



# Finite geometric spaces, Steiner systems and cooperative games

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## Abstract

Some relations between finite geometric spaces and cooperative games are considered. The games associated to Steiner systems, in particular projective and affine planes, are considered. The properties of winning and blocking coalitions are investigated.

## 1 Introduction

A first study on relations between projective planes and cooperative games is in [38], where the points of the Fano plane are considered to be the players of a cooperative game and the lines are winning coalitions. A formalization of semi-simple cooperative games as a set of players with a set of winning coalitions is in [33]. Some relations between projective planes and cooperative games are introduced in [31]. The research of blocking coalitions gave rise to the geometric theory of blocking sets, studied by various authors (see, e.g., [2], [3], [4], [5], [6], [8], [9], [10], [11], [12], [22], [23], [24], [35], [37]).

In this paper we wish to deepen some aspects of the relationship between semi-simple cooperative games and finite geometric spaces. We introduce the concepts of cooperative games associated to geometric spaces and geometric spaces associated to semi-simple cooperative games. In particular the rank 2 geometries, with particular reference to Steiner systems, are investigated.

Some new results on blocking coalitions are presented.

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