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To Angela con cari saluti
& Guardalo per sempre.
Tiene
Tasso

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Dedicated to the memory of Simos Ichtiaroglou

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Modeling Power Prices in Competitive Markets

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Power prices dynamics in deregulated markets appears variable and unpredictable with jumps, spikes, and high, non constant, volatility. Empirical distributions of log-returns are characterized by large values of the standard deviation as well as non-zero skewness and very high kurtosis. In this paper we discuss a reduced-form methodology to describe the dynamics of electricity prices in order to capture the statistical properties observed in real markets. Particular attention will be devoted to regime-switching models which seem good candidates to incorporate the main features of power prices as the seasonality component, the occurrence of stable and turbulent periods, as well as jumps and spikes. Regime-switching models offer, indeed, the possibility to introduce various mean-reversion rates and volatilities depending on the state of the system thus enhancing the flexibility of the reduced-form approach. An empirical analysis performed on market data is provided to test the adaptability of the discussed models in replicating the first four moment of the empirical log-returns distributions.

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1. Introduction

Until early 90's the electricity sector has been a vertically integrated industry in which all the phases of the process, the generation of electric power, transmission, distribution and retailing, were a well organized monopoly. In the last decade several countries all around the world decided to undertake a liberalization process in order to bring competition to the previously monopolistic market. In deregulated markets electricity is a traded commodity and its price is determined according to the rule of supply and demand. As a consequence, the power prices dynamics appears variable and unpredictable with jumps and spikes, and high, non constant, volatility. Empirical distributions of log-returns are characterized by large values of the standard deviation as well as non-zero skewness and very high kurtosis. To a deeper insight, the presence of stable and turbulent periods can be also observed: prices experience normal stable periods in which they fluctuate around some long-run mean, and turbulent periods in which the dynamics is characterized by higher values of the mean-reversion parameter and of the volatility. Randomness in the time evolution of prices implies electricity price risk, which can be hedged if and only if we can dispose of good models to describe the main characteristics of the dynamics. In this paper we propose a reduced-form methodology [1, 2] to model electricity prices dynamics in order to reproduce the statistical properties of prices observed in real markets. Within this approach, regime-switching models seem good candidates to describe the main characteristic of the prices dynamics [3–6]. Regime-switching models offer in fact the possibility to introduce various mean-reversion rates and volatilities depending on the state of the system thus enhancing the adaptability of the modeling procedure. This approach is flexible enough to distinguish the normal stable motion from the turbulent one, and to reproduce jumps and spiky dynamics. Furthermore, regime-switching models can be very useful for financial applications and for energy risk-management: option prices, as well as forward and futures prices, can be obtained as solutions of well defined partial differential equations and are smooth functions of the spot price [7, 8]. Finally, reduced-form models can be incorporated in a demand-supply framework in a quite natural way. Within a demand-supply context, we show that electricity prices dynamics in deregulated markets can be described by modeling the movements of the power margin level in a stochastic environment [9].

The paper is organized as follows. Section 2 presents basic and stylized facts about electricity: the mean-reversion property and the spike phenomenon are then described from a structural perspective of the power market. The reduced-form methodology is discussed in Section 3: four basic models are described in details, namely a jump-diffusion model, and three regime-switching models. The empirical analysis is developed in Section 4 in which the models are estimated on market data. A comparison between the empirical moments of the log-returns distributions and the estimated model distributions is discussed. Section 5 describes the possibility to include the proposed models within a demand-supply framework. Some comments conclude the paper.

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2. Basic facts

In attempt to model a phenomenon, it is very important to put in evidence the structural features of the phenomenon itself on the basis of the observed realizations. Secondly, the basic underlying variables must be identified, and then, the dynamics driving the time evolution of such variables must be specified. The objective is to obtain a good compromise between model parsimony and adequacy to capture the main characteristics of the phenomenon observed in the real world. The observation of the phenomenon is therefore of crucial importance in defining all these steps. In the case of power prices we can use, as empirical basis, the realized market prices. Figures 1 and 2 show the historical behavior of daily base load power prices, calculated as arithmetic averages of the 24 hourly market prices (week-end days have been discarded), respectively at the European Energy Exchange (EEX), the Scandinavian Nord Pool Elspot, and in two American markets, namely at the New England market (NEPOOL) and in the Texas market. The empirical analysis reveals that

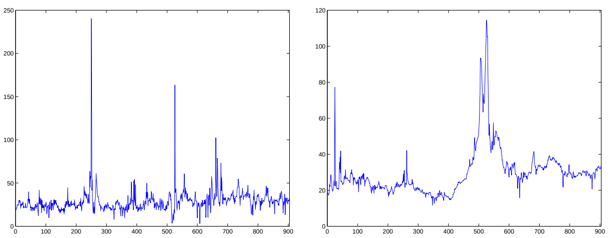

FIG. 1. Historical behavior of prices at EEX (left) and at Nord Pool (right) since January 2, 2001 until June 19, 2004.

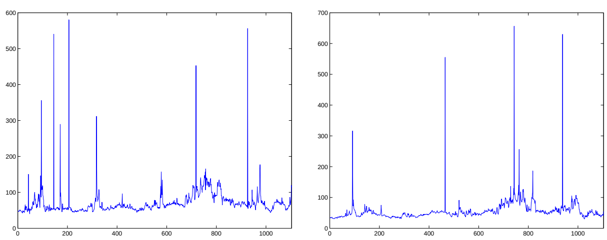

FIG. 2. Historical behavior of prices at EEX (left) and at Nord Pool (right) since January 2, 2001 until June 19, 2004.

observed prices are variable and unpredictable with seasonality, mean-reversion, high volatility, jumps and pronounced spikes. To a deeper insight, the presence of stable and turbulent periods can be also observed: prices experience normal stable periods in which they fluctuate around some long-run mean, and turbulent periods in which the dynamics is characterized by higher values of the mean-reversion and of the volatility parameters. Furthermore, the motion is affected by jumps and short-lived spikes of very large magnitude. The historical behavior of log-returns, calculated as daily changes in natural logarithm of the base load prices, is depicted in Figures 3 and 4. Table

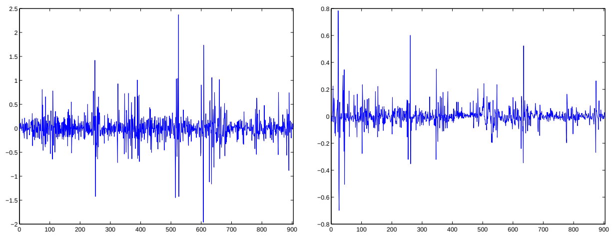

FIG. 3. Historical behavior of log-returns at EEX (left) and at Nord Pool (right) since January 2, 2001 until June 19, 2004.

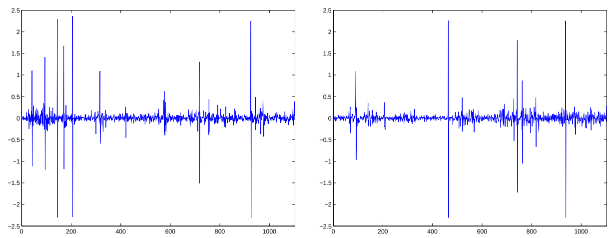

FIG. 4. Historical behavior of log-returns at NEPOOL (left) and in the Texas market (right) since October 22, 2002 until January 15, 2007.

1 displays the descriptive statistics of price returns. All these markets exhibit large prices fluctuations and the presence of jumps and spikes is revealed by non-normal empirical distributions with very high values of the standard deviation as well as non-zero skewness and large values of the kurtosis. Figures 5 and 6 show the empirical distributions of log-returns and a comparison with the fitted normal distribution is proposed. There is evidence that log-returns are non-

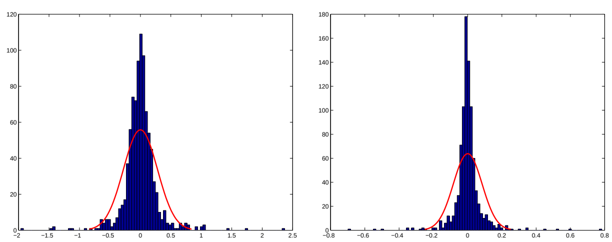

FIG. 5. Empirical distribution of log-returns at EEX (left) and at Nord Pool (right) since January 2, 2001 until June 19, 2004.

normally distributed, and the presence of jumps and

Table 1. Descriptive statistics.

	EEX	Nord Pool	NEPOOL	Texas
Start	Jan 2, 2001	Jan 2, 2001	Oct 22, 2002	Oct 22, 2002
End	June 19, 2004	June 19, 2004	Jan 15, 2007	Jan 15, 2007
n	904	904	1103	1103
Min	-1.9627	-0.6983	-2.3140	-2.3058
Max	2.3694	0.7837	2.3657	2.2591
Mean	0.0005	0.0006	0.0008	0.0004
Std. dev.	0.2797	0.0837	0.2277	0.1905
Skew	0.4677	0.5440	0.3239	-0.1211
Kurt	16.8189	26.6337	66.8421	91.5021

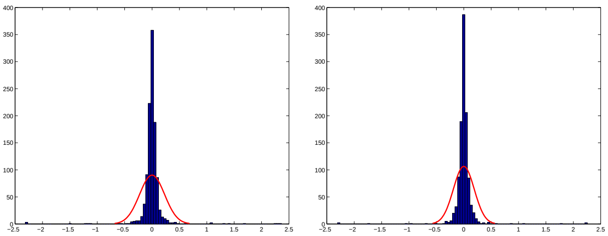

FIG. 6. Empirical distribution of log-returns at NEPOOL (left) and in the Texas market (right) since October 22, 2002 until January 15, 2007.

spikes is revealed by very high values of the daily volatility as well as by large values of the kurtosis. The rule of demand and supply has dramatically increased the volatility of price returns: daily volatilities of about 30% are frequent. Figures 5 and 6 show that prices fluctuations are characterized by very high, non constant volatility.

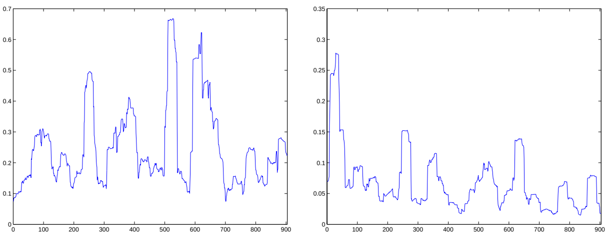

FIG. 7. Historical volatility at EEX (left) and at Nord Pool (right) since January 2, 2001 until June 19, 2004.

2.1. Electricity: a very special commodity

The unusual behavior of power prices can be explained on the basis of some well known properties

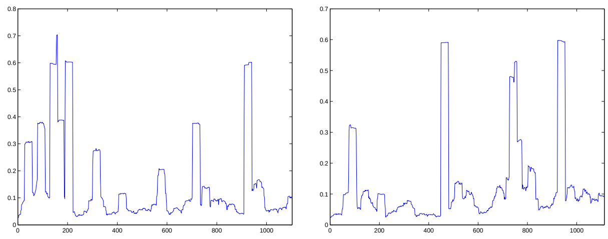

FIG. 8. Historical volatility at NEPOOL (left) and in the Texas market (right) since October 22, 2002 until January 15, 2007.

of electricity. Electricity is in fact a very special commodity: with the exception of hydroelectric power, it cannot be stored and must be generated at the instant it is consumed. As reported in Table 2, the Nord Pool market is the unique market in our analysis which has a significant hydropower production. Nevertheless, all these markets show large prices fluctuations, with extreme jump behavior observed at NEPOOL and in the Texas market. It has been pointed out that, due to the storage capability, hydroelectric energy generation can dampen prices fluctuations and reduce the volatility in the short term, but not in the long term [4]. If we look at the standard deviation of log-returns, Nord Pool exhibits the lowest value (8.37%), significantly below the values observed in the other markets. The interaction

Table 2. Power generation by source in 2002.

	Germany	Nordic Countr.	New Eng.	Texas
Hydro	4 %	47 %	5 %	0 %
Nuclear	28 %	24 %	27 %	9 %
Conv. thermal	62 %	27 %	61 %	88 %
Other sources	6 %	2 %	7 %	3 %

between demand and supply of electricity is very peculiar: the demand is highly inelastic and very sensitive to the temperature and weather conditions; the generation process, i.e. the supply of electricity, is assured by generators with low marginal costs to cover the base load, as hydroelectric plants, nuclear power plants, and coal units. To meet peaks in the demand, emergency units (oil and gas fired plants) with high marginal cost are to be put on operation. Supply curves exhibit therefore a time variable kink after which offer prices rise almost vertically [6]. Figure 9 illustrates the price formation in a demand-supply context. Whenever the load (demand) crosses the offer curve in the rapidly raising part of the curve, electricity prices may assume very high values:

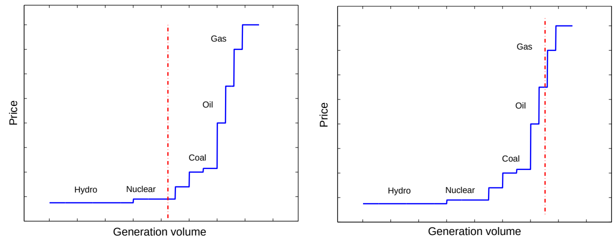

FIG. 9. A schematic supply stack with an hypothetical demand curve superimposed in the flat part of the supply (left) and in the almost vertical part of the supply (right).

a spike occurs when the load intersect the offer curve in the almost vertical part of the curve. This may be due to peaks in electricity demand, forced and planned outages, or shortages in electricity generation as consequences of fluctuations of fuel prices. Finally, electricity must be transported in a transmission network and the perfect operation of the power system is assured if and only if the instantaneous and continuous balancing between the electricity injected into the grid and the energy withdrawn is achieved, if electricity flows along each power line does not exceed the maximum transmission capacity, and if the frequency and the voltage of electricity on the grid is kept within a narrow range [10]. Grid congestions and outages may occur whenever the above quoted constraints are violated. Power prices are therefore very sensitive to the demand, outages and grid congestions: shortages in electricity generation move randomly the position of the kink in the offer curve and jointly with random peaks in electricity demand may determine short-lived spikes of very large magnitude.

3. Modeling power prices

A good representation of the spot electricity prices dynamics is crucial to value power derivatives, as well as to design supply contract, and to define risk management strategies. To model power prices, several approaches have been proposed in literature, and they are characterized by different purposes and finalities [11]. In the so called *cost-based models*, the ultimate objective is to obtain power prices by minimizing the cost of power generation to meet demand in a certain region under operational and environmental constraints. In a similar way, *fundamental equilibrium models* tend to obtain power prices as a solution of well defined equilibrium (between supply and demand) problems. Although both these approaches focus on using some primary factors (fuel prices, temperature, outages) as drivers of power prices, such models are not designed to capture the price dynamics. In the so called *reduced-form models*, the objective is to replicate the statistical properties of electricity prices observed in real markets. They produce stochastic dynamics to describe seasonality, mean-reversion, jumps and spikes, high kurtosis, and regime-switching. Within this framework, standard financial techniques can be used for building risk-management strategies and for pricing energy derivatives. *Hybrid models* fuse the benefits of different methodologies: stochastic techniques are used to describe the dynamics of the underlying drivers (as temperature, fuel prices, and outages); the fundamental methodology to represent demand-supply relations. Equilibrium prices dynamics can be then obtained by the interplay between demand and supply. In this paper we mainly concern with models which can be used in developing risk management tools in order to hedge power price risk. Reduced-form models and hybrid models seem to be good candidates.

Following Lucia and Schwartz [12], we model the dynamics of the natural logarithm of the spot price. By spot price we mean the day-ahead base load price, calculated as arithmetic average of the 24 hourly market prices. We denote by $P(t)$ the spot price at time t of one megawatthour (MWh) of electricity and by $s(t)$ its natural logarithm,

$$s(t) = \ln P(t). \tag{1}$$

We assume that the dynamics of $s(t)$ can be viewed as the sum of two processes

$$s(t) = f(t) + x(t), \tag{2}$$

where $f(t)$ is a highly predictable component accounting for the seasonality effects and $x(t)$ is the random component reflecting unpredictable movements of the prices. To account for the annual and semiannual periodicity of power prices, due to the fact that prices are higher in winter time and in summer time, we assume that the deterministic component of the motion is given by

$$f(t) = \mu + a_0 \cos\left(a_1 + \frac{2\pi t}{261}\right) + b_0 \cos\left(b_1 + \frac{4\pi t}{261}\right), \tag{3}$$

where $\mu = \ln h_0$. In the empirical analysis the parameters μ, a_0, a_1, b_0, and b_1, are determined fitting $f(t)$ to market prices using least squares.

In the remaining part of this Section we will present in some details four models describing the dynamics of the $x-$component accounting for the stochastic movements of power prices.

3.1. A jump-diffusion approach

To describe prices fluctuations in the Nord Pool market, Lucia and Schwartz [12] proposed the following affine mean-reverting diffusion process with constant volatility

$$dx(t) = -\alpha_0 x(t)dt + \sigma_0 dw(t), \tag{4}$$

where α_0 is the mean-reversion rate and σ_0 the volatility parameter; $w(t)$ is a one-dimensional Brownian motion. The model capture the mean-reverting behavior observed in real market but it does not allow for jumps and spikes. To overcome this difficulty and to include stochastic volatility, jump-diffusion processes can be used [13, 14]. In this paper we test on market data the following jump-diffusion model:

Model 1

$$dx(t) = -\alpha_0 x(t)dt + \sigma_0 dw(t) + Jdq(t), \tag{5}$$

where $q(t)$ is a Poisson process with constant intensity λ. In the empirical analysis we assume that the Brownian motion and the Poisson process are independent, and that the random jump amplitude J is distributed according to a normal random variable, $J \sim N(0, \sigma_J)$, with zero mean and standard deviation σ_J. The model describes the mean-reverting behavior of power prices around the stable level μ: normal fluctuations around this point are described by the diffusive component of the motion; Poisson jumps account for unpredictable and pronounced movements of power prices due to shortages in electricity generation, forced outages, and peaks in electricity demand. Even if they are mathematically tractable also in the case including mean-reversion, jump-diffusion models are not well suited to describe electricity prices dynamics: a strong mean-reversion component is necessary to make price spikes shortly-lived.

3.2. A two-regime approach

Regime-switching models offer the possibility to introduce various mean-reversion rates and volatilities depending on the state of the system. This approach is flexible enough to be used in modelling electricity prices dynamics in order to distinguish the normal stable motion from the turbulent and spike regime. We assume that the switching mechanism between the states is governed by un unobservable Markov process. In the two-regime approach, one regime drives the stable motion during normal periods and the second regime is used to account for turbulent periods with high volatility, high values of mean-reversion rate, jumps and short-lived spikes. In this way we can model the prices dynamics in different sectors of the supply stack. The first model we propose is characterized by a two-regime switching dynamics in which the stochastic movements of electricity prices are modelled according to mean-reverting diffusion processes both in the base state and in the excited state [15]. The dynamics can be then cast in the following form:

Model 2

$$dx(t) = \begin{cases} -\alpha_0 x(t)dt + \sigma_0 dw_0(t) \\ -\alpha_1 x(t)dt + \sigma_1 dw_1(t). \end{cases} \tag{6}$$

The first equation describes the normal motion of the system and the second one drives the dynamics in the excited state. Both regimes are characterized by their own mean-reversion rates, α_0 and α_1, and volatility parameters, σ_0 and σ_1. $w_0(t)$, $w_1(t)$ are uncorrelated one-dimensional Brownian motions.

The second model (Model 3) we propose describes a two-regime switching dynamics in which the stochastic movements of electricity prices in the base state are modelled according to a mean-reverting diffusion process; the second regime accounts for turbulent periods of the market in which short-lived jumps and spikes are modelled as Poisson jumps in a mean-reverting jump-diffusion dynamics [5]. The dynamics is given therefore by:

Model 3

$$dx(t) = \begin{cases} -\alpha_0 x(t)dt + \sigma_0 dw_0(t) \\ -\alpha_1 x(t)dt + \sigma_1 dw_1(t) + Jdq(t). \end{cases} \tag{7}$$

$q(t)$ is a Poisson process with constant intensity λ. We assume that the the random jump amplitude J is distributed according to a normal random variable, $J \sim N(0, \sigma_J)$, with zero mean and standard deviation σ_J. The Brownian motions and the Poisson process are independent and independent of the jump amplitude. This parametrization tends to separate the normal diffusive dynamics from the jump regime in a such a way that any turbulence in the power market can be captured by the dynamics of the second regime. The switches between regimes in both models are controlled by the following one-period transition probabilities matrix

$$\pi = \begin{pmatrix} 1-\gamma dt & \eta dt \\ \gamma dt & 1-\eta dt \end{pmatrix}, \tag{8}$$

where γdt denotes the transition probability for the switching from the base state to the second regime in the infinitesimal time interval $[t, t+dt]$ and ηdt is the probability for the opposite transition. γ and η are assumed to be constant. In the matrix, the diagonal terms give the probability of remaining in any given state during the time interval $[t, t+dt]$; the off-diagonal terms represent the transition probability to the other state in the same time interval. The above parametrization allows for multiple jumps and the duration of turbulent periods as well the duration of spikes can be more than one day. Although the model can describe the price movements in terms of one stable regime and one unstable short-lived regime, we will show in the following that the empirical analysis performed on market data reveals that the probability of the price system to remain in the same state is high for the base regime as well for the jump regime. Mean-reversion rates and volatilities are allowed to assume different values in different regimes and the empirical analysis confirms that in the turbulent regime such parameters are larger than the respective values in the base state.

3.3. A three-regime approach

In the three-regime model proposed by Huisman and Mahieu [3], the third regime is used to revert back prices to the stable state when the system is in the excited state. The dynamics of electricity prices is described by:

Model 4

$$dx(t) = \begin{cases} -\alpha_0 x(t)dt + \sigma_0 dw_0(t) \\ \mu_1 dt + \sigma_1 dw_1(t) \\ -\alpha_{-1} x(t)dt + \sigma_{-1} dw_{-1}(t) \end{cases} \tag{9}$$

where w_0, w_1, and w_{-1} are uncorrelated one dimensional Brownian motions. The first equation describes the normal regime in which prices follows a mean-reverting diffusion process; the second-one tries to model spikes using an arithmetic Brownian motion, and the third equation drives prices to revert back to the normal regime after a spike has occurred. The transitions from one state to another is governed by the following transition probabilities matrix

$$\pi = \begin{pmatrix} 1-\gamma dt & 0 & 1 \\ \gamma dt & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \tag{10}$$

The above parametrization means that once the system is in the excited state at time t, it decays with probability one to the base state by a strong mean-reverting dynamics. In this model multiple jumps are not allowed.

4. Empirical analysis

Our data set consists of day-ahead base load prices, calculated as arithmetic averages of the 24 hourly market prices, at the European Energy Exchange (EEX), at the Scandinavian Nord Pool, at NEPOOL and at the Texas market. The seasonal component (3) has been estimated using least squares, and the estimated parameters are reported in Table 3.

Table 3. Estimated parameters of the seasonal component.

	EEX	Nord Pool	NEPOOL	Texas
μ	3.3010	3.3185	4.2054	3.9593
a_0	0.0793	0.1991	0.0036	0.1558
a_1	1.0354	0.1347	-5.2782	1.7437
b_0	-0.0166	0.0553	0.0654	0.0471
b_1	-4.0326	7.1777	3.3015	3.2083

4.1. Model 1

In the empirical analysis the time-step Δt is assumed to be one day and we adopt the following discretised version of the jump-diffusion model:

$$x(t+1) = (1-\alpha_0)x(t) + \sigma_0\epsilon_0(t) + JM(t), \tag{11}$$

where $\epsilon_0(t) \sim N(0,1)$, and $M(t)$ is a random variable assuming the value 1 with probability λ, and the value 0 with probability $1-\lambda$. The parameters of the model have been estimated by maximum likelihood, and the results are shown in Table 4, where we reported, for each market under investigation, the values of the model parameters, the values of the loglikelihood (LL) and the values of the Schwartz criterion (SC). Table 5 displays the first four moments of the model distribution of log-returns obtained averaging over 5000 simulated paths randomly generated using estimated parameters. Even if the agreement with the

Table 4. Model 1 estimation results. Standard errors are between parentheses.

	EEX	Nord Pool	NEPOOL	Texas
α_0	0.2384	0.0076	0.0340	0.0454
	(0.024)	(0.004)	(0.009)	(0.009)
σ_0	0.1337	0.0299	0.0650	0.0667
	(0.006)	(0.001)	(0.002)	(0.002)
λ	0.1693	0.2113	0.0720	0.0497
	(0.024)	(0.021)	(0.009)	(0.009)
σ_J	0.5334	0.1690	0.7988	0.7815
	(0.045)	(0.011)	(0.064)	(0.090)
LL	-2825.4	-1677.9	-3582.5	-3249.5
SC	5678.0	3383.1	7193.1	6527.0

Table 5. Simulated moments using Model 1. Standard errors are between parentheses.

	EEX	Nord Pool	NEPOOL	Texas
Mean	0.0003	0.0003	0.0004	0.0003
	(0.0005)	(0.0007)	(0.0008)	(0.0006)
Std.dev.	0.2730	0.0832	0.2253	0.1878
	(0.0140)	(0.0045)	(0.0200)	(0.0186)
Skew	0.0088	0.0137	-0.0228	-0.0622
	(0.3811)	(0.4436)	(1.3282)	(1.6983)
Kurt	8.8968	11.2864	33.6626	42.9418
	(1.2113)	(1.3949)	(6.9095)	(9.4282)

first two moments is very interesting, the (model) distributions of log-returns show very low values of the kurtosis with respect to the observed values in each market. We recall the importance that a given model captures the first four moments of the empirical distribution. Namely, skewness is related in particular to the properties of upward versus downward moves; the value of the kurtosis describes the tails of the distribution and is particularly important in the case of power prices in which extreme events may occur [10].

4.2. Model 2

The discretised version of Model 2 we use in the empirical analysis is given by

$$x(t+1) = \begin{cases} (1-\alpha_0)x(t) + \sigma_0\epsilon_0(t) \\ (1-\alpha_1)x(t) + \sigma_1\epsilon_1(t) \end{cases} \quad (12)$$

where $\epsilon_0(t)$, $\epsilon_1(t) \sim N(0,1)$ are i.i.d. normal random variables. Regime-switching models can be estimated by maximum likelihood. Following Hamilton [16], in a multi-regime model the likelihood function can be expressed as a linear combination of the likelihoods of the single regimes. The estimation has been performed using the Hamilton filtering technique [17], and the results are shown in Table 6. In Table 7 we reported the first four moments of the model distribution of log-returns obtained averaging over 5000 simulated paths randomly generated using estimated parameters.

4.3. Model 3

The discretised version of Model 3 we use in the empirical analysis is given by

$$x(t+1) = \begin{cases} (1-\alpha_0)x(t) + \sigma_0\epsilon_0(t) \\ (1-\alpha_1)x(t) + \sigma_1\epsilon_1(t) + JM(t). \end{cases} \quad (13)$$

Table 6. Model 2 estimation results. Standard errors are between parentheses.

	EEX	Nord Pool	NEPOOL	Texas
α_0	0.2074	0.0055	0.0317	0.0405
	(0.023)	(0.004)	(0.009)	(0.008)
σ_0	0.1294	0.0293	0.0628	0.0635
	(0.005)	(0.001)	(0.002)	(0.002)
α_1	0.4357	0.0990	0.7059	0.4736
	(0.068)	(0.032)	(0.099)	(0.101)
σ_1	0.5066	0.1622	0.5932	0.6029
	(0.034)	(0.010)	(0.048)	(0.056)
$1-\gamma$	0.9466	0.9099	0.9648	0.9729
	(0.012)	(0.013)	(0.007)	(0.006)
$1-\eta$	0.7785	0.6942	0.6390	0.6394
	(0.048)	(0.049)	(0.064)	(0.072)
LL	-2770.2	-1623.3	-3513.2	-3170.8
SC	5581.2	3287.3	7068.4	6383.6

Table 7. Simulated moments using Model 2. Standard errors are between parentheses.

	EEX	Nord Pool	NEPOOL	Texas
Mean	0.0003	0.0003	0.0004	0.0003
	(0.0004)	(0.0004)	(0.0004)	(0.0004)
Std.dev.	0.2770	0.0832	0.2179	0.1851
	(0.0220)	(0.0058)	(0.0263)	(0.0241)
Skew	0.0224	0.0510	0.0292	-0.0010
	(0.3107)	(0.3220)	(0.7826)	(1.0103)
Kurt	9.6630	10.7365	29.2307	33.2943
	(1.4570)	(1.5427)	(7.1919)	(8.0073)

As in the previous case, the parameters of the model have been estimated by maximum likelihood, and the estimation has been performed using the Hamilton filtering technique. The results are shown in Table 8. Table 9 displays the first four moments of the model distribution of log-returns obtained averaging over 5000 simulated paths randomly generated using estimated parameters. The results show a very interesting agreement: in all the markets under investigation the value of the kurtosis is fairly similar to the observed one and the empirical skewness is situated at about one half of the standard error. The inclusion of Poisson jumps seems to better capture the randomness of power prices due to unpredictable outages and peaks in electricity demand.

Table 8. Model 3 estimation results. Standard errors are between parentheses.

	EEX	Nord Pool	NEPOOL	Texas
α_0	0.1858	0.0056	0.0197	0.0167
	(0.030)	(0.005)	(0.008)	(0.007)
σ_0	0.1246	0.0223	0.0482	0.0367
	(0.005)	(0.001)	(0.002)	(0.002)
α_1	0.4719	0.0306	0.2216	0.0768
	(0.075)	(0.013)	(0.037)	(0.020)
σ_1	0.3617	0.0730	0.1397	0.1065
	(0.036)	(0.006)	(0.008)	(0.007)
λ	0.0641	0.1218	0.0850	0.0489
	(0.040)	(0.033)	(0.022)	(0.013)
σ_J	1.0914	0.2805	1.2119	1.1124
	(0.346)	(0.046)	(0.191)	(0.189)
$1-\gamma$	0.9408	0.8935	0.9476	0.9221
	(0.016)	(0.021)	(0.012)	(0.016)
$1-\eta$	0.8206	0.8635	0.8710	0.9108
	(0.046)	(0.034)	(0.030)	(0.031)
LL	-2757.1	-1584.7	-3441.5	-3079.3
SC	5568.6	3223.9	6939.1	6214.6

Table 9. Simulated moments using model 3. Standard errors are between parentheses.

	EEX	Nord Pool	NEPOOL	Texas
Mean	0.0003	0.0003	0.0004	0.0003
	(0.0004)	(0.0005)	(0.0004)	(0.0005)
Std.dev.	0.2774	0.0828	0.2164	0.1860
	(0.0285)	(0.0077)	(0.0353)	(0.0288)
Skew	0.0140	0.0473	0.1752	-0.1225
	(0.8714)	(1.1351)	(2.7574)	(3.2828)
Kurt	17.0803	23.5910	67.8732	78.4923
	(7.5577)	(6.3148)	(23.2237)	(26.0368)

4.4. Model 4

The discretised version of Model 4 we use in the empirical analysis is given by

$$x(t+1) = \begin{cases} (1-\alpha_0)x(t) + \sigma_0\epsilon_0(t) \\ x(t) + \mu_1 + \sigma_1\epsilon_1(t) \\ (1-\alpha_{-1})x(t) + \sigma_{-1}\epsilon_{-1}(t) \end{cases} \tag{14}$$

where $\epsilon_0(t)$, $\epsilon_1(t)$, and $\epsilon_{-1}(t) \sim N(0,1)$ are i.i.d. normal random variables. As in the previous case, the parameters of the model have been estimated by maximum likelihood and using the Hamilton filtering technique. The results are shown in Table 10. Table 11 displays the first four moments of the model distribution of log-returns obtained averaging over 5000 simulated paths randomly generated using estimated parameters.

Table 10. Model 4 estimation results. Standard errors are between parentheses.

	EEX	Nord Pool	NEPOOL	Texas
α_0	0.2065	0.0059	0.0500	0.0414
	(0.023)	(0.006)	(0.009)	(0.008)
σ_0	0.1470	0.0464	0.0748	0.0735
	(0.004)	(0.002)	(0.002)	(0.002)
μ_1	-0.1118	-0.0176	0.5337	0.4949
	(0.086)	(0.039)	(0.156)	(0.173)
σ_1	0.6630	0.2133	0.7853	0.7031
	(0.064)	(0.029)	(0.106)	(0.130)
α_{-1}	0.6393	0.3049	0.8541	0.7565
	(0.092)	(0.102)	(0.054)	(0.060)
σ_{-1}	0.4992	0.2305	0.3591	0.2911
	(0.052)	(0.030)	(0.052)	(0.050)
$1-\gamma$	0.9313	0.9525	0.9717	0.9784
	(0.012)	(0.009)	(0.006)	(0.006)
LL	-2806.1	-1737.6	-3563.9	-3213.6
SC	5659.9	3522.9	7176.8	6476.3

Table 11. Simulated moments using model 4. Standard errors are between parentheses.

	EEX	Nord Pool	NEPOOL	Texas
Mean	0.0003	0.0002	0.0004	0.0003
	(0.0004)	(0.0004)	(0.0003)	(0.0003)
Std.dev.	0.2859	0.0856	0.2317	0.1791
	(0.0203)	(0.0068)	(0.0299)	(0.0256)
Skew	-0.0717	0.1769	0.2183	0.9361
	(0.4220)	(0.6862)	(0.7102)	(0.8280)
Kurt	12.2632	18.0892	39.8483	44.3733
	(2.2642)	(3.9153)	(9.3625)	(12.3473)

Even if the third regime can be used to add flexibility to the model, the statistical properties of the simulated trajectories are less appealing with respect to those obtained using Model 3. Some generalizations of the three-regime model to allow for multiple jumps can be then considered to better capture the main characteristics of the power prices dynamics in deregulated markets [5].

Finally, we point out that more general dynamics can be included into a regime-switching approach to generate spikes of well defined properties. A three-regime switching model in which the spikes phenomenon is described by a stochastic FitzHugh-Nagumo excitable dynamics has been recently proposed in literature [18].

5. Towards a demand-supply model of electricity prices dynamics

Reduced-form models are not capable of incorporating non-price information. We propose therefore an equilibrium methodology in which the dynamics of power prices is determined by the interplay between demand and supply. Barlow [19] proposed an equilibrium model characterized by a fixed, i.e. time-independent, supply curve of the form

$$P(t) = \begin{cases} \left[\left(a_0 - q(t)\right)/a_1\right]^{1/c} & \text{if } q(t) < a_0 - ba_1 \\ b^{\frac{1}{c}} & \text{if } q(t) \geq a_0 - ba_1 \end{cases} \tag{15}$$

where $q(t)$ is the load at time t, and a_0, a_1, and c are constant parameters. Since the demand is fairly inelastic and it can be represented by a quasi-vertical line, Barlow assumes that

$$q(t) = D(t), \tag{16}$$

where $D(t)$ is a stochastic process independent of the power price $P(t)$. In the proposed model, $D(t)$ is driven by a mean-reverting diffusion process and and the presence of jumps and spikes is assured by the nonlinearity of the supply function. To introduce a time-dependent supply function we can assume, as shown in Figure 10, that the functional form of the offer curve is given by

$$P(t) = h_0 \exp\left[\frac{q(t) - k(t)}{h_1}\right], \tag{17}$$

where $k(t)$ defines the kink position in the offer curve at time t, h_0 and h_1 are normalization parameters [9]. The electricity demand is assumed to be highly inelastic and expressed by

$$q(t) = D(t) \tag{18}$$

for some stochastic process $D(t)$ which is independent of the power price. The equilibrium between demand and supply is assured if

$$P(t) = h_0 \exp\left[\frac{D(t) - k(t)}{h_1}\right] \equiv h_0 \exp(-z(t)), \tag{19}$$

where

$$z(t) = \frac{k(t) - D(t)}{h_1} \tag{20}$$

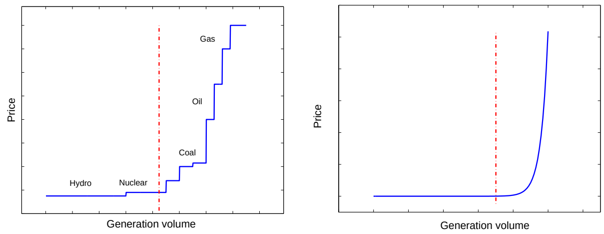

FIG. 10. A schematic supply curve (left) and the exponential representation of the offer (right).

measures the (normalized) *power margin level* at time t, that is the (normalized) difference between the demand level and the kink position in the offer curve at time t. In this approach, the power margin level is assumed as explanatory variable for describing the dynamics of power prices and interpreting the occurrence of the spike phenomenon in electricity markets: we expect that power prices may experience spikes when the margin level approaches zero and becomes negative, that is when the load is greater than the kink value in the offer curve. Power prices are very sensitive to the demand, outages and grid congestions: shortages in electricity generation move randomly the position of the kink in the offer curve and jointly with random peaks in electricity demand may determine short-lived spikes of very large magnitude. The dynamics of $z(t)$ must account for seasonality of the demand and for random fluctuations of the demand level around some long-run mean, for unpredictable supply outages and shortages in electricity generation (moving the position of the kink in the offer curve) as well as for peaks in electricity demand. During normal periods the process $z(t)$ is positive, and we expect that electricity prices experience spikes when $z(t)$ becomes negative and intersect the offer curve in the exponentially raising part of the curve. Seasonal effects in the electricity demand can be then captured if we decompose $z(t)$ as

$$z(t) = f(t) + x(t), \tag{21}$$

where $f(t)$ is a highly predictable component accounting for the seasonality and $x(t)$ is the random component reflecting unpredictable movements of the power margin level. Including h_0 in the exponential representation of prices, we get

$$P(t) = \exp\left(\tilde{f}(t)\right) \exp\left(-x(t)\right), \tag{22}$$

where

$$\tilde{f}(t) = \ln h_0 - f(t). \tag{23}$$

Jump-diffusion models with mean-reversion can be used to describe the dynamical behavior of the power margin around some stable level: normal fluctuations around this point are described by the diffusive component of the motion; Poisson jumps account for unpredictable and pronounced movements of the power margin level due to shortages in electricity generation, forced outages, and peaks in electricity demand. A regime-switching approach is also useful to distinguish the normal stable motion from the turbulent regime. For example, in a two-regime model one regime can be used to drive the stable motion during normal periods and can be described by a mean-reverting diffusion process with constant volatility to account for random fluctuations of the power margin level around the long-run mean. The second regime can used to describe turbulent periods in which peaks in electricity demand, supply outages, and shortages in electricity generation may occur.

6. Concluding remarks

Fundamental variables, as fuel prices, temperature and outages, can be taken into account at this step of the modeling process. Indeed, a stochastic model of the temperature can be used to model the time-evolution of the load curve. On the other side, the dynamics of the supply function can be described by using, as primary drivers, fuel prices. A model to describe planned and forced outages is necessary to complete the time behavior of the supply stack. Several models have been proposed in literature [2, 11]. To make the models realistic more and more fundamental variables can be incorporated: highly complex hybrid models are subject to a significant modeling risk and a balance between model parsimony and adequacy to capture the main characteristics of the power prices dynamics is required to price power derivatives.

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