Dealing with variability of natural soils

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Statistical soil characterization of Italian sites for reliability analyses

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ABSTRACT: The paper is a review of the most commonly used techniques and methods for the estimation of inherent variability and uncertainties concerning soil physical and mechanical properties estimation. The methods illustrated in the following as, among others, intraclass correlation coefficient method, the Vivatrat procedure, the Cusum method and the Bartlett modified method are described theoretically and applied to three types of Italian soils in order to give useful suggestions for common geotechnical design activity. Furthermore reliability approach is also presented and some classical methods are dealt with here as FORM (first order reliability method) and SORM (second order reliability method). Finally an example of the reliability analysis application to a pile design is presented and discussed. Accordingly the evaluation of the reliability index of the global safety factor is carried out in order to show that the variation the safety factor can suffer on its level of reliability by taking into account the variability structure of physical and mechanical soil properties.

1 INTRODUCTION

The most relevant aspect in geotechnical design is the evidence that physical and mechanical properties of natural soils and rocks are not known before the explorative campaigns are performed.

Whatever is the effort to accomplish a high quality in soil and rock characterization, as to take numerous data from a large amount of samples, the data sets collected are just statistically representative of the effective volume of soil and rock involved in construction. Moreover the characterization of natural soils and rocks for geotechnical engineering purposes is strongly affected by uncertainties and variability that shall be identified and quantified in order to make a good design as the safety aspect is concerned.

Accordingly the estimation of errors and uncertainties in sampling, testing and calculation steps and the recognition of the inherent variability structure of soil and rock properties should be a common practice. This paper deals with statistical procedures for soil variability investigation and characterization and discusses different sources of uncertainties in order to accomplish an adequate knowledge of reliability level in geotechnical design.

Examples from statistical characterization of three Italian soil types are described in the next sections. Moreover two reliability methods are presented in order to apply statistical soil characterization to the evaluation of the reliability level of geotechnical foundation design.

2 GENERAL CONSIDERATIONS ON DEVELOPMENT IN GEOTECHNICAL DESIGN

Over the past years geotechnical design activity was generally based on the "working stress design" approach (WSD) according to which the measure of the safety is the global safety factor (F) defined as the ratio between resistances (C) and actions (D): $F = C/D$.

Such approach, which played a central role in technical calculation is nowadays overcome because of:

– It causes ambiguity in defining the safety level of design according to assumptions on what actions and resistances should be accounted for.
– The global safety factor values for each problem are defined differently from country to country and generally they are calibrated by means of experiences made in specific fields.
– The global safety factor is characterized by the uncertainties, errors and variability coming from resistances and actions. Thus it is the measure of the ignorance about the design safety.

The studies of soil and rock materials developed over the last decades have demonstrated that homogeneous lithotypes show significant variability whenever sophisticated field and laboratory experiments were performed.

These evidences and the exigency of an up to date "Safety" concept according to the acquired knowledge on reliability methods and statistical procedures, gave birth and development to the new European technical provisions (named Eurocodes). Their applications throughout European countries have been spreading out the "partial safety factor approach" which pays attention on the "failure probability estimation" based upon reliability principles and methods. For Geotechnical interests EN 1997-1 (1997) and EN 1998 (2003) shall be considered.

At this stage it seems useful to state the more the scientific knowledge in geotechnical field develops the more the estimation of reliability of data increases its relevance. Accordingly it appears to be still valid the chain suggested by Harr (1977) as a metaphor of geotechnical designing process:

Sampling – Testing – Formulas – Experience = To build with confidence

Every time one of the steps becomes weaker than others the confidence in building activity decreases. Hence the level of confidence is dependent on the worst performance among the rings of the chain. Thus sophisticated design models, which represent an interesting field of research and application, should be employed whenever large and complete experimental campaigns take place.

The Eurocodes clearly stress the importance of the previous rule in spite of old national technical provisions but do not manage neither discuss the criteria for choosing physical models among numerous theoretical approaches which are available for the same geotechnical design. They are continuously refined by means of the enhancement of monitoring procedures and databases of measurements while the most imprecise and inadequate methods tend to be abandoned. Besides, up to now, only the engineering judgment and experience can be advocated to make the best model be chosen according to the design problem and the soil characteristics.

3 VARIABILITY AND UNCERTAINTY IN SOIL CHARACTERIZATION

Here some aspects of variability and uncertainty estimation concerning homogeneous soil deposits are analyzed. This paper starts from the hypothesis that whatever laboratory and in situ tests are performed the number of measures can be considered statistically meaningful and the sampling strategies employed can be regarded as efficient (Rethati 1988) provided that sounding and testing phases are correctly scheduled and designed.

Unfortunately on these last two issues, which are of extreme importance in designing practice, a limited part of scientific community is interested.

Let us consider the geotechnical variables as spatial distributed variates. If we assume that a data set is made up of measurements related to the depth (then the measured variable is considered as one dimensional spatial variate), such values k(z) can be divided into two components: a deterministic trend (spatial mean) t(z) and residual values (random fluctuations about the trend) w(z):

$$k(z) = t(z) + w(z) \tag{1}$$

There are many possible trends for the spatial mean over the depth t(z): it can be constant, linearly variable or it can follow an higher order mathematical law. Moreover, also the random part w(z) can be constant or variable over depth.

In order to study the variability structure of the measure k(z) it is necessary to focus the attention on the residuals w(z). These latter values shall be isolated by the trend and analyzed by looking for their spatial distribution provided that they have a zero mean value and a standard deviation σ to be calculated.

Accordingly the fluctuation scale or distance is an important measure of the distance over which the k(z) values show a strong spatial correlation. It is analytically expressed by:

$$\delta = 2\int_0^\infty \rho(z)dz \tag{2}$$

where $\rho(z)$ is the autocorrelation function for a distance "z". In order to calculate such statistic, measurements at narrow distance lag are needed. Thus for such study field test measurements are needed.

Besides, the following steps shall be undergone before assessing the mean trend, the variance and the fluctuation scale:

- To detect homogeneous formations by "ad hoc" procedures.
- To eliminate errors and uncertainties from measures in order to deal with inherent variability of soil data. This point cannot be undertaken because of the impossibility of distinguishing the three contributions to the dispersion extent.
- To analyze the data set consistency by identifying outlier values and deciding on their "destiny".

Over the years, many procedures have been developed to accomplish the item 1. Two methods among others are worthy to be mentioned:

- Wickremesinghe & Campanella (1991)
- Gill (1970)

Such methodologies are referred to as the statistical methods for *analysis of variance*. They test for differences between means of two or more populations whose elements are described in terms of various contributing "effects" by means of a linear model. The applicability of any particular model is determined both by the specific experimental design involved and by our underlying concept of the different effect components. In the case of one-dimensional measurements (commonly over depth) the "single-factor fixed-effect" model is appropriated. According to this model the *j*th observation of the *i*th zone (X_{ij}) can be expressed as the sum of three components:

$$X_{ij} = \mu + \alpha_i + e_{ij} \tag{3}$$

where μ is the mean of the entire population; α_i is the effect associated with its particular zone (i) and e_{ij} is the random error associated with each individual observation.

If mean values of sub-zones are significantly different, the variance of the combined zones, which reflects both the zone effect and the random error, will be larger than the variance of the separate groups, which reflects the random error alone.

Accordingly the first method uses the statistic called intraclass correlation coefficient RI (Webster & Beckett 1968). Wickremesinghe & Campanella (1991) developed a procedure which divides the measurement profile in "windows" of fixed width and the exposed portion of the data within the window is examined, with the central point of the window being potential boundary.

The extension of the window will be varied at each iteration of the procedure. At each iteration, assuming s_1 and s_2 as variances of the two samples lying above and below the "window" centre indicated with C, for the two samples with equal number of data points, n, a pooled combined variance (SC) is calculated:

$$SC = \frac{n \cdot (s_1 + s_2)}{2n - 1} \tag{4}$$

Then the intraclass correlation coefficient, RI, is given by the expression:

$$RI = \frac{SB}{SB + SC} \tag{5}$$

where SB is the variance of the combined sample of size 2n. At optimal layer boundaries RI will attain a peak. On the basis of the applications of this procedure carried out by the authors, the peak value of 0.8 indicates the presence of a layer boundary at the center of the window whereas at lower peak values (about 0.7–0.75) similar soil interfaces cannot be excluded at all.

The second method (Gill 1970) is based on the comparison with two independently computed estimates of variance:

$$B = \frac{1}{z-1}\left[\sum_{i=1}^{z} n_i \left(M_i - M\right)^2\right] \quad (6)$$

and:

$$W = \frac{1}{N-z}\left[\sum_{i}^{z}\sum_{j=1}^{n_i}\left(X_{ij} - M_i\right)^2\right] \quad (7)$$

where:
B = between-zone variance;
z = number of zones;
n_i = number of observations in zone (i);
M_i = mean of zone (i);
M = mean of the entire population;
W = pooled variance between zones;
N = total number of observations;
X_{ij} = *j*th observation in the *i*th zone.
Thus a zonation index can be computed as follows:

$$R = \frac{B - W}{B} \quad (8)$$

For homogeneous data set B = 0 and R is undefined, whereas for an optimal zonation W = 0 and R = 1. Furthermore any proposed zonation which does not yield $B \geq W$ should be regarded as arbitrary and not reflecting real zones.

Another issue to manage is related to the presence of "anomalous" values inside of data set measures. A value belonging to a data set is defined "anomalous" whether it is far less or far more than others. The anomalous values affect statistical moments of the data set but they are seldom discussed or eliminated on the basis of rational and systematic approaches. In order to detect anomalous values many are the available procedures (Cherubini *et al*. 1986; Rethati 1988).

Here two of them are illustrated:

- Box plot (Velleman & Hoaglin 1981)
- Vivatrat (Vivatrat 1979)

The Vivatrat technique can be applied directly to measurements which are spatially distributed, whereas the Box plot procedure treats single values of data sets without considering their spatial distribution.

The Box plot can be considered an efficient tool for detecting anomalous values and for providing the variability representation by means of a few indexes: the upper and lower adjacent values and the interquartile range. The adjacent upper and lower values can be defined as those measurements belonging to the data set that follow the expressions below:

$$\text{Upper adjacent value} = \text{Data set value} \leq \text{Upper Quartile} + 1.5 \cdot \text{IQR}$$
$$\text{Lower adjacent value} = \text{Data set value} \geq \text{Lower Quartile} - 1.5 \cdot \text{IQR}$$

where IQR is the interquatile range defined as:

$$\text{IQR} = \text{Upper Quartile} - \text{Lower Quartile}$$

All of the values which lie beyond the adjacent upper value and the adjacent lower value respectively can be considered "outliers". This occurrence does not really mean that such values must be eliminated even though they must be detected. However it is worth noticing that the dispersion of a data set, which is commonly measured by the standard deviation, can be strongly affected by anomalous data. Instead the estimate of the central tendency of a data set can be accomplished by two statistics that are the median and the trimean, which show not to be affected by the outliers.

The trimean is defined as follows:

$$\text{Trimean} = \frac{\text{Upper Quartile} + \text{Lower Quartile} + 2 \cdot \text{Median}}{4}$$

In the cases of measurement profiles performed over one direction (commonly depth), it could be useful to apply a “filtering” method in order to improve the geotechnical interpretation of the entire profile into mechanical soil variable. At this scope the Vivatrat procedure can be efficiently employed.

It consists of trying to select peaks in measurement profile that can be considered mechanically significant and eliminate the others. Such procedure results in flattening the profile and it can be described by the following items:

- To plot the unfiltered measures versus depth.
- To divide measures in layers of extension D: it is suggested to vary from 0.5 m to 2.5 m.
- To calculate the mean “μ” and the standard deviation “s” for each of the identified sub-layer.
- To calculate the representative dispersion “S_r”, which is defined as the minimum value among the following expressions:

$$S_r = \frac{1}{2}(S_{i+1} + S_i) \tag{9}$$

$$S_r = \frac{1}{2}(S_{i-1} + S_i) \tag{10}$$

$$S_r = \frac{1}{2}(S_{i+1} + S_{i-1}) \tag{11}$$

where S_{i-1}, S_i and S_{i+1} are the standard deviations calculated for sub-layers $i-1$, i and $i+1$ respectively.

- To eliminate the measures which lie beyond the following limit values:

$$\mu_i \pm A \cdot S_r \tag{12}$$

where μ_i is the mean value within the sub-layer i, S_r is the characteristic standard deviation and A is the coefficient of the limiting band which can be assumed belonging to the range (0.5; 2.5). The A and D values can be calibrated by varying them from 1 m to 2 m.

4 DATA TREATING TECHNIQUES

As far as the Equation 1 is concerned the subdivision of measurement values into trend and fluctuation parts represents the starting point for statistical data treatment aimed at spatial random structure investigation. The random field theory states that fluctuations can be modeled as a zero-mean stationary random field. This means that the trend removal, commonly performed by means of least square regression procedures, should produce a zero-mean stationary random field. Nevertheless the stationarity assumption must be verified whereas only in the weak sense.

At the scope two useful techniques are suggested here:

- Modified Bartlett test (Phoon et al. 2003)
- Cusum test (Caulcutt 1983)

The first technique was proposed and illustrated by Phoon et al. (2003) and its explanation is here briefly presented. The modified Bartlett test is based on the classical one (Kanji 1993), which consists on comparing couple of windows in terms of sample variances. The width of the windows which correspond to the number of measurements considered along the profile are iteratively increased. Thus the statistic of Bartlett’s test can be expressed as follows:

$$B_{stat} = \frac{2.30259 \cdot (m-1)}{C}\left[2\log(s_1^2) - \left(\log(s_1^2) + \log(s_2^2)\right)\right] \tag{13}$$

where m is the number of values included in two adjacent windows, s_1^2 and s_2^2 are the variances of two adjacent windows; s^2 is the total variance:

$$s^2 = \frac{s_1^2 + s_2^2}{2} \tag{14}$$

and C is a constant given by:

$$C = 1 + \frac{1}{2(m-1)} \tag{15}$$

To ensure that the sample variances in each window can be reasonably accurately estimated there are few rules to be satisfied:

– The minimum number of data in each window is 10 (Lacasse & Nadim 1996).
– The Bartlett statistic is computed by comparing two sample variances, thus the total point number in the soil record n must exceed 2 m.
– The number of points in one scale of fluctuation k is taken to lie between 5 and 50.
– The Bartlett statistic profile should be compared with a peak value B_{max} that the statistic should not overpass. The critical value of B_{max} presented by Phoon et al. (2003) corresponds to a 5% level of significance of the test, which is the most commonly used.

The second technique is proposed to verify the weak stationarity of the residuals from a measurement set. It is called Cusum (whose name is the abbreviation of "cumulative sum").

It is based on a graphical representation of the fluctuations by means of plotting the calculated sum of the data deviations from the trend function. Then a test at two significance levels (5% and 1%) shall be performed to verify that this sum is less than a critical value. Given a profile of measures, the steps to undertake can be summarized as follows:

1. To remove the trend to get deviations at each depth.
2. To add up at each depth the preceding deviations, to get the "Cusum".
3. To plot the Cusum against the batch number.
4. To reveal the changes in the slope of the graph by means of visual inspection: the part of the graph within two slope changes is called "section" of the Cusum graph. The slope of each section is related to the "mean fluctuation" for the data set in that section. The positive slope tells us that the mean fluctuation is above the trend whereas a negative slope means it is under the trend. The mean fluctuation should be much closer to the local means of the data set than the overall mean.
5. To plot the slopes of the sections against the batch number and to calculate the localized standard deviation according to the following expression:

$$\text{Localized standard deviation} = \sqrt{\frac{\sum (x_c - x_{c-1})^2}{2(n-1)}} \tag{16}$$

were x_c is the deviation obtained subtracting the fluctuation value from the Cusum value at each depth; x_{c-1} is the deviation preceding the x_c and n is the entire number of the batch. Thus, even if the process mean fluctuation has changed n times during the batch, the process variability structure could be changed or not.
6. In order to estimate whether the fluctuations are stationary the statistical test must be performed. Such test uses a statistic which is the ratio between the maximum value of the Cusum throughout the entire batch and the localized standard deviation.
7. Finally the calculated values of the statistic must be compared with the critical values of the test at two significance levels.
8. If the test value is lower than the critical value, the fluctuations can be considered a stationary random field and their random structure can be investigated.

5 EXAMPLES OF SOIL CHARACTERIZATION IN ITALIAN SITES

Examples of statistical applications in Italian soil characterization are illustrated here that are taken from the authors' experiences. They concern two types of in situ measurements that are cone penetration tests (CPTs), piezocone tests CPTUs) and down hole tests (DHs). These geotechnical in field techniques rely on different principles but provide the same advantage from a statistical point of view because they give measurement profiles with depth (in the case of cone penetration tests, these measures are practically continuous).

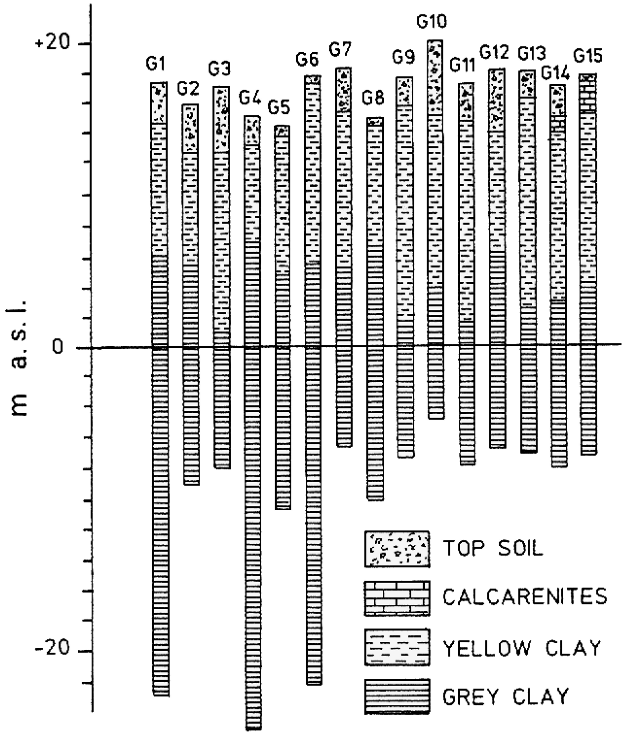

Figure 1. Stratigraphies from cone penetration testing.

This feature allows to correctly investigate the variability structure of physical and mechanical properties of bored soil layers and allows to improve old methodologies and possibly formulate new procedures for statistical in situ measurement interpretation.

The case studied relate to three sites. Two of them are set in southern Italy (Apulia region):

- Taranto district, where clayey soils were investigated (Cafaro & Cherubini 2002).
- Brindisi town, where silt mixture soils were taken into account (Cherubini & Vessia 2006).

and one site is set in the central part of Italy (Tuscany region):

- Fivizzano city set where alluvial soils were studied (Cherubini & Vessia 2004).

5.1 *Taranto clayey soils*

In the case of Taranto clays, they are stiff overconsolidated clays of mainly illitic mineralogical composition. Figure 1 shows the stratigraphies corresponding to the performed boreholes which show the presence of two clay horizons.

As a matter of fact, this soil is characterized on a regional scale, by a brownish-yellow upper horizon and a grey lower horizon. Both clays can be classified as clayey silt to silty clay on the basis of the correlation between fines content (FC) and "soil behaviour type index", I_c (Robertson & Fear 1995).

The vertical strength variability in Taranto clay has been defined by means of statistical treatment of data coming from CPTs. Cafaro et al. (2000) have analyzed the scale of fluctuation with depth of five cone bearing profiles for each horizon of the Taranto clays.

Figure 2 shows the values assumed by the fluctuation function with increasing lag distance for the G1 cone bearing profile within each horizon. In Table 1 the trend and the scale of fluctuation of the cone bearing profiles along the five boreholes for both the upper (a) and the lower (b) clay are listed.

The residuals were obtained by removing a low-order polynomial trend, no higher than a quadratic, by regressing the cone bearing values using the ordinary least-squares (OLS) method. The first problem, at this step, is identifying the most suitable trend removal technique. Since the set of residuals, which are used to calculate the fluctuation scale, varies according to the trend removal technique employed,

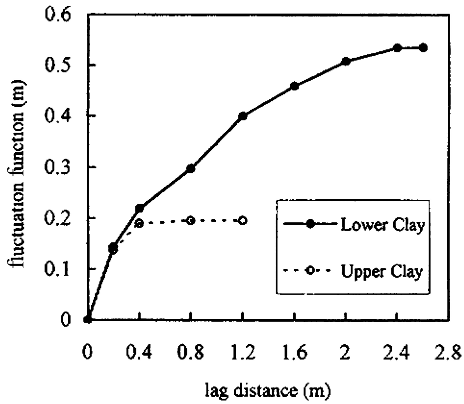

Figure 2. Scale of fluctuation of G1 cone bearing profile.

Table 1. The trend and the scale of fluctuation of cone bearing within upper and lower clay.

Cone penetration	Trend	Scale of fluctuation (m)
Upper clay		
G1	$y = 54.671x^2 - 21.21x + 5301$	0.195
G3	$y = 12.44x^2 + 113.06x + 2950$	0.401
G6	$y = 40.713x^2 - 439.7x + 5601$	0.207
G7	$y = 73.690x^2 - 172.2x + 9753$	0.401
G15	$y = 11.027x^2 + 212.3x + 2541$	0.436
Lower clay		
G1	$y = 149.11x + 4732$	0.536
G3	$y = 319.58x + 1722$	0.287
G6	$y = 201.29x + 3700$	0.720
G7	$y = 201.14x + 4036$	0.269
G15	$y = 203.34x + 3699$	0.185

quality of a set of residuals shall be controlled as concern its stationarity property. It was achieved by "Cusum analysis" (Caulcutt 1983).

Figure 3 shows the Cusum plot of residuals from the regression equation against depth, for both linear and quadratic trend removal, for the G1 cone profile within the upper clay. It can be seen that it is clearly preferable to remove a quadratic trend. Thus, as expected, the average fluctuation of the cone profiles within the upper clay is higher whenever a linear trend respect to a quadratic trend is used (Table 2).

Moreover the autocorrelation function of q_c was investigated according to the following expression (Campanella et al. 1987; Jaksa et al. 2000):

$$\rho_k = \sum_{i=1}^{N-k} (X_i - X_{av})(X_{i+k} - X_{av}) / \sum_{i=1}^{N} (X_i - X_{av})^2 \tag{17}$$

where N is the number of data, X_{av} is the mean of the data X_i, and the autocorrelation is calculated for different lags k.

Figure 4 shows the autocorrelation functions from both original and detrended data for the G3 lower cone profile. Among the expressions proposed by Vanmarcke (1977) for the autocorrelation function model, the estimated autocorrelation curve for the case studied follows the equation below:

$$\rho_z = e^{-a(z)} \cos(2\pi bz) \tag{18}$$

where $a = 1.18\ m^{-1}$, $b = 0.419\ m^{-1}$ and z is the correlation distance. This fitting curve, although roughly in line with the values calculated from detrended data, seems to indicate the same correlation distance as that coming from the experimental data points.

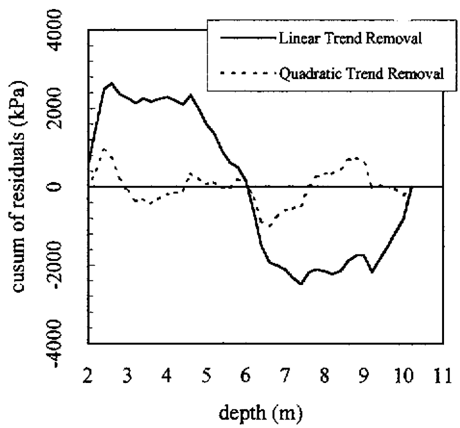

Figure 3. Cusum plot of cone bearing residuals for both linear and quadratic trend removal (G1 upper profile).

Table 2. Influence of the trend removal on scale of fluctuation of cone bearing within upper clay.

Cone profile	Scale of fluctuation (m) for linear trend removal	Scale of fluctuation (m) for quadratic trend removal
G1	0.42	0.20
G3	0.40	0.40
G6	0.60	0.21
G7	0.96	0.40
G15	0.47	0.44

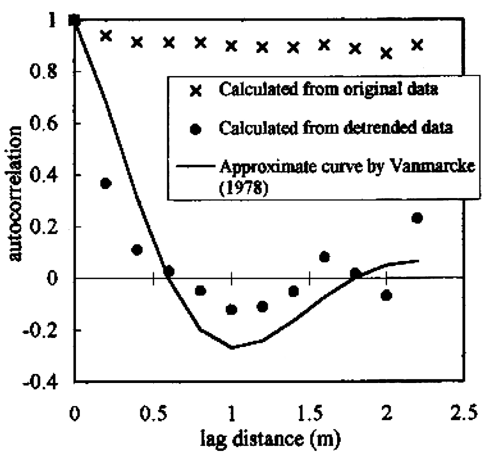

Figure 4. Autocorrelation function for G3 lower cone profile.

Indeed the decay of the autocorrelation values calculated for detrended data leads to a correlation distance of about 0.65 m; basically the same distance as that deduced from the approximate curve. Furthermore geostatistical boundaries of the two Taranto clays were investigated by means of intraclass correlation coefficient RI (Wickremesinghe & Campanella 1991).

Figure 5 shows the comparison between RI profiles coming from processing the q_c, f_s and I_c profiles along G1. The main peak is near the lithological boundary for the profiles coming from q_c and f_s, but not for the one coming from I_c. However, the secondary peaks of the RI profile coming from I_c correspond better to the secondary peaks of the other profiles.

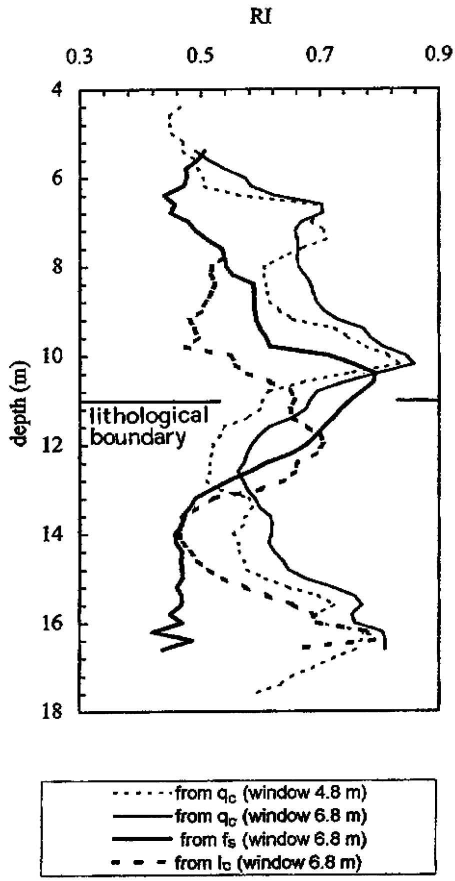

Figure 5. Intraclass correlation coefficient profiles of cone bearing, sleeve friction and soil behaviour index along G1.

These differences may depend on the fact that the strength trend in the upper clay layer is more complex than that in the lower clay layer, making the normalization process difficult. Clearly a geomechanical boundary is identified between strata by means of the intraclass correlation coefficient calculated on either the cone bearing or sleeve friction profile. This is likely not possible with the same results on an I_c profile, due to the uncertainties over its normalization.

Moreover, the distance between boundaries obtained by processing the cone bearing profile and the sleeve friction profile, in the same G1 borehole, is no more than 40 cm. This offset, at a depth of 11 m, can be neglected from an engineering point of view. The increasing of the calculation window for q_c from 4.8 m to 6.8 m does not seem to affect the position (depth) or the value of the peaks.

5.2 *Brindisi silt mixture*

The second case studied concerns silt mixtures from Brindisi urban area. The area where Brindisi is set, geologically belongs to Calabrian Age and is made up of silty clay and clayey silt soils whose properties are quite more variable than those from Bradanic Foretrough (Cherubini et al. 1987).

Such silt mixture can be lithologically classified from the bottom to the surface as clayey silts, silty clay and gray-blue clayey-sandy silt up to yellow sands respectively. Table 3 summarizes physical parameters of soils from Brindisi area. For each property the calculated coefficients of variation are about the upper limit values of the coefficients of variation range reported by Phoon et al. (1995).

The measures of the effective strength parameters are determined by means of direct shear tests that gave a mean value of 35 kPa for effective cohesion and 23° for shear strength angle. Unconsolidated undrained triaxial tests show undrained shear resistance s_u varying from 100 kPa for samples at 10 m

Table 3. Physical parameters for Brindisi silts and clays.

Parameter	Mean value	Coefficient of variation
Natural water content w_o	28%	26.4%
Unit volume weight γ_n	20 kN/m^3	4.9%
Silt content	46%	23.5%
Clay content	36%	20%
Liquid limit, LL	43%	21.8%
Plastic limit, PL	18%	20.7%

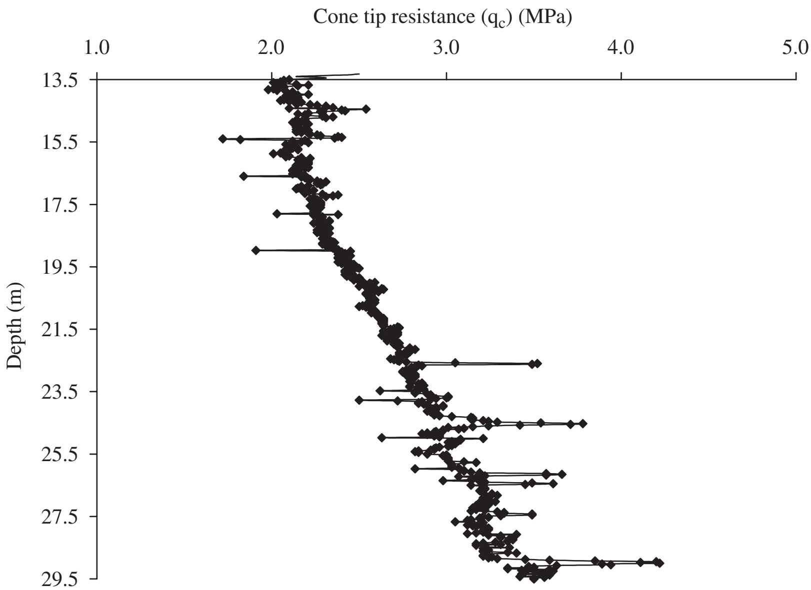

Figure 6. Cone tip resistance profile from CPTU in Brindisi area.

depth to 250–270 kPa for samples at 25 m depth. According to the performed piezocone tests (CPTUs), the q_c trend confirms that these soils are lightly overconsolidated. Profiles of q_c from CPTUs can be usefully employed to estimate the spatial mean trend and the variability structure of undrained shear resistance s_u. It is affected by the variability of $q_c - \gamma z$ and of N_k (cone factor) according to the following expression:

$$s_u = \frac{q_c - \gamma z}{N_k} \tag{19}$$

where N_k variability structure results to be independent of $(q_c - \gamma z)$'s.

One of the q_c profiles shown in Figure 6 is made up of two layers: the sandy upper layer which rests on alternation of clayey silt and silty clays. Since the top sandy layer is not frequently found in Brindisi area here just the lower part of the profile is studied in order to characterize the undrained shear strength of the silt mixtures where pile foundations would be set. In this case the Robertson index I_c (Robertson & Fear 1995) values in the lower part of the profile fall within the range of silt mixtures, that is clayey silt to silty clays. Hence the first step of the work is to try to smooth $q_c - \gamma z$ profile that is rich in peaks. Two paths can be followed:

- Try to detect one or more boundaries from homogeneous soil sub-layers or
- To smooth the profile by eliminating the odd measurements called "outliers" that affect the variability structure of $q_c - \gamma z$. Those values can be related to the errors of measurement execution.

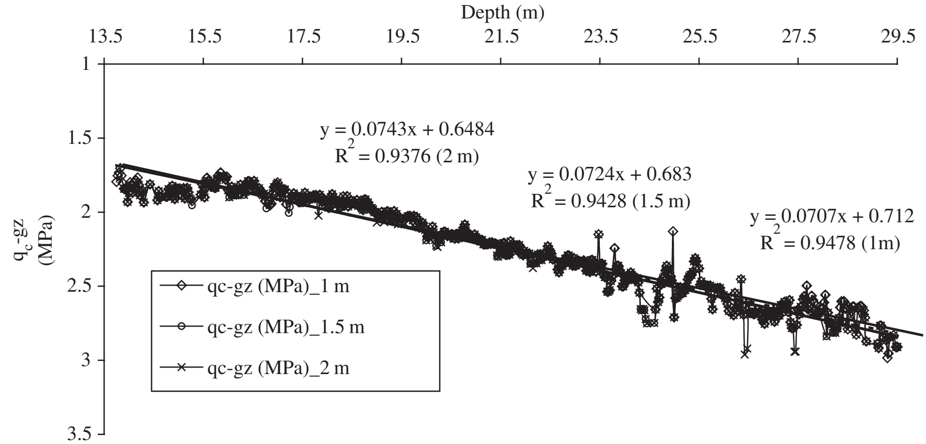

Figure 7. Vivatrat procedure to eliminate outliers in $q_c - \gamma z$ profile.

The first option is undertaken with no results by means of intraclass correlation coefficient RI, because of the uniform distribution of the silty mixture over the depth of investigation.

The second option has given good results by means of the Vivatrat filtering procedure (Vivatrat 1979). This procedure was employed to filter $q_c - \gamma z$ values.

Figure 7 shows the filtering effect for different values of D and A. As can be seen the trend functions recognized by means of the three sampling windows are similar with a similar correlation index R^2. The Vivatrat procedure was then compared with results of Box plot procedure which was successfully employed by Cafaro & Cherubini (2002) and Cherubini & Vessia (2004). Results (which are not presented here) show similar trend functions to the Vivatrat ones but with slightly lower correlation indexes. Hence Vivatrat results are then employed in this variability analysis.

At this stage the second step of the variability study is to assess the structure of the inherent variability of $q_c - \gamma z$. Accordingly it is necessary to verify the weak stationary of $q_c - \gamma z$. This purpose is accomplished by the modified Bartlett test (Phoon et al. 2003). A continuous Bartlett statistic profile is provided by moving a sampling window over the residuals of $q_c - \gamma z$ (Fig. 8).

The scale of fluctuation drawn from $q_c - \gamma z$ profile is 1.6 m. That value belongs to the range indicated by Phoon et al. (1995) that is 0.1 m–2.2 m for clayey soil. In order to perform the Bartlett modified test, three rules are taken into account as explained in section 4. For the case studied m, k and n parameters are $m = 13$; $k = 6$; $n = 218$. The critical value of B_{max} presented by Phoon et al. (2003) corresponds to a 5% level of significance of the rejecting of the null hypothesis of weak stationarity. For single exponentially correlated soil profile and for m, k and n defined above, it is:

$$B_{crit} = (0.36k + 0.66)\ln\left(\frac{n}{k}\right) + 1.31k - 1.77 \tag{20}$$

Figure 9 shows the results of modified Bartlett test application: all of the residuals are under the B_{crit} so that the variability study carried out on residuals is correct. The autocorrelation function from the $q_c - \gamma z$ residuals was estimated and compared with autocorrelation models suggested by literatures (Phoon et al. 2003). A single exponential form for autocorrelation function was estimated, according to the Figure 10:

$$R = e^{-\lambda|x|} = e^{-1,2345x} \tag{21}$$

To complete the description of the variability structure of $q_c - \gamma z$, according to the simple exponential autocorrelation function of $q_c - \gamma z$, Vanmarcke (1984) suggests the expression for the Variance reduction function Γ^2 which is:

$$\Gamma^2 = 2 \cdot \left(\frac{R}{T}\right)^2 \cdot \left(\frac{T}{R} - 1 + e^{-T/R}\right) \tag{22}$$

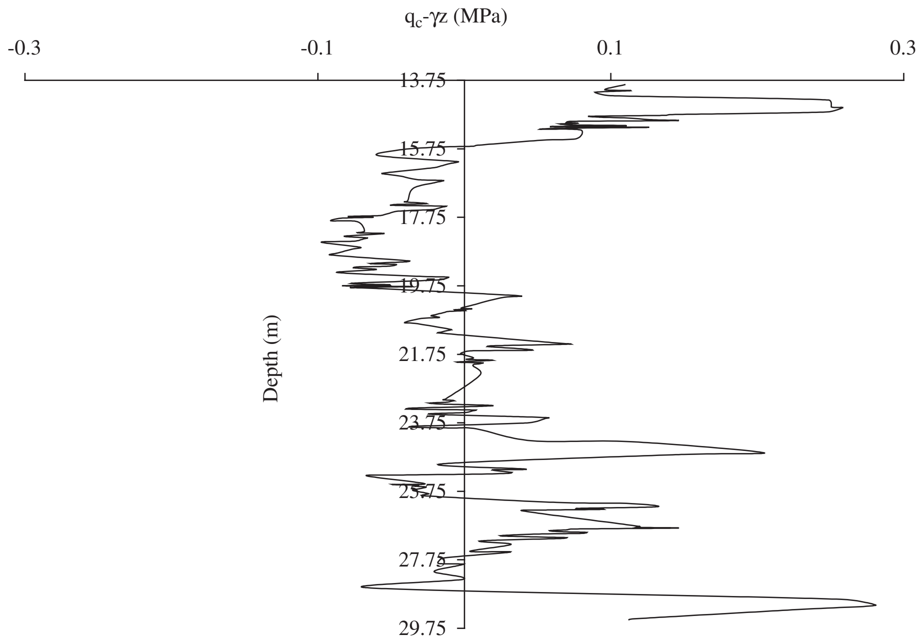

Figure 8. $q_c - \gamma z$ residual trend with depth.

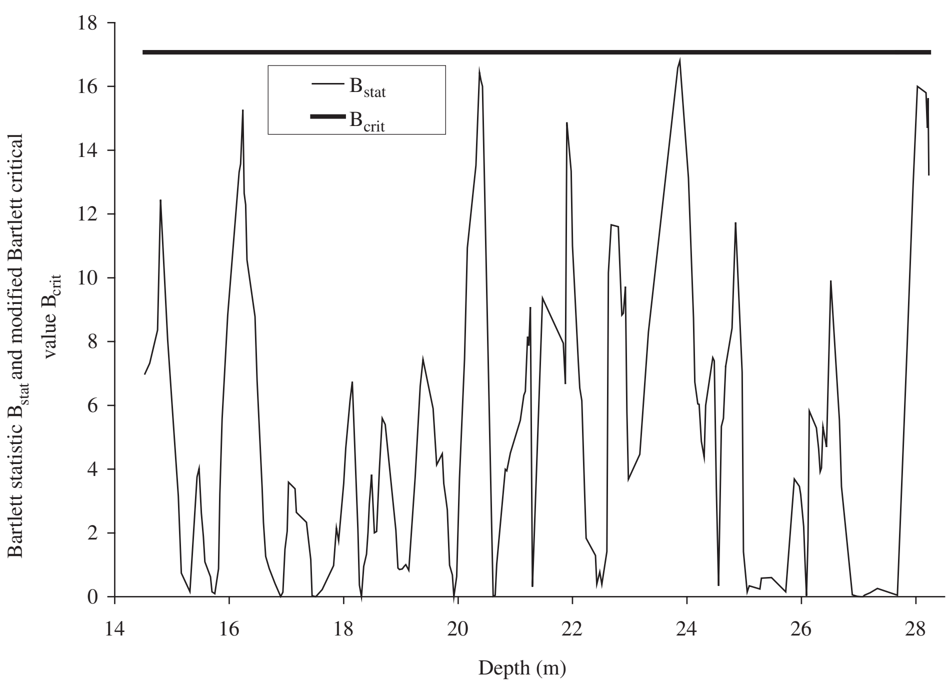

Figure 9. Continuous Bartlett statistic B_{stat} and Modified Bartlett critical value B_{crit} corresponding to single exponential autocorrelation function.

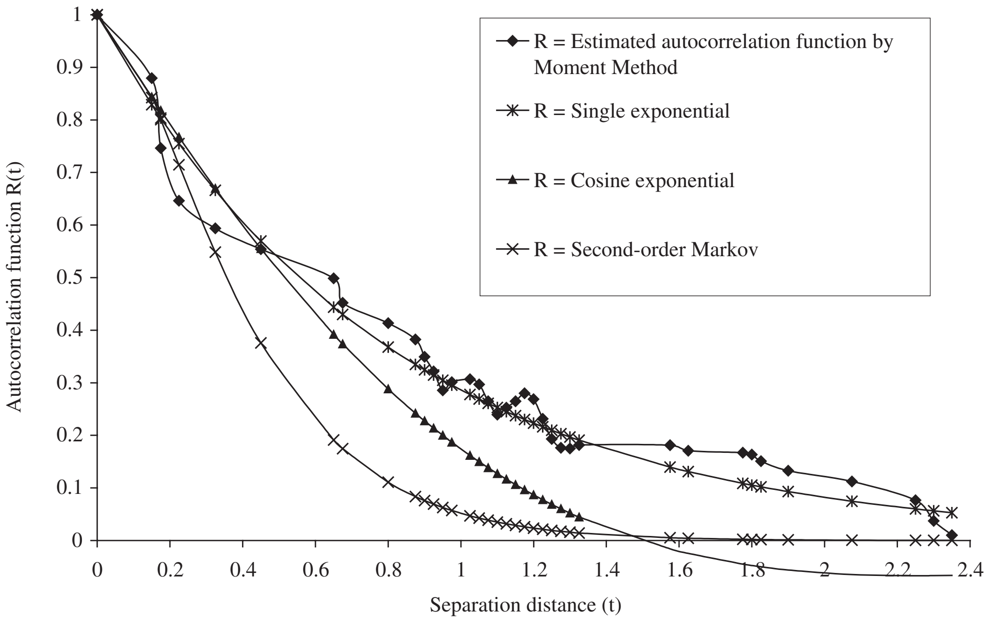

Figure 10. Autocorrelation function model and estimation.

where R is the autocorrelation distance of a one-dimensional variability structure and T represents the size of the average volume which means the length of the foundation structure to be designed.

Moreover the variance of this special average can be correlated to the point variance σ^2 by means the variance reduction function Γ^2 according to Vanmarcke (1984):

$$\sigma_T^2 = \Gamma_T^2 \sigma^2 \tag{23}$$

5.3 *Alluvial deposits at Fivizzano*

The third case studied concerns two Down-Hole tests (DHs) carried out for dynamically characterization of superficial deposits at Fivizzano town, set in Lunigiana area. These DHs are part of a larger project known as "VEL" supported by the Tuscany Region with the scope of evaluating the local seismic effect throughout the region. One dimensional analyses were carried out in Fivizzano town over two seismically profiles characterized by these Down-Hole tests (Cherubini et al. 2004) called DH1 and DH2. The two experiments were carried out over 34 m and 61 m respectively.

The sampling showed two lithologies represented by DH1 and DH2, as can be seen in Table 4. The deterministic interpolation of a travel-time plot to calculate the P and SH wave velocities is shown in Table 5. For the probabilistic study proposed here, relationships between P and SH wave velocities and depth were built by the regression curve method.

The analysis considered only soil layers which means that in the DH2 the rocky "Macigno" formation was not taken into account. This choice is due to the fact that for numerical analysis, this formation represents the bedrock so that it will not be considered as a layer but as an interface. Before applying the regression curve method and the residual treatment the Box Plot procedure (Velleman & Hoaglin 1981) was employed in order to clear the data set from the outliers. After that, a research of geostatistical interfaces by means of intraclass correlation coefficient RI was carried out.

As can be seen in Figure 11, the maximum peak value of RI of P wave time arrivals for both DH1 and DH2 were recorded at the same depth recognized by means of sampling activity (Table 4) that is 28 m and 37 m respectively.

It is interesting to note that the SH wave arrival time does not show the presence of the interface in travel-time curves as P wave arrival times do. This is referred to in literature (Pergalani & Signanini 1984) because the SH waves reveal fewer interfaces than P waves. After doing this, the trend functions were investigated by means of a regression curve method using linear and quadratic curves. The Cusum

Table 4. Lithological characterization of two boreholes in Fivizzano.

Depth (m)	Lithological profiles reconstructed from sampling activity
DH1	
0–0.3	Fill soil
0.3–29.2	Alluvial deposit
29.2–34	"Canetolo" clay and limestones
DH2	
0–1.3	Fill soil
1.3–37	Alluvial deposits
37–60	"Macigno" formation

Table 5. P and SH velocity values according to the deterministic analysis.

	Wave velocity deterministic values drawn from travel-time curve	
Depth (m)	V_P (m/sec)	V_{SH} (m/sec)
DH1		
0–2	390	275
2–13	770	455
13–29	1575	640
29–34	1960	890
DH2		
0–13	1100	605
13–18	1330	870
18–28	1280	700
28–35	1540	780
35–39	2150	1000
39–51	2220	1400
51–61	2570	1780

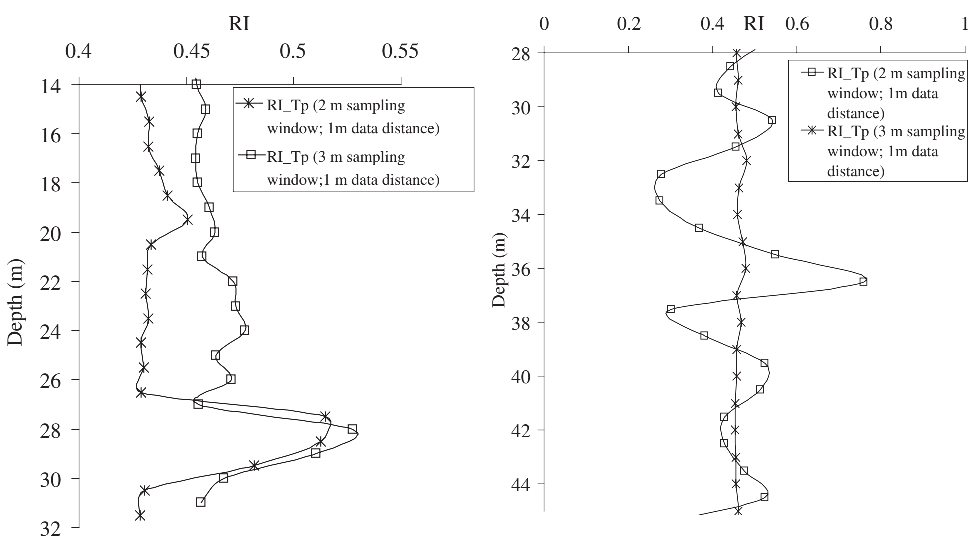

Figure 11. RI calculated values versus depth: the peak maximum values mean a geostatistical interface.

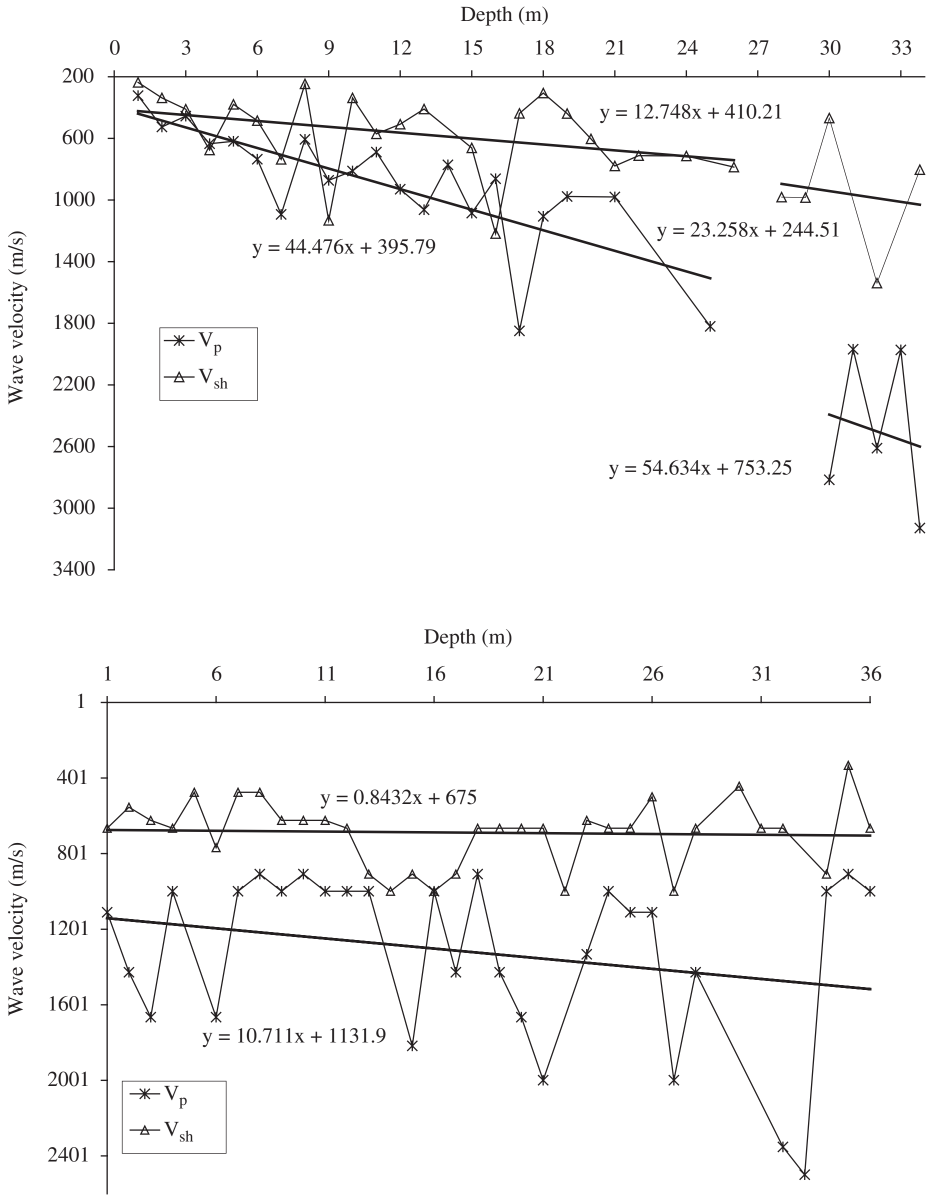

Figure 12. DH1 (upper graph) and DH2 (lower graph) trend functions sketched on the basis of geostatistical analysis.

analysis was then performed to test the stationary condition of the residual random field. Results in terms of trend functions for P and SH wave velocities are illustrated in Figure 12 and their representative values are listed in Table 6. The residuals were used to calculate the velocity fluctuations.

The residual random field is described by means of the Coefficient of Variation, the scale of fluctuation which is shown in Table 6 and the autocorrelation function not reported here. The scale of fluctuation is estimated by means of Vanmarcke's simplified method (1977) and the autocorrelation function is calculated by Equation 17. It is worth pointing out that the calculation of the autocorrelation function and the scale of fluctuation need numerous data. It implies that for the second layer in DH1 (from 28 m to 36 m depth) where there are a few data the estimation of the autocorrelation function and its scale of fluctuation were not carried out.

Table 6. P and SH mean velocity values, coefficients of variation and scale of fluctuation according to the geostatistical analysis.

Depth	Wave velocity values drawn from probabilistic analysis		Local coefficient of variation (m) (%)		Scale of fluctuation (m) and variance function (a dimensional)		Reduced coefficient of variation (%)	
(m)	V_P (m/sec)	V_{SH} (m/sec)	CV_{Vp}	CV_{Vsh}	δ_{Vp}/Γ	δ_{Vsh}/Γ	CV_{Vp}	CV_{Vsh}
DH1								
0–28	910	570	26.9	41.7	1.8/0.25	1.4/0.22	6.8	9.2
28–34	2500	960	20.4	40.2	–	–	–	–
DH2								
0–36	1320	690	32.9	25.1	2.0/0.24	2.6/0.28	7.9	7.0

Finally results of the study can be here briefly synthesized:

– differences are recorded in mean velocity values between deterministic and probabilistic procedure (unless from probabilistic standpoint linear functions were calculated and multiple mean velocity values can be used over the entire depth instead of one representative value from the deterministic approach);
– the variabilities about the trend are reduced by the variance function (Vanmarcke 1977) in order to account for the spatial averaging of the residual random field (Table 6).

6 MONITORING AND VERIFICATION OF GEOTECHNICAL STRUCTURE PERFORMANCE

A fundamental question to formulate about the engineering modelling could be expressed as follows: "It is possible to verify the capacity of numerical model for adequately representing phenomena and predicting geotechnical structure response?"

"Adequate" should mean the ability for a model of considering only those variables that are relevant within the studied phenomenon. From a philosophic standpoint two interesting positions on scientific knowledge could be recalled:

– Popper (1970), in contrast to the concept of verification proposed by scientists from the 20th school of Vienna, states that a model can be considered scientifically valid whether it can be falsified. Indeed the scientific knowledge is like a pile dwelling whose foundation can be deepened after every falsification process.
– Feyerabend (1984), exponent of the scientific anarchism, thinks scientific theories cannot be validated by means of comparison and although a theory would fail, it always does not seem to be the best way to leave it at all.

However the issue seems to be unsolvable from a philosophical point of view while could be easier managed by the engineering standpoint through statistical tools in order to seek a solution to the "validation" process.

Let us consider a data set made up of numerous measurements. The point is to apply statistics to validate calculation models (i.e. settlement theoretical formulation) comparing their results with data from monitoring activity. In order to accomplish the scope it is needed to take account of errors and uncertainties which are often of a large extent.

Moreover the engineering solution reveals some weaknesses as:

– The lack of large amount of monitored data which should be useful to collect.
– The lack of detailed description of monitoring conditions like the soil characteristics, the layer setting, the geometrical characteristics of the foundations considered. Thus monitored data which are time by time selected and collected do not guarantee either the reliability of measures and, at the same time, either the validation of calculation models.
– The lack of monitored data related to different soils from different countries. This problem comes from the poor detailing of monitoring conditions and produces loss of generality in validation process.

In spite of difficulties explained above, over the last twenty years, validation studies were attempted by means of monitored data from real size models.

Several measurements of settlements concerning the serviceability limit state (SLS) (Orr & Cherubini 2003) are available (carried out especially in the USA) whereas the bearing capacity measurements of driven piles are seldom performed and they can be less reliable than the settlements.

As the settlement measures of shallow foundation on sandy soils are concerned, the validation process is developed on the variable K, which is the ratio between calculated value (Q_{calc}) of settlements and the measured ones (Q_{mis}):

$$K = \frac{Q_{calc}}{Q_{mis}} \tag{24}$$

The Q_{calc} relates to different formulations by which settlements on the same type of soils are estimated and, for each formulation, Q_{calc} relates to different foundation dimension settlements. The amount of Q_{calc} represents a data set of measurements which are normalized respect to the corresponding Q_{mis}.

For the validation process a relevant number of monitored data and numerous calculation models are needed in order to treat K data set by statistical tool. At this scope two statistics can be employed for evaluating data set properties:

- The accuracy which is connected to the central trend or the mean value of K data set.
- The precision which is associated with the dispersion of K data set.

The accuracy corresponds quantitatively to the classical mean value or to the trimean (this last statistic results to be more robust than the mean because it is scarcely affected by anomalous values belonging to the data set).

The precision is connected with the standard deviation or with the coefficient of variation as well as to the interquartile range.

The more the mean or trimean of K is about the unity the more the calculation model is accurate; whereas the less the coefficient of variation or the interquartile range the more the calculation model is precise.

An interesting point to wonder about, which is beyond the scope of this paper, is to investigate the nature of anomalous data, called “outliers”, which are values fairly lower or higher than the others.

Let us now consider a data set of K values: the first step of the validation process consists of deciding whether or not the “outliers” shall be eliminated. Such step can be undergone by means of convenient procedures as proposed by some researchers (Briaud & Tucker 1988; Cherubini & Orr 2000). Two indexes were defined to synthetically measure both accuracy and precision, which is the Ranking Index:

$$RI = \mu\left|\left[\ln\left(\frac{Q_{calc}}{Q_{mis}}\right)\right]\right| + s\left[\ln\left(\frac{Q_{calc}}{Q_{mis}}\right)\right] \tag{25}$$

or the Ranking Distance:

$$RD = \sqrt{\left[1 - \mu\left(\frac{Q_{calc}}{Q_{mis}}\right)\right]^2 + \left[s\left(\frac{Q_{calc}}{Q_{mis}}\right)\right]^2} \tag{26}$$

where μ is the mean and s is the standard deviation.

Whenever the mean of K data set is less or higher than the unity (which means that calculation methods results to be conservative or non conservative respectively) it is possible to use a coefficient to correct the calculation model: it is lower than the unity if the K mean is higher than it and vice versa.

This procedure shows that the correction coefficient for accuracy pertains to the errors of the models employed for the settlement prediction whereas the variability and the uncertainty relating to the input data affect the precision values.

7 FACTOR OF SAFETY VS RELIABILITY APPROACHES

7.1 *Basic considerations*

The limit state design approach (LSD) introduced by Eurocodes represents a sort of revolution respect to the working stress design (WSD) approach which was commonly used all over the Europe. Two main methodological differences between LSD and WSD can be pointed out:

– The WSD employs global safety factors calibrated by the different design experiences performed throughout European countries whereas the LSD considers partial safety factors calibrated by statistical approaches and applied to action and strength distributions.
– The LSD introduces "characteristic values" concept for design values of loads and strengths defined on statistical bases (i.e. EN 1990 2004) while the WSD deals with "nominal values" for design parameters without taking care of how they are determined.

To such a revolution in technical provisions a great contribution was given by the developments achieved in estimating and quantifying both the inherent variability of physical-mechanical properties of natural soils and the nature and the sources of uncertainties affecting experimental measurements for civil engineering design. As they concern the uncertainties due to systematic errors in measurements and to transformation laws, they shall be considered more accurately because they contribute on final dispersion of designing variables in different manners. Measurement errors can be distinguished in two contributions as the sample disturbance and the phenomenological model used to transform the measured parameter in design variables.

Even though the two contributions of measurement errors are difficult to be treated separately an attempt to evaluate the model errors can be empirically performed by comparing theoretical predictions (accomplished by the model investigated) with real size measurements (i.e. settlements).

Furthermore especially for geotechnical structures, it is useful to use the probabilistic approaches for soil characterization in order to evaluate the failure probability of design according to different failure mechanisms (called design models) which can occur over the lifetime of the structure under different loading conditions. Such an approach is well known as Reliability-based design. In such approach the estimation of the probability density function (pdf) of design variables plays an important rule and numerous statistical tests were developed at this aim. Nonetheless very often the hypothesis of Gaussian distribution is assumed without verifying its correctness. In spite of this practice as far as mechanical properties of soils and rocks are concerned the Lognormal or the Beta distribution better fit geotechnical data than the Gaussian one (Baecher 2003).

In the next paragraph a brief explanation of theoretical bases of classical methods for reliability assessment is illustrated and a geotechnical application is carried out for the case of a horizontally loaded pile.

7.2 *Classical reliability methods*

Structural and geotechnical reliability problems are usually described by the so-called limit state function g(x). The argument x of the function g is a random vector $X = (X_1, X_2, \ldots, X_n)$ consisting of basic random variables defining loads, material properties, geometrical quantities, etc. as well as some other properties considered as deterministic.

The hypersurface corresponding to the equation $g(x) = 0$ is called the *limit state surface*. As a reliability measure the probability of failure is used:

$$p_F = \int_{\{g(x)<0\}} f_X(x)dx \tag{27}$$

Here f_X denotes a multidimensional joint probability density function (p.d.f.) of the random vector X. In the special case if X is a Gaussian random vector with uncorrelated components X_i, $i = 1, \ldots, n$, a linear transformation of the coordinate system, known as the standardisation, is convenient to use:

$$y_i = \frac{x_i - E(X_i)}{\sigma_{X_i}}, \; i=1,\ldots, n \tag{28}$$

where E(Xi) and σ_{Xi} are the expected value and the standard deviation of the random variable Xi, respectively. The corresponding mapping of the limit state surface $g(x) = 0$ is as follows:

$$G(\mathbf{y}) \equiv g(\mathbf{x}(\mathbf{y})) = 0 \tag{29}$$

If moreover, the limit state function g is a linear one, than G will remain linear.

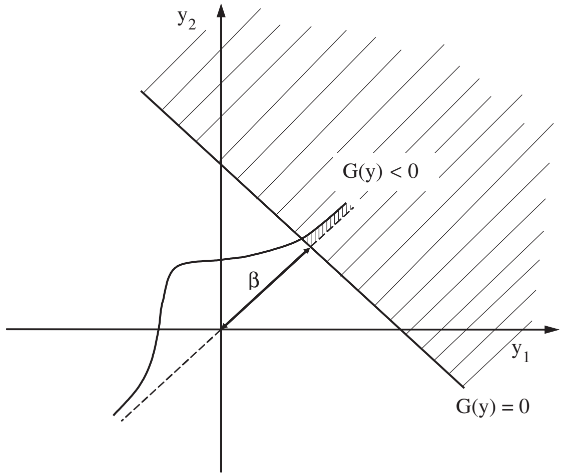

Figure 13. Graphical representation of β geometrical meaning.

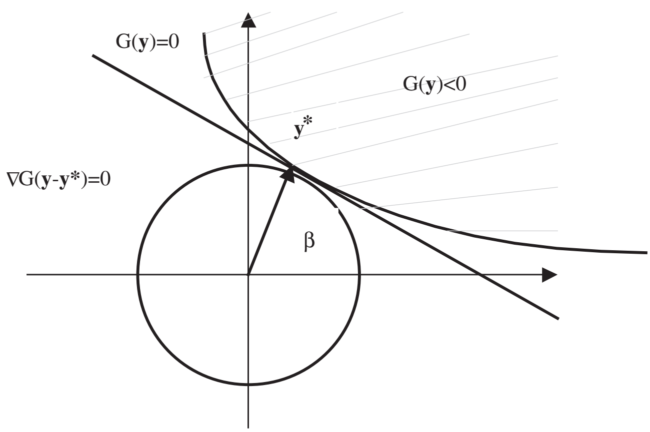

Figure 14. Graphical representation of limit state surface approximation according to the first-order reliability method.

By utilizing the property that the family of Gaussian probability distributions is closed with respect to linear combinations, it could be easily demonstrated that:

$$p_F = \int_{\{G(\mathbf{y})<0\}} \phi_0(\mathbf{y})d\mathbf{y} = \Phi_0(-\beta) \tag{30}$$

(provided that $p_F < 0.5$) where ϕ_0 is n-dimensional standard Gaussian probability density function, Φ_0 is one-dimensional standard Gaussian probability cumulative function, and β is the distance of the hyperplane $G(\mathbf{y}) = 0$ from the origin called *reliability index*. This result is schematically presented in Figure 13.

7.2.1 *FORM and SORM methods*

In the most practically interesting cases if either non-Gaussian probability densities or non-linear limit state functions appear the exact value of p_F is hardly to be obtained. Then approximate methods have to be used. Among these methods the FORM (first-order reliability method) and the SORM (second-order reliability method) are most commonly in use (Rackwitz & Fiessler 1978, Hohenbichler et al. 1987, Ditlevsen & Madsen 1996).

Therefore in the FORM approximation (see Fig. 14), the limit state surface in the standard normal space is replaced with the tangent hyperplane at the point $\mathbf{y}^*$ (called *design point*) with the minimum distance from the origin which is β.

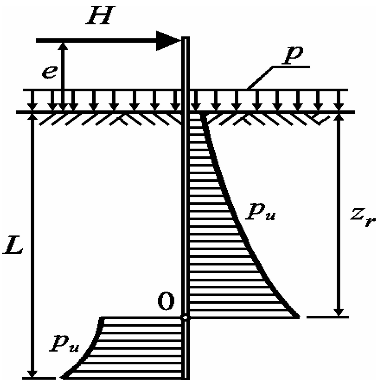

Figure 15. A scheme of rigid pile embedded in soil and subjected to lateral load.

In the SORM approximation, the limit state surface is fitted with a quadratic surface (see Fig. 14) in the vicinity of the design point $\mathbf{y}^*$ and the right-hand side of eqn. (30) is multiplied by a certain correction factor (Breitung 1984), affected by curvatures of the hypersurface $G(\mathbf{y}) = 0$ at the point $\mathbf{y}^*$. This gives the value of p_{FSORM}. Next the reliability index β_{SORM} can be computed by inverting the following relationship:

$$\beta_{SORM} = -\Phi_0^{-1}(p_{FSORM}) \tag{31}$$

The most important problem in the FORM and the SORM lies in finding the minimum-distance point $\mathbf{y}^*$, i.e. the design point. Several sophisticated algorithms for this problem were developed (Liu & Der Kiureghian 1991).

To evaluate the influence of individual parameters on the reliability index β, some sensitivity parameters α_i can be defined as follows:

$$\alpha_i = \frac{1}{\|\mathbf{y}^*\|} \frac{\partial \beta}{\partial y_i}\bigg|_{y=y^*} \tag{32}$$

where partial derivatives are evaluated with respect to $y_1,...y_n$ coordinates in the standard normal space. In the case of stochastically independent variables X_1, X_2,...X_n coefficients α_i can be interpreted as a sensitivity measures of individual random physical parameters.

7.3 *An example of reliability design: a rigid pile subjected to lateral loads*

As an example of reliability analysis accomplished by means of the FORM and the SORM, let us consider the problem of the evaluation of the bearing capacity of a rigid pile embedded in a homogeneous non cohesive soil deposit and subjected to lateral loads (Fig. 15).

The mechanism associated with the failure assumes a rotation of the pile around the centre O due to external load as well as reaction of surrounding soil. Assume that H_u and M_u, denoting the ultimate lateral load and ultimate value of the moment respectively, cause an ultimate ground resistance $p_u(z)$ on the depth z. Treating the pile as a strip of width D (or diameter D in the case of a pile of circular cross-section) and length L the following equilibrium equations can be written:

$$H_u = \int_0^{z_r} p_u(z) D dz - \int_{z_r}^{L} p_u(z) D dz \tag{33}$$

$$M_u = H_u e = -\int_0^{z_r} p_u(z) D z dz + \int_{z_r}^{L} p_u(z) D z dz \tag{34}$$

The Equations 33 and 34 have to be solved with respect to unknown parameters z_r and H_u. It is evident that the solution of those equations needs specifying the ground resistance distribution, $p_u(z)$ along the pile. Here the distribution proposed by Brinch Hansen (1961) has been applied. This approach utilizes the limit state theory as well as rigid ideally-plastic model of subsoil. Using the equations derived by

Brinch Hansen (see Brinch Hansen 1961) it can be found (see Puła 1997 2004) that the equation for the rotation centre z_r can be written as follows:

$$\begin{aligned}&a_o + a'_o c + (a_1 + a'_1 c)z_r + (a_2 + a'_2)z_r^2 + (a_3 + a'_3)z_r^3 + \\ &b_1 \ln(D + a_q z_r) + b'_1 c \ln(D + a_c z_r) = 0\end{aligned} \tag{35}$$

where c denotes the cohesion of soil and a_o, a'_o, a_1, a'_1, a_2, a'_2, a_3, a'_3, b_1, b'_1, a_q, a_c are coefficients depending on soil properties, namely the friction angle φ and the unit weight γ, as well as load parameters like overburden pressure p and eccentricity e (see Fig. 15). Hence the equation for the ultimate force H_u can be derived (Pula 2004):

$$H_u = a''_o + a'''_o c + (a''_1 + a'''_1 c)z_r + (a''_2 + a'''_2 c)z_r^2 + b''_1 \ln(D + a_q z_r) + b'''_1 c \ln(D + a_c z_r) \tag{36}$$

As before the coefficients in Equation 41 are also functions of soil properties and load parameters.

Some preliminary numerical studies have evidently showed high sensitivity of the ultimate loading force H_u to a value of the subsoil friction angle φ.

Then investigation of random fluctuations due to inherent variability of φ seems to be a vital problem. In deterministic computations as a measure of safety a "total safety factor" is usually considered, which is defined as the ratio of the ultimate lateral force H_u and the applied lateral force H_a:

$$F = \frac{H_u}{H_a} \tag{37}$$

If some soil properties are random variabilities then a natural question arises: how reliable the total safety factor is.

Accordingly appropriate reliability problem can be formulated as follows: find the probability p_F that the applied loading H_a exceeds the ultimate lateral loading H_u:

$$p_F = P\{H_a > H_u\} = P\left\{\frac{H_u}{H_a} < 1\right\} = P\{F < 1\} \tag{38}$$

(both values H_u and H_a are positive). The main difficulty in evaluating probability from Equation 38 is the lack of dependencies between the random soil parameters and the ultimate force H_u in an explicit form. As it can be seen from Equations 35 and 36, the evaluation of H_u requires the expression for z_r.

This difficulty can be overcome by applying some symbolic computations as well as some power series expansions (Pula 1997). Next probability of failure computations are carried out by means of the SORM method (Hohenbichler et al. 1987). This approach, called in the sequel "symbolic algorithm", seems to be rather precise, however it produces a lot of numerical difficulties, before obtaining the final results.

Some preliminary computations, carried out by "symbolic algorithm" associated with the SORM method allowing sensitivity factors computations for each random variable, have shown that the random variability of soil friction angle φ as well as random variability of applied lateral load H_a mostly affect reliability indices in this problem. Therefore in the first part of the computations only these two variables were treated as random. The assumptions concerning variables associated to the problem are summarized in Table 7.

In order to evaluate reliability index corresponding to a given value of total safety factor F (Equation 37) the following computational steps should be applied:

1. Select a value of the total safety factor F.
2. Assume that all random variables involved are constant and equal to their expected values. Applying these values evaluate the ultimate load H_u utilizing equilibrium Equations 33 and 34.
3. For the value of F selected in step 1, by means of Equation 37, find value of the load H_a.
4. Assuming that H_a is a random variable with the expected value computed in step 3 and the friction angle is a random variable with characteristics given in Table 7, evaluate reliability index β corresponding to probability of failure p_F given by Equation 38.
5. Repeat the steps above for several values of total safety factor F.

Selecting several values of the total safety factor F can be considered them versus corresponding reliability indices β. The results in the form of reliability indices β are summarized in. Table 8. Note that in order to have the reliability index greater than 3 (the ISO 2394 (1998) standard requires 3.8 for moderate consequences of a failure) it is necessary to fix the total safety factor equal to 3.2.

Table 7. Parameter characteristics involved into the problem considered.

No.	Parameter	Probability distribution	Expected value	Coefficient of variation
1.	Angle of internal friction φ	lognormal	33.6°	15%
2.	External lateral load H_a	lognormal	8–35 kN	15%
3.	Overburden pressure p	constant (nonrandom)	8.8 kNm^{-2}	
4.	Eccentricity e	constant (nonrandom)	8.64 m	
5.	Unit weight γ	constant (nonrandom)	20.2 kNm^{-3}	
6.	Pile diameter D	constant (nonrandom)	0.36 m	
7.	Pile embedding L	constant (nonrandom)	2.9 m	

Table 8. Reliability indices β corresponding to specified values of total safety factor F.

Total safety factor F	Expected value of the lateral force H_a [kN]	Reliability index β by "symbolic algorithm"
1.2	21.92	0.37
1.4	18.79	0.74
1.6	16.44	1.08
1.8	14.61	1.39
2.0	13.15	1.69
2.2	11.95	1.96
2.4	10.96	2.21
2.6	10.12	2.45
2.8	9.39	2.68
3.0	8.77	2.89
3.2	8.22	3.10

Results listed in Table 8 are associated with the length of the pile equal to L = 2.9 m.

By now reliability computations in the framework of this example have been performed without any spatial averaging. On the other hand it is well-known that spatial averaging of soil properties random fields leads to more realistic values of reliability indices. Then in the next step the spatial averaging of the internal friction angle random field has been applied. According to this, a new random variable $\varphi_{\bar{L}}$ is defined as follows:

$$\varphi_{\bar{L}} = \frac{1}{\bar{L}} \int_{\bar{L}} \varphi(z)\,dz \tag{39}$$

where $\varphi(z)$ is the random function, which describes random variability of the friction angle φ with the depth z and L is the pile length (in cases where L is treated as random variable $\bar{L}$ is understood as the expected value of L). The function $\varphi(z)$ is assumed to be stationary with constant mean value m_φ and constant point variance σ_φ^2. The variance of $\varphi_{\bar{L}}$ can be computed as:

$$VAR\left[\varphi_{\bar{L}}\right] = \sigma_{\bar{L}}^2 = \gamma(\bar{L})\sigma_\varphi^2 \tag{40}$$

In the present example the Gaussian autocorrelation function has been selected, which is considered as one of the most suitable for describing soil properties (Rackwitz 2000). The Gaussian autocorrelation function implies the following variance function:

$$\gamma(\bar{L}) = \frac{\frac{\pi}{\delta}\bar{L}\,\mathrm{erf}\left(\frac{\sqrt{\pi}}{\delta}\bar{L}\right) - 1 + \exp\left(-\frac{\pi}{\delta^2}\bar{L}^2\right)}{\frac{\pi}{\delta^2}\bar{L}^2} \tag{41}$$

where δ is a fluctuation scale. Here, in the case of non-cohesive soils in vertical direction it was assumed as about 1 m or less. Next computations were carried out for three different values of δ. Resulting values of the variance function are given in Table 9.

Table 9. Values of variance function for three different values of fluctuation scale.

Averaging length $\overline{L}$ [m]	Values of the fluctuation scale		
	$\delta = 0.6$ [m]	$\delta = 0.8$ [m]	$\delta = 1.0$ [m]
	Values of the variance function		
2.9	0.1933	0.2516	0.3070

Table 10. The effect of spatial averaging on reliability indices ($\text{cov}\{L\} = 0.02$).

Total safety factor F	Reliability index β without spatial averaging	Reliability index β with spatial averaging ($\delta = 0.6$ m)	Reliability index β with spatial averaging ($\delta = 0.8$ m)	Reliability index β with spatial averaging ($\delta = 1.0$ m)
1.2	0.37	0.76	0.70	0.65
1.4	0.73	1.42	1.30	1.22
1.6	1.07	2.01	1.85	1.73
1.8	1.38	2.56	2.35	2.20
2.0	1.67	3.06	2.82	2.64
2.2	1.94	3.53	3.25	3.05
2.4	2.19	3.96	3.66	3.43
2.6	2.44	4.37	4.04	3.79
2.8	2.67	4.77	4.40	4.13
3.0	2.90	5.13	4.75	4.46

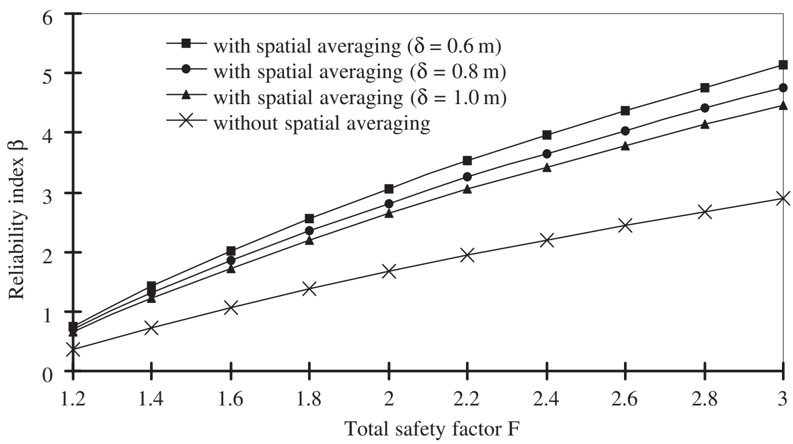

Figure 16. The influence of spatial averaging on the values of reliability indices β.

Table 10 shows resulting reliability indices. In the second column indices without spatial averaging are presented. This column is the same as the third column in Table 9. Columns third, fourth and fifth show the reliability indices obtained when spatial averaging has been applied. The results are also presented in graphical form in Figure 16.

Obtained results evidently demonstrate high influence of spatial averaging on reliability measures when safety of rigid piles is considered. Note that the total safety factor F of level 2.4 gives relatively high values of reliability indices (greater than 3.4), if the averaging is applied.

On the other hand it is worth mentioning that the effect of the value of fluctuation scale is remarkable. This means that the value of fluctuation scale has to be carefully selected and supported by laboratory testing.

8 CONCLUSIONS

Throughout the paper a brief review of statistical methods and procedures for dealing with the inherent variability of natural soils are presented. Moreover the reliability assessment of geotechnical design is proposed by means of a reliability based-design of a pile foundation.

The approach considered is clearly operative to achieve the declared objective of applying statistical tools for solving practical design problems concerning soil properties characterization and geotechnical design models, taking into account the high variability and uncertainties affecting soil physical-mechanical properties. This means that in geotechnical problems resistances are considered as statistical estimates. Apart from the procedures recalled two main practical difficulties arises:

- The necessity of extensive experimental campaigns with a sufficient number of measurements of good quality in statistical sense. This is not often achieved.
- The possibility of validation of calculation models by means of real size measures in order to select the best models to get previsions closer to the real behaviour of geotechnical structures.

Within the previous limitations procedures for statistical approach were illustrated. Some applications on soils from South and Central part of Italy are discussed and statistical techniques are proposed in order to show their efficiency in statistical characterization of physical mechanical parameters to be employed in geotechnical design. As a matter of fact typical values for the scale of fluctuation, the variance, the mean and the variance function of the studied soils can be usefully applied in design when poor experimental campaigns are performed. These approaches are nowadays not commonly performed in geotechnical design activity unless it should be mandatory whenever relevant geotechnical structures are designed.

Moreover reliability based design is also discussed and one application is shown. It illustrates step-by-step reliability measures computations versus the total safety factor when the bearing capacity of a rigid pile under lateral load is considered. The efficient procedure, despite of complicated and non-linear dependences between random variables resulting by applying the Brinch Hansen approach, was obtained due to the FORM and SORM application. The inherent variability of soil friction angle as well as uncertainty in the precise pile embedding are considered.

Comparison of the total safety factors versus reliability indices shows a vital role of spatial averaging as well as the importance of precise evaluation of the fluctuation scale value. The presented computation gives a kind of link between working stress design and the limit state design.

The previous issue is also stressed by considering the point variance reduction standard deviations of soil strength parameters by incorporating spatial averaging. This reduction leads to a significant increase in reliability indices (decrease in failure probabilities). This is a step forward in making reliability measures more realistic in the context of well-designed (according to standards) foundations.

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