case, if no hidden variable is assumed, we deduce that the set of all conditional probability distributions is not an exponential family. Otherwise, for a given Φ_{t-1}, we can regard $(S_t \mid \Phi_{t-1})$ as a hidden discrete random variable, taking values $i = 1, 2$ with conditioned probabilities $\rho_{it|t-1}$. In this hypothesis the joint probability distribution is

$$p(y_t \mid y_{t-1}, S_t \mid \Phi_{t-1}) = \sum_i \delta_i(S_t \mid \Phi_{t-1})\rho_{it|t-1}f_{it} \quad (10)$$

It is a normal mixture with hidden variable.

As it is well known, Information Geometry endows a family of probability densities (Statistical Manifold) with a Riemannian metric (the Fisher-Rao metric) and a geometrical structure, induced by a couple of dual affine connections having two types of geodesics (e-geodesics and m-geodesics) and consequently two types of projections (e-projections and m-projections).

In the paper [1] we proved the following Propositions:

Proposition 1): The set of conditional distributions (10) is an *exponential family* S_t, which depends on a hidden variable moving with the information at the previous time Φ_{t-1}.

Proposition 2): When Φ_{t-1} is not fixed, we obtain a submanifold (*curved exponential family*) M of dimension 6 in the product space $S_{t-1} \times S_t$.

Proposition 3): The *observed data submanifold* is

$$D_t = \{\eta \mid \eta_1 = y_t, \eta_2 = y_t^2, \eta_{1i} = \alpha_i, \eta_{2i} = \alpha_i y_t, \eta_{3i} = \alpha_i y_t^2\} \quad (11)$$

where α_i takes any real values satisfying $\alpha_i > 0$ and $\sum_i \alpha_i \leq 1$.

D_t is a linear submanifold, as Figure 2 shows.

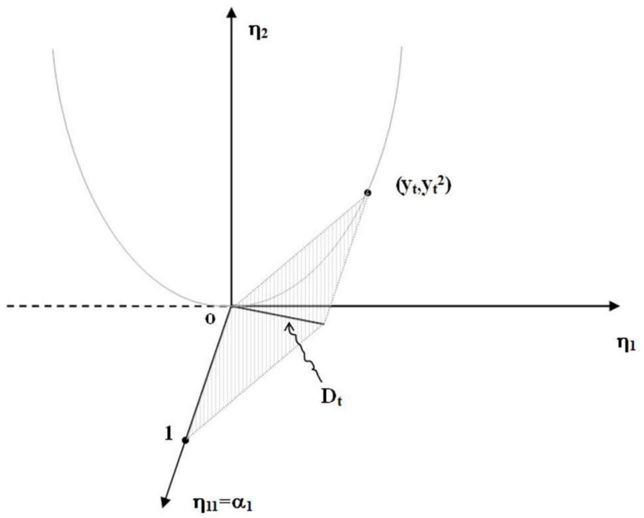

FIG. 2: Observed data submanifold D for fixed t

From this we deduce the last:

Proposition 4): The (EM)-algorithm, applied by statisticians to estimate the parameters of a model, is equivalent to the (em)-algorithm defined in Information Geometry by the e-projections and the m-projections.

Possible applications: The theoretical results proved in the cited work allow us to construct new statistical tests to detect the regime-switching in real phenomena, like price behavior, using geometric quantities as the dimension or the curvature of the statistical manifold of the parameters.

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