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To Angela con cari saluti
Guardalo per sempre
Tiene
Tasso

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Dedicated to the memory of Simos Ichtiaroglou

RESEARCH PAPERS

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Modeling Spikes in Natural and Social Phenomena

Angela De Sanctis* and Carlo Mari†
Department of Quantitative Methods and Economic Theory,
"G. d'Annunzio" University of Chieti-Pescara, ITALY
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We propose a general methodology to model natural and social phenomena characterized by stochastic dynamics in which the occurrence of random spikes can be observed. The method uses excitable dynamics in a multi-regime switching approach to describe dynamical systems where the motion randomly switches between a stable dynamics and an excited dynamics. In particular we discuss a two-regime switching model in which the stable motion is described by mean-reverting diffusion process and the spikes dynamics by a stochastic FitzHugh-Nagumo model.

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1. Introduction

Many natural and social phenomena show sudden and unpredictable changes of their dynamical behavior. In such cases the motion seems to be characterized by random switches between a stable regime and an excited regime in which the occurrence of spikes of very large magnitude can be observed. Solar cycles (sunspots), planetary magnetospheres, lithosphere dynamics and earthquakes, brain dynamics [1, 2] as well as power prices observed in deregulated markets represent relevant examples [3–8]. The presence of spikes is a typical property of excitable systems and it is observed in a wide range of natural phenomena, as, among the others, lasers physics, chemical reactions, ion channels, neural systems and climate dynamics [9]. Figure 1 show at left the historical dynamics of electricity prices observed at the Californian electricity market (SP15) since October 22, 2002 until January 27, 2007. Data refer to daily base load power prices [10], calculated as arithmetic averages of the 24 hourly market prices (week-end days have been discarded). At the right side, the historical behavior of the total seismic activity observed in California since 1960 to 1985 is shown. Data refer to weekly cumulated intensities of earthquakes with magnitude 3.0 or more [11].

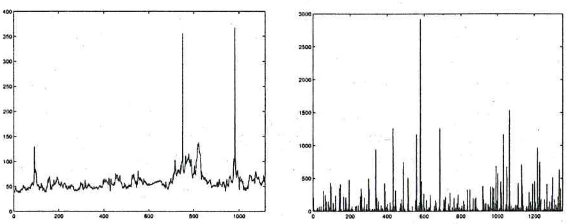

FIG. 1. Left side: California electricity prices since October 22, 2002 until January 27, 2007. Right side: historical behavior of the Californian total seismic activity.

In both cases the empirical analysis reveals a multi-regime dynamics with random switches from one regime to another. In particular we can observe normal stable periods in which the dynamics is characterized by fluctuations around some long-run mean, and turbulent periods in which the dynamics is affected by more pronounced fluctuations with jumps and short-lived spikes of very large magnitude.

In this paper we propose a two-regime switching approach in order to capture the principal statistical properties of the empirical dynamics. Regime-switching models offer indeed the possibility to introduce various mean-reversion rates and volatilities depending on the state of the system thus allowing to model the stable motion and the spike regime in a very flexible way. We assume that the switching mechanism between the states is governed by un unobservable Markov process. In the two-regime model we discuss, one regime drives the stable motion and it is described by a mean-reverting diffusion process. The spike regime is then modelled according to a stochastic FitzHugh-Nagumo dynamics [12, 13]. Even if spikes in the FHN-model are characterized by the same height and the same time duration [14, 15], we will show that the regime-switching mechanism allows us to modulate these quantities in such a way to produce stochastic spikes of random heights and random durations.

In the following Section we introduce the methodology and we discuss in some detail the random spikes generation mechanism. Some comments about future research conclude the paper.

2. The general model

In the general model we propose, the normal regime is described by a mean-reverting diffusion process to account for random fluctuations around some long run mean. The spike regime is modelled according to a well defined transformation of the voltage solution of a stochastic FitzHugh-Nagumo (FHN) dynamics. The model can be cast therefore in the following form,

*E-mail: a.desanctis@unich.it
†E-mail: c.mari@unich.it

$$\begin{cases} dx(t) = \big(\mu_0 - \alpha_0 x(t)\big)dt + f(x(t))dw_0(t) \\ x(t) = \psi(x_s(t)) \end{cases} \quad (1)$$

where $x_s(t)$ denotes the fast variable of a FHN-dynamics of the type

$$\begin{cases} \epsilon dx_s = \big(x_s - \frac{1}{3}x_s^3 - y\big)dt \\ dy = \big(x_s + a\big)dt + \sigma dw_1(t) \end{cases} \quad (2)$$

and ψ is an arbitrary transformation of this function. We also assume that the random noise $w_1(t)$ in the FHN-dynamics and $w_0(t)$ are uncorrelated Brownian processes. The switches between regimes are controlled by an unobservable Markov process, and we cast the transition probabilities matrix in the following form

$$\pi = \begin{pmatrix} 1-\xi dt & 1-\eta dt \\ \xi dt & \eta dt \end{pmatrix}, \quad (3)$$

where ξdt denotes the transition probability for the switching from the base state to the spike regime in the infinitesimal time interval $[t, t+dt]$, and $1-\eta dt$ is the probability for the opposite transition. ξ and η are assumed to be constant. In the above matrix, the diagonal terms give the probability of remaining in any given state during the time interval $[t, t+dt]$, and the off-diagonal terms represent the transition probabilities to the other state in the same time interval. We expect that the probability to remain in the stable regime as well as the probability to revert back to the stable regime are quite high. The above model has been proposed to describe the stochastic movements of natural logarithm of power prices in competitive markets [16] by assuming constant volatility in the diffusion process and a linear transformation of the FHN-dynamics, namely $\psi(x_s) = c_1 + c_2 x_s$, for the spiky regime. This approach seems quite flexible to describe dynamical systems in which the motion randomly switches between a stable dynamics and an excited dynamics. Although in the FHN-dynamics the oscillations have the same height and the same duration, in the two-regime model the heights and the durations of the spikes can be modulated by controlling the parameters ξ and η. As the probability of the system to remain in the spike regime reduces, the duration of the spike reduces itself and we can observe the formation of spikes of different heights. To better put in evidence this mechanism, Figures 2 and 3 show simulated trajectories for the x_s-dynamics and for the corresponding x-dynamics by using the following positive square-root process

$$\begin{cases} dx(t) = \big(\mu_0 - \alpha_0 x(t)\big)dt + \sigma_0\sqrt{x(t)}dw_0(t) \\ x(t) = \exp\big(c_1 + c_2 x_s(t)\big). \end{cases} \quad (4)$$

The observed behavior can be explained by considering that, if the parameter η reduces, the system spends less time in the excited regime and, in general, the trajectory cannot complete the whole oscillation before reverting back to the stable motion.

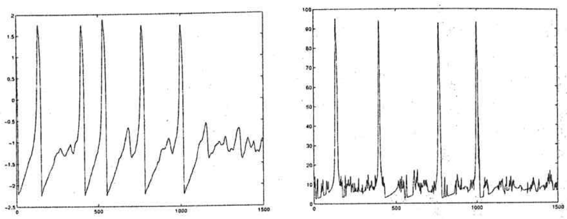

FIG. 2. x_s-dynamics (left), and x-dynamics (right) in the exponential case: the values of the transition probabilities are 95% of remaining in the stable state and 95% of remaining in the spike regime.

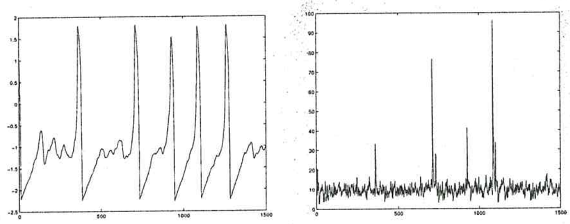

FIG. 3. The values of the transition probabilities are 95% of remaining in the stable state and 5% of remaining in the spike regime.

In particular, Figure 2 shows five spikes in the free FHN-dynamics. By assuming that the probability to remain in the excited state is 95%, four spikes survive in the x-dynamics and their heights and time amplitudes do not vary in a significant way. The phenomenon of randomizing heights and time durations is more evident in Figure 3, where the probability of remaining in the excited state is assumed to be of 5%. The five spikes in the FHN dynamics become four in the x-dynamics, and they are characterized by very different heights and very short durations. Figure 4 shows simulated trajectories for the x_s-dynamics and for the corresponding x-dynamics if a linear transformation of the FHN-dynamics of the type $\psi(x_s) = c_1 + c_2 x_s$ is used. In this case we can distinguish three different kinds

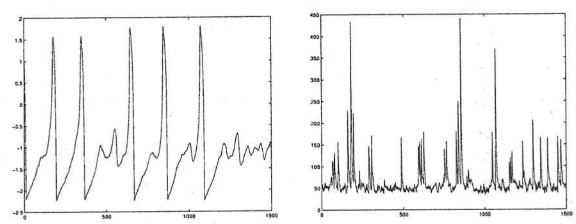

FIG. 4. x_s-dynamics (left), and x-dynamics (right) in the linear case: the values of the transition probabilities are 97% of remaining in the stable state and 1% of remaining in the spike regime.

of dynamics: the first one is a stable dynamics around some long-run mean, the second one is characterized by medium size jumps, and the third is a spiky dynamics. A jump occurs whenever the system switches to the FHN-dynamics, and isolated random spikes are generated if the transition happens when the excitable dynamics is performing the large oscillation around the stationary state [16]. In Table 1 are summarized the values of the dynamical parameters used in the simulation. The first two columns refer to model 4, and the remaining ones show the value of the dynamical parameters when a linear transformation of the FHN-dynamics is used.

Table 1. Simulation parameters.

Stable dyn.	Spikes dyn.	Stable dyn.	Spikes dyn.
$\mu_0 = 3.00$	$\epsilon = 0.10$	$\mu_0 = 10.0$	$\epsilon = 0.10$
$\alpha_0 = 0.30$	$a = 1.08$	$\alpha_0 = 0.20$	$a = 1.08$
$\sigma_0 = 0.50$	$\sigma = 0.08$	$\sigma_0 = 0.55$	$\sigma = 0.08$
	$c_1 = 2.97$		$c_1 = 280$
	$c_2 = 0.90$		$c_2 = 100$

3. Concluding remarks

In this paper we have proposed a general methodology to model dynamical systems showing very complex behavior. In particular we discussed the possibility to describe in a flexible way dynamical systems in which the motion randomly switches between a stable and an excited dynamics where jumps and spikes may occur. The methodology uses Markovian transitions from diffusion processes to nonlinear stochastic excitable dynamics, and this switching mechanism produces spikes of random heights and random durations. The proposed approach can be generalized to excitable systems transmitting excitation among themselves, producing synchronization, multistability and chaos phenomena. From this point of view, the method can be extended introducing in the spike regime, a pair of FHN-dynamics with diffusive coupling [17] as follows

$$
\begin{aligned}
\epsilon_1 \dot{x}_s &= x_s - \frac{{x_s}^3}{3} - y + k(v - x_s) \\
\dot{y} &= x_s + a_1 \\
\epsilon_2 \dot{v} &= v - \frac{v^3}{3} - z + k(x_s - v) \\
\dot{z} &= v + a_2.
\end{aligned}
$$

It is a well know fact that, if $k > 0$, the rest state is globally stable: a finite perturbation to the rest state causes the excitation of an element, which is transmitted to the other, then the synchronization occurs till the elements return to the rest state. On the contrary, if $k < 0$, various types of cyclic firing patterns emerge including chaotic firing [18].

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