Markov switching of the electricity supply curve and power prices dynamics

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Abstract

Regime-switching models seem to well capture the main features of power prices behavior in deregulated markets. In a recent paper we proposed an equilibrium methodology to derive electricity prices dynamics from the interplay between supply and demand in a stochastic environment. In particular, assuming that the supply function is described by a power law where the exponent is a two-state strictly positive Markov process, we derived a regime switching dynamics of power prices in which regime switches are induced by transitions between Markov states.

In this paper we provide a dynamical model to describe the random behavior of power prices where the only non-Brownian component of the motion is endogenously introduced by Markov transitions in the exponent of the electricity supply curve. In this context, the stochastic process driving the switching mechanism becomes observable, and we will show that the non-Brownian component of the dynamics induced by transitions from Markov states is responsible for jumps and spikes of very high magnitude. The empirical analysis performed on three Australian markets confirms that the proposed approach seems quite flexible and capable of incorporating the main features of power prices time-series, thus reproducing the first four moments of log-returns empirical distributions in a satisfactory way.

Keywords: electricity prices, stochastic processes, regime-switches, spikes.

1 Introduction

Regime-switching models are widely used in literature to describe the random character of power prices in deregulated markets. From the pioneeristic work by Huisman and Mahieu [1], several studies have been proposed in order to capture the main features of the observed time behavior of power prices (see [2]-[6] and references therein for a comprehensive overview of the topics). As a consequence of the deregulation and liberalization process, electricity prices are determined by the interaction between demand and supply and they exhibit a very erratic dynamics. Fig.1 shows the historical path of daily base-load power prices in three Australian markets, namely at the New South Wales market, at the Tasmania and at the Victoria markets.

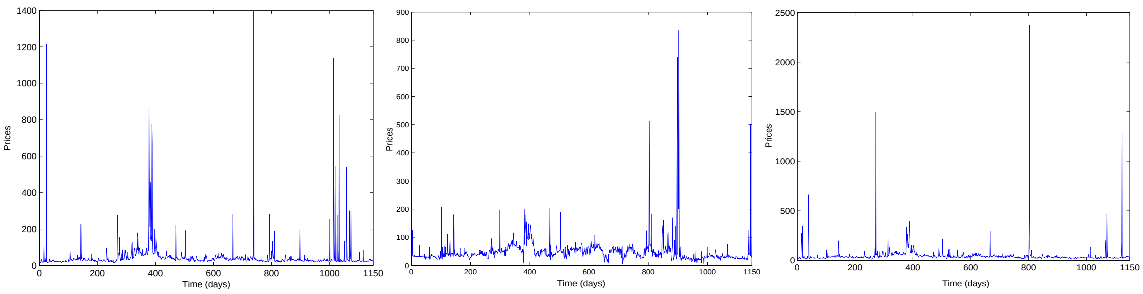

Figure 1: Historical behavior of power prices at the New South Wales market (left), at the Tasmania market (central) and at the Victoria market (right) from January 1, 2006 to May 31, 2010. Market prices are available at www.aemo.com.au and they are provided as average daily prices without weekend days.

The resulting time series exhibit multi-regime dynamics where stable periods can be distinguished by turbulent periods in which unanticipated, short-lived extreme price changes may occur. After a jump, power prices are forced back to their normal level by some mean-reversion mechanism inducing them to fluctuate around the long-run average. Regime-switching models offer the possibility to introduce various mean-reversion rates and volatilities in order to distinguish the dynamics in different states and they seem good candidates to capture the most important observed features of historical data.

In a recent article [7], we proposed an equilibrium methodology to derive regime switching dynamics directly by the interplay between demand and supply in a stochastic environment. Under the hypothesis that the functional form of the electricity supply curve is described by a power law in which the exponent is a two-state strictly positive Markov process, we derive the dynamics of power prices from the equilibrium between demand and supply.

In this paper we propose a dynamical model to describe the evolution of power prices in which the only non-Brownian component of the motion is introduced in a natural way by Markov transitions in the exponent of the electricity supply curve. Various approaches have been proposed in literature to model jumps and spikes in power prices dynamics. Poisson jumps [8]-[9], and other spiking mechanisms, as for example excitable dynamics [10], have been introduced in an exogenous way to reproduce observed behavior of electricity prices in deregulated markets. We investigate the possibility that the non-Brownian component of the dynamics induced by transitions from Markov states in the exponent of the supply curve can be responsible for observed jumps and spikes. This paper completes therefore the work presented in [7], and it provides a dynamical example where the Markov process driving the switching mechanism is observable. Such a process can be therefore estimated from market data and, in the empirical part of the paper, we provide a model estimation using maximum-likelihood techniques.

The paper is organized as follows. In the next Section we briefly review some basic concepts about the proposed methodology, and we derive the main equation characterizing the dynamics of power prices. Section 3 presents a dynamical model to describe power prices dynamics; an empirical analysis on market data is also provided in order to test the adaptability of the model. The analysis, performed on the New South Wales power market, on the Tasmania and the Victoria markets, reveals that the non-Brownian component of the motion is responsible for the high values of the kurtosis and it seems to well capture the spiking behavior of power prices. The followed approach is quite flexible and capable of incorporating the main features of observed prices time-series, thus reproducing the first four moments of the empirical distributions in a satisfactory way. Some comments conclude the paper.

2 Basic facts and results

Let us briefly recall the main ideas and the basic concepts of the methodology in order to make the analysis self-contained (for a deeper insight see [7]).

It is well known that electricity is a very special commodity: with the exception of hydroelectric power, it cannot be stored and must be generated at the instant it is consumed. In general the generation process is assured by generators with low marginal costs to cover the base load, as hydroelectric plants, nuclear power plants, and coal units. To meet peaks in the demand, emergency units (oil and gas fired plants) with high marginal costs are to be put on operation [11]. The supply curve (stack function) exhibits therefore a time variable kink after which offer prices start rising almost exponentially [12]. The demand is highly inelastic and very sensitive to the temperature and weather conditions. Whenever the market volume (demand) crosses the stack function in the rapidly raising part of the curve, electricity prices may assume very high values: a spike occurs when the demand intersects the offer curve in the almost vertical part of the curve. This may be due to unpredictable peaks in electricity demand, and/or random movements of the supply curve reducing the power offer in a significant way [13]. We assume therefore that the offer curve is described by:

$$P(t) = h_0(t) \left(\frac{q(t)}{a(t)} \right)^{\beta(t)} \tag{1}$$

where h_0 is a deterministic function accounting for power-price seasonality, $q(t)$ represents the market volume at time t, and $a(t)$ is a scale process responsible of random movements of the offer curve. $\beta(t)$ is a discrete, strictly positive Markov process assuming only two values, β_0 and β_1, with $\beta_0 < \beta_1$ and $\beta_1 > 1$. Both processes, $\beta(t)$ and $a(t)$, can capture random movements of the offer curve due to unpredictable changes in the generation process as outages and grid congestions as well as random power generation from renewable sources. Anyway we will show that, without introducing any spiking mechanism in an exogenous way, the process $\beta(t)$ can be responsible of jumps and spikes of very high magnitude. To give an example, in Fig.2 we simulated a situation in which at the same level of the market volume the switching mechanism in the exponent of the offer curve produces very high prices. This may happen, for example, whenever unpredictable outages reduce the offer in a significant way thus determining price spikes also in the case of normal market volumes.

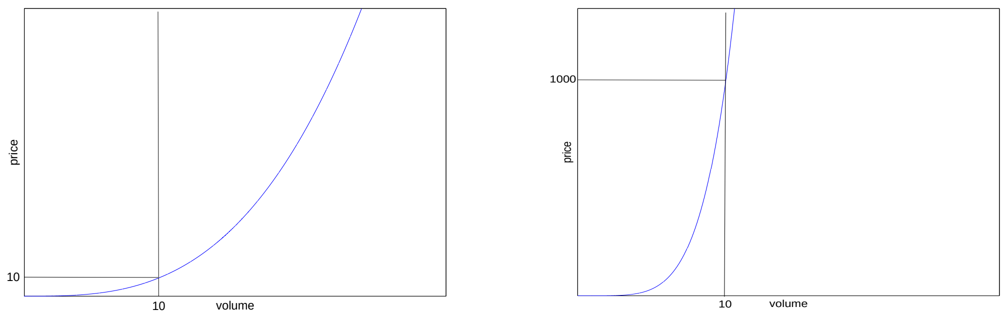

Figure 2: Understanding the spike phenomenon as a consequence of the switching mechanism in the offer curve: a schematic supply stack with a hypothetical demand curve superimposed. In correspondence of the normalized volume $q/a = 10$, the price becomes two order of magnitude greater when the Markov exponent switches from $\beta_0 = 3$ to $\beta_1 = 5$ (for simplicity h_0 has been posed equal to 1).

In the following we will prove that transitions between Markov states in the offer curve induce regime-switching dynamics on power prices and introduce a non-Brownian component of the motion which can be responsible for the spiking mechanism. To see this let us recall that the demand of electricity is fairly inelastic and it can be represented by a quasi-vertical line. We assume that it can be approximated by,

$$q(t) = D(t), \tag{2}$$

for some stochastic process $D(t)$ which is independent of the power price [14]. The equilibrium between supply and demand is assured if

$$P(t) \;=\; h_0(t)\,\left(\frac{D(t)}{a(t)}\right)^{\beta(t)}, \tag{3}$$

so that log-prices can be represented by the following process:

$$\log P(t) \;=\; \log h_0(t) \,+\beta(t)\,[\log D(t) - \log a(t)]\,. \tag{4}$$

The seasonal component of electricity prices is more pronounced with respect to other commodities due to the sensitivity of the demand to climate conditions, such as the temperature or the number of daylight hours. In order to consider seasonality effects, we assume that $\log P(t) = \log h_0(t) + p(t)$ where $p(t)$ is the stochastic component of log-prices. Without loss of generality we also assume that $\log D(t)$ and $\log a(t)$ have the same deterministic component $f_x(t)$, i.e. $\log D(t) = f_x(t) + x(t)$ and $\log a(t) = f_x(t) + y(t)$. We finally get:

$$p(t) \;= \beta(t)\,[x(t) - y(t)]\,, \tag{5}$$

where $x(t)$ accounts for the stochastic behavior of the demand, and $\beta(t)$ and $y(t)$ are responsible for random movements of the offer curve. The dynamics of power prices is then obtained from the time evolution of both processes, $x(t)$ and $y(t)$, and from the dynamics of the Markov process $\beta(t)$. We assume that the two-state process $\beta(t)$ is independent of both process $x(t)$ and $y(t)$, and that the transition probability matrix can be cast in the following form:

$$\pi = \left(\begin{array}{cc} 1-\gamma dt & \eta dt \\ \gamma dt & 1-\eta dt \end{array}\right), \tag{6}$$

where γdt denotes the transition probability for the switching from the base state β_0 to β_1 in the infinitesimal time interval $[t, t+dt]$ and ηdt is the probability for the opposite transition. As shown in [7], if the supply curve makes a transition from state i to state j in the infinitesimal time interval $[t, t+dt]$, that is if $\beta(t)$ switches from state i to state j $(i, j = 0, 1)$, we obtain

$$dp(t) \;= \beta_j\,[dx(t) - dy(t)] + \frac{\beta_j - \beta_i}{\beta_i} p(t), \tag{7}$$

where β_i is the value of the Markov exponent of the supply curve in the state i and β_j is the Markov exponent in the state j. Apart from the dynamics of $x(t)$ and $y(t)$, the switching mechanism in the supply curve introduces the non-Brownian component of the motion given by the last term in equation (7). In this paper we investigate the possibility that such a component can be responsible for the spikes generation. To do this we assume that the dynamics of both processes, $x(t)$ and $y(t)$, is described by a mean-reverting diffusion process with constant volatility. As proved in [7] the random evolution of power prices can be cast therefore in the following final form:

$$dp(t) = \big(\mu_{ij} - \alpha_{ij}\,p(t)\big)\,dt + \sigma_{ij}\,dW_{ij} + \frac{\beta_j - \beta_i}{\beta_i} p(t). \tag{8}$$

Equation (8) clearly shows that the ratio between β_j and β_i is observable in the dynamics of power prices and it can be estimated from market data. In the next Section we provide an empirical analysis in order to test the adaptability of the model to market data.

3 The model

In this Section we model the random behavior of power prices according to the stochastic process (8). If we look at equation (8) we can observe that the dynamics of transitions between Markov states (transitions $i \to j$ and $j \to i$) can be modelled independently from the dynamics of the stable regime (transition $i \to i$) and of the turbulent regime (transition $j \to j$). It is only during transitions $i \to j$ and $j \to i$ that the Markov parameters β_i and β_j become observable, specifically in the ratio β_i/β_j. The following parametrization is therefore adopted:

$$dp(t) = \begin{cases} \big(\mu_0 - \alpha_0\, p(t)\big)dt \,+\, \sigma_0\, dW_0 \\[2ex] \big(\mu_{01} - \alpha_{01}\, p(t)\big)dt \,+\, \sigma_{01}\, dW_{01} + \dfrac{\beta_1 - \beta_0}{\beta_0}\, p(t) \\[2ex] \big(\mu_1 - \alpha_1\, p(t)\big)dt \,+\, \sigma_1\, dW_1 \\[2ex] \big(\mu_{10} - \alpha_{10}\, p(t)\big)dt \,+\, \sigma_{10}\, dW_{10} + \dfrac{\beta_0 - \beta_1}{\beta_1} p(t) \end{cases} \tag{9}$$

in which the Brownian motions W_0, W_{01}, W_1, and W_{10} are assumed mutually independent. Even if the process $\beta(t)$ is a two-state Markov process, the dynamics of power prices can be described in terms of four distinct processes: the first one governs the dynamics in the base state; the second describes the dynamics of transitions from state 0 to state 1; the third accounts for the dynamics in the excited state, and the fourth is responsible for transitions from state 1 to the base state 0. In this dynamical representation the transition probability matrix is described by the following 4×4 matrix, directly obtained by the two-state matrix (6):

$$\pi = \begin{pmatrix} 1-\gamma & 0 & 0 & 1-\gamma \\ \gamma & 0 & 0 & \gamma \\ 0 & 1-\eta & 1-\eta & 0 \\ 0 & \eta & \eta & 0 \end{pmatrix}. \tag{10}$$

In the next part of this Section we estimate the model on three Australian markets, namely the New South Wales market, the Tasmania and the Victoria markets to test the adaptability of the model to market data. We will show that the followed approach seems capable of incorporating the main features of observed power prices, thus reproducing quite well the first four moments of the empirical distributions of log-returns.

3.1 The empirical analysis

The data set consists of day-ahead base-load prices, calculated as daily averages of market prices from January 1, 2006 to May 31, 2010. Market prices are available at www.aemo.com.au and they are provided as average daily prices without weekend days. The behavior of electricity prices time-series is depicted in Fig.1. Seasonal effects can be captured by adopting the following decomposition:

$$\log P(t) = f_p(t) + p(t), \tag{11}$$

where $f_p(t) \equiv \log h_0(t)$ is a deterministic function of time. To account for the semiannual periodicity, due to the fact that power prices may be higher in winter time and in summer time, we assume that the seasonality component of the motion is given by:

$$f_p(t) = b_0 + b_1 \frac{t}{261} + b_2 \cos\left(b_3 + \frac{2\,\pi\,t}{261}\right) + b_4 \cos\left(b_5 + \frac{4\,\pi\,t}{261}\right), \tag{12}$$

in which a linear trend has been included. We estimated the parameters of the deterministic component by fitting $f_p(t)$ to market data using ordinary least-squares (OLS) techniques. The results are reported in Table 1.

	N. South Wales	Tasmania	Victoria
b_0	3.6319	3.8562	3.7417
b_1	-0.0270	-0.0540	-0.0686
b_2	0.1278	0.1293	0.1486
b_3	-3.1661	-2.1071	-2.7574
b_4	0.1204	0.1144	0.0836
b_5	6.6588	6.9652	6.3807

Table 1: Estimated parameters of the seasonal component.

In Fig.3 the deterministic component has been superimposed on the historical path of market log-prices.

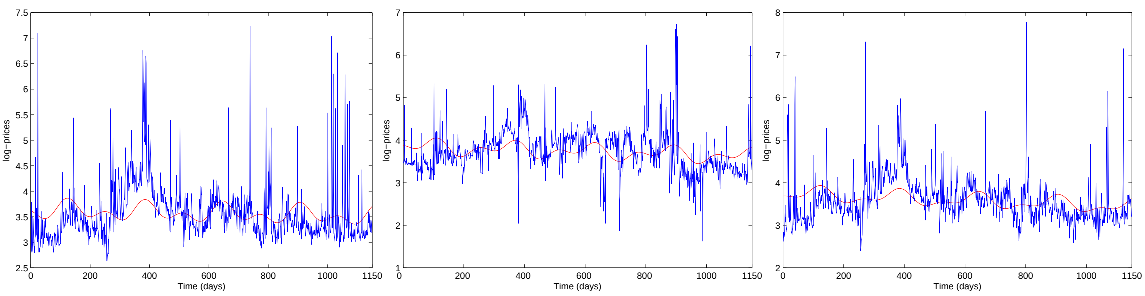

Figure 3: Daily log-prices and their seasonal components (thick red lines) for the New South Wales market (left), for the Tasmania market (central) and for the Victoria market (right) since January 1, 2006 until May 31, 2010.

Fig.4 shows the historical behavior of deseasonalized log-returns. The rule of demand and supply has strongly increased the volatility of price returns: deregulated markets exhibit, in general, large price fluctuations, and the presence of jumps and spikes is revealed by non-normal empirical distributions with very high values of the kurtosis. The descriptive statistics of log-returns observed in the markets under investigation is displayed in Table 2.

We observe that the Tasmania market presents a positive skewness unlike the Victoria and the New South Wales markets in which the skewness assumes negative values. Such a statistical parameter is related to the properties of upward versus downward jumps. For example, negative skewness indicates that price drops (downward jumps) have on average a greater weight than upward jumps. To account for the sign and the magnitude of the

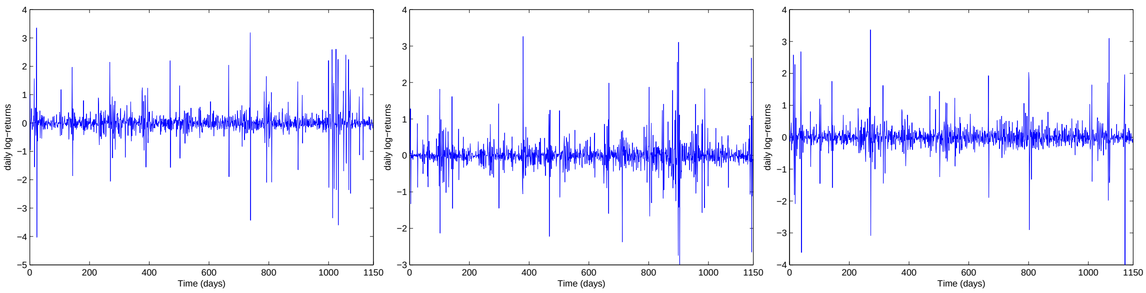

Figure 4: Deseasonalized log-returns at the New South Wales market (right), at the Tasmania market (central) and at the Victoria market (right), from January 1, 2006 to May 31, 2010.

	N. South Wales	Tasmania	Victoria
Mean	0.0003	- 0.0001	0.0006
Std.Dev.	0.4994	0.4295	0.4426
Skewness	- 0.7105	0.2739	- 0.3666
Kurtosis	21.8111	19.0504	26.0731

Table 2: Descriptive statistics.

skewness parameter as well as for the high values of the kurtosis, a parsimonious version of the model (9) can be proposed. First of all, to ensure that the reverting back dynamics (the fourth regime) drives the system toward the stable regime to fluctuate around the long-run mean, we impose that $\alpha_0 = \alpha_{10}$ and $\mu_0 = \mu_{10}$. Further, we assume that both, the dynamics of the excited regime and the dynamics governing the transition between the stable regime and the excited regime, have the same mean-reversion parameter, $\alpha_{01} = \alpha_1$. To account for positive and negative skewness, a different choice regarding μ_{01} and μ_1 is adopted. The markets under observation exhibit positive skewness (Tasmania market) and negative skewness (New South Wales and Victoria markets). Since the non-Brownian component in the dynamics is proportional to log-prices, to get positive skewness one possibility is to require that μ_1 is positive and greater than μ_0, thus enhancing the average weight of upward jumps. To do this we require that in the case of the Tasmania market, the only exhibiting positive skewness, we pose $\mu_{01} = \mu_0$ without constraining μ_1. On the contrary, for the New South Wales and the Victoria markets, both with negative skewness, we require $\mu_1 = \mu_0$ without constraints on μ_{01}.

The model has been estimated by maximum-likelihood using the Hamilton filtering technique [15]-[16]. Following this approach, the likelihood function in multi-regime dynamics can be expressed as a linear combination of likelihoods of the single regimes. The estimation results are summarized in Table 3.

The proposed model seems to well capture the main features of power prices dynamics observed in real markets. In particular, the model distinguishes the stable regime from the turbulent one in the sense that the values of the mean-reversion parameter and of the volatility are lower in the stable regime with respect to the excited one. Furthermore the ratio β_1/β_0 is greater than one, thus revealing that the switching mechanism in the supply curve works well in arising the exponent when the system make a transition from the base

	N. South Wales	Tasmania	Victoria
$\mu_0 = \mu_{10}$	-0.0291	-0.0085	-0.0278
	(0.0058)	(0.0049)	(0.0070)
$\alpha_0 = \alpha_{10}$	0.1436	0.0538	0.1537
	(0.0161)	(0.0170)	(0.0171)
σ_0	0.1462	0.1118	0.1675
	(0.0051)	(0.0081)	(0.0094)
μ_{01}	0.4194	-0.0085	0.3302
	(0.0596)	(0.0049)	(0.1560)
$\alpha_{01} = \alpha_1$	0.6148	0.8870	0.6094
	(0.1067)	(0.1131)	(0.1104)
σ_{01}	0.3025	0.3135	0.4125
	(0.0520)	(0.0595)	(0.0997)
μ_1	-0.0291	0.2572	-0.0278
	(0.0058)	(0.1202)	(0.0070)
σ_1	1.3752	0.9787	1.3647
	(0.1373)	(0.1013)	(0.1444)
σ_{10}	0.4984	0.3539	0.4075
	(0.0487)	(0.0356)	(0.0551)
β_1/β_0	2.0146	1.6385	1.6820
	(0.1487)	(0.1128)	(0.1846)
$1-\gamma$	0.9177	0.8184	0.9185
	(0.0114)	(0.0308)	(0.0229)
$1-\eta$	0.4794	0.4101	0.4261
	(0.0592)	(0.0610)	(0.0635)
LL	5.8013	-6.0138	54.7355

Table 3: Estimation results. Standard errors are between parentheses.

state to the excited state. Table 4 displays the first four moments of the model distribution of log-returns, obtained by averaging over 5000 simulated paths randomly generated using estimated parameters. The statistical analysis of simulated trajectories reveals a very interesting agreement with experimental data: without introducing Poisson jumps in the model as in [13] and [7], the first four moments of log-returns distributions are very close to the empirical ones. In particular, the proposed model allows for very high values of the kurtosis.

To complete the empirical analysis, Fig.5 compares the historical path of market prices with some simulated trajectories generated by the estimated model using Monte Carlo techniques, and Table 5 reports the first four moments of the simulated trajectories. The model seems capable to capture the main features of power prices dynamics observed in real markets.

4 Concluding remarks

Within the context of an equilibrium methodology where the supply curve follows a power law in which the exponent is a two-state strictly positive Markov process, we proposed a

	N. South Wales	Tasmania	Victoria
Mean	0.0006	0.0004	0.0009
	(0.0005)	(0.0005)	(0.0005)
Std.Dev.	0.4948	0.4383	0.4472
	(0.0457)	(0.0333)	(0.0418)
Skewness	- 0.4524	0.5868	- 0.3724
	(0.5765)	(0.4965)	(0.6884)
Kurtosis	20.3313	16.2756	21.8586
	(4.2475)	(3.1351)	(5.5528)

Table 4: Simulated moments. Standards errors are between parentheses.

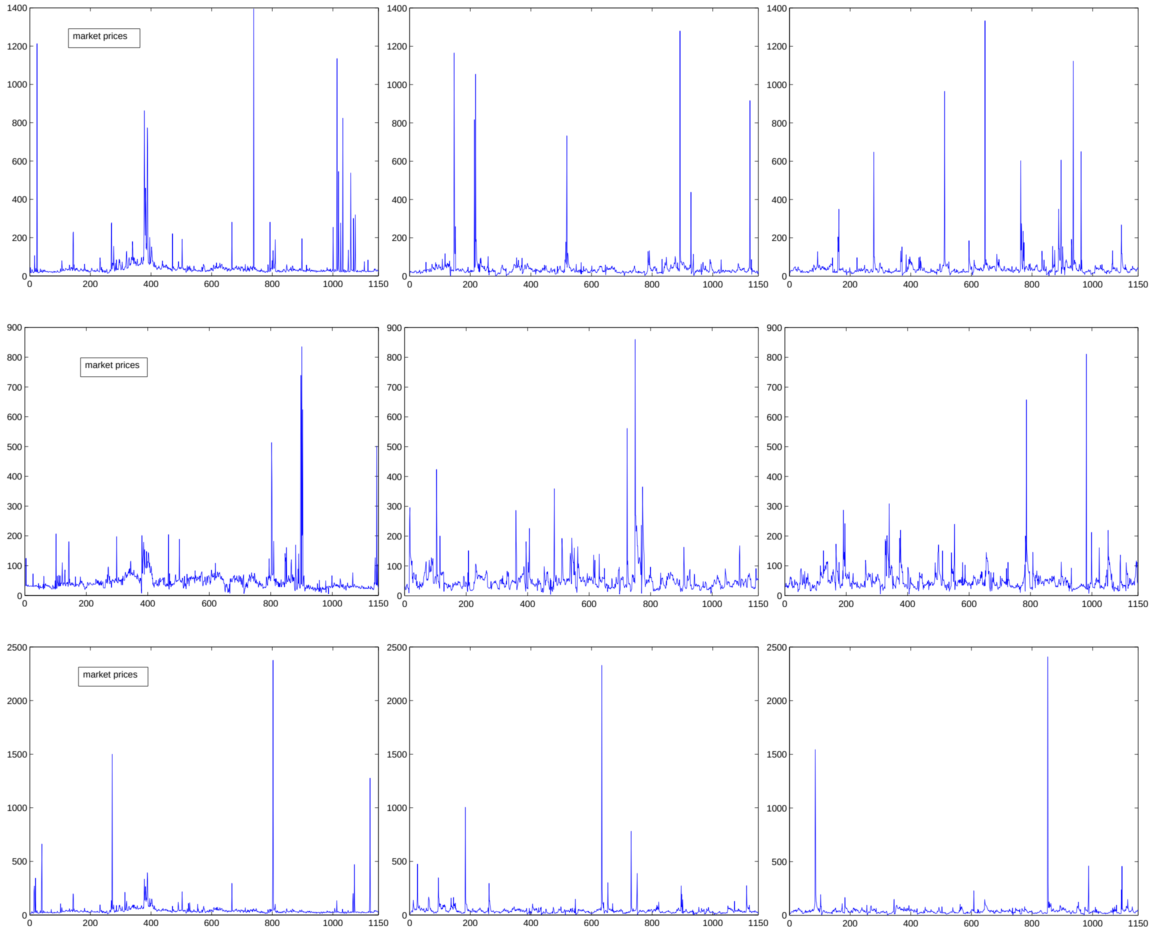

Figure 5: Historical prices and simulated price trajectories at the New South Wales (upper), at the Tasmania (middle), and at the Victoria (lower) markets.

regime-switching model to capture the main features of power prices in deregulated markets. Switches in the dynamics are induced by transitions of the offer curve between Markov exponents. We also showed that the model accounts also for jumps and spikes without introducing spiking mechanisms in a exogenous way. As discussed in the text, the switching

N. South Wales	Mean	Std.Dev.	Skewness	Kurtosis
Observed	0.0003	0.4994	-0.7105	21.8111
Simulated 1	0.0008	0.4980	-0.6379	21.0198
Simulated 2	0.0006	0.4778	-0.7410	20.1859

Tasmania	Mean	Std.Dev.	Skewness	Kurtosis
Observed	-0.0001	0.4295	0.2739	19.0504
Simulated 1	-0.0001	0.4271	0.3332	18.5960
Simulated 2	0.0001	0.4214	0.3352	18.2837

Victoria	Mean	Std.Dev.	Skewness	Kurtosis
Observed	0.0006	0.4426	-0.3666	26.0731
Simulated 1	0.0006	0.4493	-0.3176	25.8648
Simulated 2	0.0007	0.4495	-0.2873	26.4104

Table 5: Descriptive statistics of observed and simulated trajectories.

mechanism in the supply function induces in a natural way a non-Brownian component which is responsible for jumps and spikes of very high magnitude. The model is flexible enough to account for non-zero skewness and very high values of the kurtosis. The empirical analysis reveals that the proposed dynamics reproduces market data in a satisfactory way: the first four moments of the model distributions of log-returns are very close to the empirical ones. It is worth notice that a good representation of observed price movements is crucial to value power derivatives and to define risk management strategies. In particular fitting the first four moments of the empirical distributions is very important for pricing purposes in which the calculus of expectations values is a crucial task [17].

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