Social Preferences and Price Cap Regulation

ALBERTO IOZZI
University of Rome "Tor Vergata"

JONATHAN A. PORITZ
IBM Zurich Research Laboratory

EDILIO VALENTINI
University of Chieti

Abstract

This paper analyzes the allocative properties of price cap regulation under very general hypotheses on the nature of society's preferences. We propose a generalized price cap that ensures the convergence to optimal (second best) prices in the long-run equilibrium for virtually any form of the welfare function. Hence, the result of the convergence to Ramsey prices of Laspeyres-type price cap regulation is a particular instance of our more general result. We also provide an explicit and relatively easy to calculate and implement generalized price cap formula for distributionally weighted utilitarian welfare functions, as suggested by Feldstein (1972a).

Alberto Iozzi, University of Rome "Tor Vergata", Dept. SEFEMEQ, Via di Tor Vergata s.n.c., I-00133 Rome, Italy (Alberto.Iozzi@UniRoma2.It). Jonathan A. Poritz, IBM Zurich Research Laboratory, Säumerstrasse 4, CH-8803 Rüschlikon, Switzerland (jap@zurich.ibm.com). Edilio Valentini, University of Chieti "G. D'Annunzio", Dept. of Quantitative Methods and Economic Theory, Viale Pindaro, 42, I-65127 Pescara, Italy, (valentin@dmqte.unich.it).

We are very much indebted to Gianni De Fraja, who read and made valuable comments on previous versions of this paper. We also wish to thank Paolo Bertoletti, Tim Brennan, Simon Cowan, Ninette Pilegaard, Ingo Vogelsang, and seminar audiences at the University of Warwick and Rome III. We received useful comments from participants at the 1999 ITS, EARIE, and SIEP Conferences and EEA Summer School. Comments and suggestions from an associate editor and two referees of this journal have contributed to substantial improvement of the paper. The usual disclaimer applies.

Received April 2000; Accepted March 20001.

Journal of Public Economic Theory, 4 (1), 2002, pp. 95–114.

1. Introduction

The properties of price cap regulation have been rather extensively analyzed in the economic literature. These regulatory schemes leave the pricing decision to the regulated firm and simply require it to choose its prices within a set of permitted prices that is exogenous to the firm's behavior. Hence, the properties of such schemes in terms of productive efficiency are those typical of fixed-price contracts: since the firm is the residual claimant of any efficiency gains, the regulatory mechanism does not alter its incentives to productive efficiency.

Price caps have also very desirable properties in term of allocative efficiency. It has been shown (Vogelsang and Finsinger 1979; Bradley and Price 1988; Brennan 1989; Vogelsang 1989; Vickers 1997) that, when the regulated firm is a multiproduct monopolist and under some other conditions, an appropriate definition of the rule that determines the set of permitted prices ensures the long-run convergence toward Ramsey prices. These are the prices that ensure the highest possible welfare for the society, when this is defined as the sum of consumers' surplus and the firm's profits and provided that the firm is able to obtain a given amount of profits. In particular, convergence to Ramsey prices occurs under Laspeyres-type price cap regulation. This requires the firm to set prices such that the weighted average of prices' changes in any period of time is smaller than a given value, where the weights are the firm's revenue shares in the previous period.

The regulatory reform undertaken in many countries since the beginning of the 1980s has seen in most cases the adoption of price cap based regulatory schemes. This large diffusion of price cap regulation was powered by the very strict correspondence between the properties of these schemes and the objectives that were set out by governments for regulatory activity. Indeed, the main objective of regulatory activity in those years was the pursuit of efficiency, with very little emphasis on distributional issues. Since efficiency may clearly be pursued by increasing productive efficiency but also through more efficient price structures and given the properties of price cap regulation detailed above, the choice of price caps as the main regulatory instruments by many governments and regulatory agencies seemed therefore very natural.

However, in recent years there has been growing concern for the social consequences of this focus on the efficiency of regulatory activity. This has also been motivated by a (probably belated) recognition of the social nature of the commodities that are supplied by most regulated firms (Waddams Price, forthcoming). The concern for the social consequences of regulation has caused growing attention to the distributional effects that regulatory activity has had in the past and also attention to a modification of the objectives of regulation, which in many cases is now required to pursue broader social objectives.

Given the relationship that has been described above between price cap regulation and efficiency, it comes as no surprise to observe that this form of regulation may have adverse distributional effects. The undesired effects of price cap regulation may be the direct result of the relationship between the prices charged by a price capped firm and Ramsey prices or, more indirectly, may be simply the effect of the pricing discretion allowed to the regulated firms. As for the first issue, it is well known that, despite their optimality, Ramsey prices may indeed have adverse distributional effects. This is because they entail higher markups on those goods with lower demand elasticity. As these are often the goods that represent a large share of low-income consumers' expenditures, Ramsey prices may adversely affect the welfare of these consumers. For instance, these effects have been analyzed in the United Kingdom by Hancock and Waddams Price (1995) and Waddams Price and Hancock (1998).[1] They analyze the noticeably regressive effect of the changes of prices observed in most regulated industries due to a movement toward a more cost-related (and, in some cases, demand-related) price structure since the adoption of price cap schemes.

On the other hand, prices charged under price cap regulation may have undesired effects because regulated firms choose prices that pursue their own objectives but go against some notion of efficiency and/or social equity held by the regulator. For instance, Oftel, the regulator for the UK telecommunications industry, recognized that, despite the fact that the average level of prices charged by British Telecom has decreased dramatically in the last 15 years, the change in the structure of these prices has primarily benefited business and high-expenditure residentials, with very little advantage accruing to low-consumption residential users (Oftel 1997). This has been a joint effect of the pricing discretion left to the regulated firm and of the different speeds at which competition has developed in the different markets of the TLC industry. British Telecom has preferred to concentrate the greater part of the price reductions in those markets, such as business users and high-consumption residential, where it faced competition by other operators. Similarly, Giulietti and Waddams Price (2000) review how firms that were subject to price cap regulation in the United Kingdom and the United States have rebalanced their prices under this constraint. They argue that the choices of the regulated firms of their price structure seem to be motivated mainly by strategic reasons and by concern with long-term issues of resetting the cap.

As discussed above, the new concern for the social consequences of regulation has also taken the form of a major review of the regulatory activity. This is what has happened in the United Kingdom, where the general attitude of the labor government toward regulation has been stated since 1998 in the Green Paper "A Fair Deal for Consumers" (Depart-

[1] See also Burn, Crawford, and Dilnot (1995).

ment of Trade and Industry 1988). In this document, the government states the intention to build a general framework of the regulatory activity that ensures that "the consumer comes first." One of the instruments to this end is an amendment of the statutory duties of regulators; it is stated that "a primary duty on the regulators to protect the interests of consumers [. . .] will help to ensure lower prices and better standard of services." The Green Paper also suggests that the government should issue statutory guidance on social objectives "to ensure regulation takes into account the need of disadvantaged consumers, including those on low incomes." The review of the regulatory activity started with this Green Paper has also continued in the following years. A Social Action Plan to tackle fuel poverty has been developed by the electricity and gas regulator between 1998 and 2000 (Offer and Ofgas 1998; Ofgem 1999a, 1999b, 2000).[2] More important, the Utilities Act 2000 has shifted the emphasis of regulation toward distributional issues. This has been primarily done through a redefinition of the main objective of the regulatory activity of Ofgem, which has now become to protect the interests of consumers and also to be concerned with the interests of low-income and disadvantaged consumers and in addition to give a new role to the (formerly separated) Consumer Council.

This redefinition of the objectives of the regulatory activity naturally calls for a reconsideration of the most appropriate instruments that may be used to pursue such objectives. The main issue that this paper addresses is whether price cap regulation is an appropriate instrument to pursue distributional objectives. At first sight, given the relationship existing between the prices set by a price capped firm and Ramsey prices and given the distributional effects of these prices, one could expect that price cap regulation is not able to guarantee the pursuit of objectives different from productive efficiency and allocative efficiency, as it would be intended by a utilitarian social planner.

In contrast, we find that price cap regulation is a suitable instrument to pursue allocative efficiency for a much wider class of social welfare functions. We describe a price cap mechanism that generalizes the mechanism currently in use and that is able to guarantee the pursuit of distributional objectives. In particular, the price cap mechanism we propose guarantees the convergence to optimal (second-best) prices in the long-run equilibrium for a very large class of society's preferences. We also show that the Laspeyres-type price cap is a special case of our more general mechanism. Hence, one has to conclude that adverse distributional effects of Laspeyres-type price cap regulation are due to the rule

---

[2]In spite of a formal separation, the electricity and gas industries had the same director general (DG) since January 1, 1999. The Utilities Act 2000 has established the Gas and Electricity Markets Authority (Ofgem), whose DG has had formally transferred all the functions of the formerly distinct DGs.

that—under a Laspeyres-type price cap—determines the set of permitted prices for the firm rather than to the price cap mechanism itself.

The generalization of the price cap mechanism we propose basically is a modification of the weights used in the price cap formula. This does not substantially alter the simplicity typical of traditional price cap schemes nor does it impose much higher informational requirements on the regulator's side. To substantiate this argument, we provide an explicit and relatively easy to calculate and implement price cap formula for distributionally weighted utilitarian welfare functions, as proposed by Feldstein (Feldstein 1972a, 1972b, 1972c) and then largely used in the optimal taxation literature. In this welfare function, the weight attached to the surplus of each consumer is her marginal social utility of income. This gives the marginal increase in the welfare of the society due to an increase of the income of that consumer. Such a modified price cap guarantees the convergence to optimal prices when the preferences of the society take this form.

Furthermore, by referring to the case of the new price cap formula that has been imposed on British Telecom since 1997, we show the usefulness of the theoretical framework derived in this paper also for the purposes of evaluating the properties of price cap based regulatory policies and of ascertaining the regulator's objectives from the choice of the price cap formula adopted. In particular, by showing the relationship existing between this price cap and the generalized price cap we propose in this paper, we show that, by adopting this new price cap formula, Oftel has chosen a regulatory instrument that is actually able to take care of the welfare of the consumers who were intended to benefit from this provision and to pursue these distributional objectives in a socially optimal manner (at least in a long-term perspective).

The structure of the paper is as follows. Section 2 describes the characteristics and the properties of the generalization of the price cap mechanism we propose under very general conditions on the nature of the preferences of the society. Section 3 contains an application of this general result for the case of a distributionally weighted additive welfare function and of consumers with quasi-linear preferences. Section 4 presents some concluding remarks and applies the theoretical framework derived in the paper to the analysis of the price cap formula imposed on British Telecom since 1997. Finally, the Appendix reformulates one of the results of the paper and provides an alternate proof when the uniqueness assumption of the solution to the problem of welfare maximisation is relaxed.

2. The Generalized Price Cap (GPC)

2.1 The Setup

We assume that there exist M markets. Let $\boldsymbol{p}^t$ be the M-dimensional vector of market prices at time t, where $t = 0,\ldots,\infty$. To simplify notation, we drop the superscript t whenever this is unambiguous. We denote by $\boldsymbol{q}(\boldsymbol{p})$ the M-dimensional vector of market demand functions, which are assumed to be continuous, downward sloping, and time invariant. Further, we assume that, for all m (i) $q_m(\mathbf{0}) \in \Re_+$ and (ii) there exists a $\Gamma > 0$ such that for all $\boldsymbol{p}' \in \Re_+^M$ satisfying $|\boldsymbol{p}'| > \Gamma$, we have $q_m(\boldsymbol{p}') = 0$.[3]

A multiproduct firm produces the M goods in each period of time. The firm is a monopolist in all the M markets. Production costs are denoted by $c(\boldsymbol{q})$, which is assumed to be continuously differentiable and constant over time. Let $c_m \equiv \partial c(\boldsymbol{q})/\partial q_m$ for $m = 1,\ldots, M$. The firm's profits are given by $\Pi(\boldsymbol{p}) = \sum_m p_m q_m(\boldsymbol{p}) - c(\boldsymbol{q}(\boldsymbol{p}))$. The firm is assumed to myopically maximize its profits in each period of time t.

Society's preferences are given by the welfare function $W(\boldsymbol{p})$, which is assumed to be continuously differentiable and quasi-convex. It is also assumed that the gradient of $W(\boldsymbol{p})$ is different from zero for any vector of prices $\boldsymbol{p}$ such that $\Pi(\boldsymbol{p}) \geq \Pi(\boldsymbol{p}^0)$ and $W(\boldsymbol{p}) \geq W(\boldsymbol{p}^0)$, where $\boldsymbol{p}^0$ denotes the price vector at time 0.[4]

A benevolent regulator offers the firm a regulatory contract that must ensure that the firm receives a minimum level of profits $\bar{\Pi} \in [0, \Pi(\boldsymbol{p}^m)]$, where $\boldsymbol{p}^m$ is the vector of unconstrained monopoly prices. Moreover, $\Pi(\mathbf{0}) < \bar{\Pi}$ and $\lim_{p\to+\infty} \Pi(\boldsymbol{p}) < \bar{\Pi}$, which together imply an interior solution for the firm's maximization problem.

2.2. The Full Information Benchmark

A fully informed regulator with the right to set prices would simply choose the prices for the goods produced by the firm that maximize the social welfare subject to the constraint that the firm obtains profits at least equal to $\bar{\Pi}$. That is,

$$\max_{p} W(\boldsymbol{p}) \quad \text{s.t. } \Pi(\boldsymbol{p}) \geq \bar{\Pi}. \tag{1}$$

Let $\boldsymbol{p}^* = (p_1^*, \ldots, p_M^*)$ be a price vector that solves this problem. $\boldsymbol{p}^*$ is implicitly given by the $m + 1$ conditions

[3]This assumption, equivalent to assuming that the choke price is positive and finite in the case of independent demands, is needed to ensure that the set of prices that gives positive revenues is compact.

[4]This purely technical assumption is necessary because of the crucial role played by the gradient of $W(\boldsymbol{p})$ in the formula of the generalized price cap. Since profits and welfare are weakly increasing over time under the GPC (as will be clear in what follows), it is sufficient to restrict this assumption only to those price vectors such that profits and social welfare are higher than at time 0, that is, to all the price vectors that may be part of the sequence of prices chosen by the regulated firm. Notice that to comply with this assumption it is sufficient (but not necessary) that the welfare function is a strictly decreasing function in prices.

$$\begin{cases} \Pi(\boldsymbol{p}^*) = \bar{\Pi} \\ \left.\dfrac{\partial W}{\partial p_m}\right|_{p^*} + \lambda \left.\dfrac{\partial \Pi}{\partial p_m}\right|_{p^*} = 0 \end{cases} \quad \text{for } m = 1, \ldots, M, \tag{2}$$

where λ is the Lagrange multiplier. We shall assume that this vector exists and is unique for any level of the firm's profits $\bar{\Pi}$ in (1).

2.3. The Properties of the Generalized Price Cap

This section describes the properties of the regulatory contract that, at any time t from 1 to infinity, constrains the firm to choose prices that satisfy the following:

$$\sum_m p_m^t \cdot \left.\frac{\partial W}{\partial p_m}\right|_{p^{t-1}} \geq \sum_m p_m^{t-1} \left.\frac{\partial W}{\partial p_m}\right|_{p^{t-1}}. \tag{3}$$

In the rest of the paper, we call this constraint (3) the Generalized Price Cap (GPC).[5] Upon acceptance of this regulatory contract, in each period t the regulated firm faces the problem of choosing prices such that

$$\begin{aligned} &\max_{p^t} \sum_m p_m^t \cdot q_m^t - c(\boldsymbol{q}^t(\boldsymbol{p}^t)) \\ &\text{s.t. } \sum_m p_m^t \cdot \left.\frac{\partial W}{\partial p_m}\right|_{p^{t-1}} \geq \sum_m p_m^{t-1} \cdot \left.\frac{\partial W}{\partial p_m}\right|_{p^{t-1}} \quad \text{for any } t = 1, \ldots, \infty. \end{aligned} \tag{4}$$

Note that for the regulator to check that the firm is complying with the contract, it only needs to know the form of the welfare function and the prices set by the firm in the previous period.

Now we can state the following proposition.

PROPOSITION 1: *Under this regulatory contract, social welfare is monotonically nondecreasing in time.*

Proof: First note that, by assumption, the gradient of $W(\cdot)$ cannot be equal to zero at any price vector charged by the firm in any period of time t. Since $W(\cdot)$ is quasi-convex, $\sum_m (p_m^t - p_m^{t-1}) \cdot \left.\frac{\partial W}{\partial p_m}\right|_{p^{t-1}} \geq 0$ implies $W(\boldsymbol{p}^t) \geq W(\boldsymbol{p}^{t-1})$. ■

This result can be illustrated with the aid of Figure 1, drawn for the simplified two-good case. In the figure, W^{t-1} is the iso-welfare curve going

[5] The direction of this inequality seems to be the opposite than expected. Notice however that this is simply due to the negative values of the derivatives of the welfare functions.

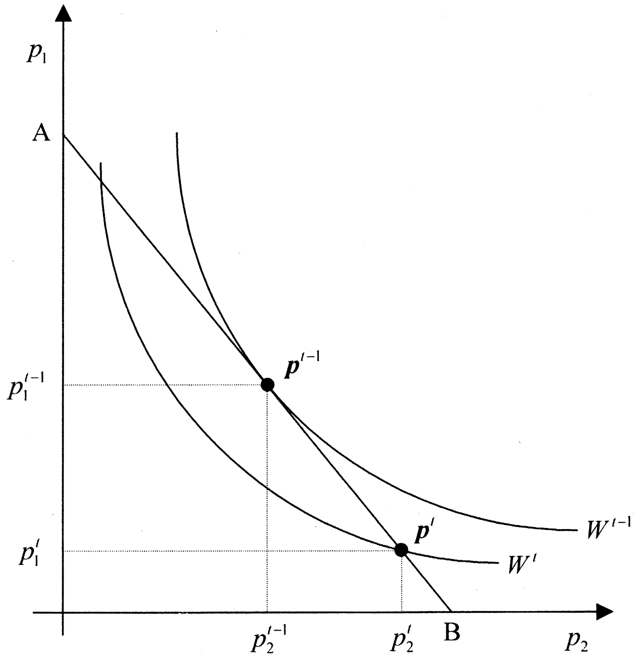

Figure 1: The Generalized Price Cap

through the price vector $\boldsymbol{p}^{t-1}$. By totally differentiating $W(\cdot)$, it is straightforward to show that the slope of W^{t-1} at $\boldsymbol{p}^{t-1}$ is $-(\partial W/\partial p_1^{t-1})/(\partial W/\partial p_2^{t-1})$. Note that this is also the slope of the price cap constraint imposed on the firm. As the prices set by the firm at time $t-1$ always satisfy the price cap constraint at time t, the tangent to $W(\cdot)$ at $\boldsymbol{p}^{t-1}$ and the price cap constraint are actually the same line. Thus, the price cap restricts the set of feasible prices for the firm at time t to those on (or below) a line tangent to the iso-welfare curve going through the prices set at time $t-1$. This is illustrated in the figure, where the line AB gives both the tangent to $W(\cdot)$ at $\boldsymbol{p}^{t-1}$ and the price cap constraint at time t.

Because of the quasi-convexity of the welfare function, the price cap constraint never lies above the iso-welfare line at $t-1$. Thus, it cannot happen that a vector of prices selected by the firm at time t and satisfying the price cap constraint reduces social welfare.

We also have the following proposition.

PROPOSITION 2: *Let $\{\boldsymbol{p}^t\}$ be the sequence of prices chosen by a regulated firm as the solution to (4). $\{\boldsymbol{p}^t\}$ converges to a unique price vector that satisfies the first-order condition for a constrained welfare maximum.*

Proof: Because of the assumptions on the vector of market demand functions $\boldsymbol{q}(\boldsymbol{p})$, the set $K = \{\boldsymbol{p} | \Pi(\boldsymbol{p}) \geq \bar{\Pi}\}$ is compact. As the firm can at worst keep its profits fixed in each successive time interval, the sequence of profits $\{\Pi(\boldsymbol{p}^t)\}$ is nondecreasing, and thus the sequence $\{\boldsymbol{p}^t\}$ lies entirely in K. Let $\boldsymbol{p}^\infty$ be an accumulation point of $\{\boldsymbol{p}^t\}$ and notice that $\Pi(\boldsymbol{p}^\infty) = \lim_{t\to\infty} \Pi(\boldsymbol{p}^t)$, again since the profits form a nondecreasing sequence. We will denote this limiting maximum profit by Π^∞ and write $K^\infty = \{\boldsymbol{p} | \Pi(\boldsymbol{p}) \geq \Pi^\infty\}$. Denote also $\Sigma^\infty = \Pi^{-1}(\Pi^\infty)$ the iso-profit surface at $\boldsymbol{p}^\infty$.

In a situation such as this, $\nabla W(\boldsymbol{p}^\infty)$ must be normal to the iso-profit surface Σ^∞ at $\boldsymbol{p}^\infty$. Suppose not: then the half-space of prices $\boldsymbol{p}$ satisfying the constraint $(\boldsymbol{p} - \boldsymbol{p}^\infty) \cdot \nabla W(\boldsymbol{p}^\infty) \geq 0$ contains points in the interior of K^∞, that is, price vectors whose profits are greater than Π^∞. The continuity of the first derivatives of W implies that for some sufficiently large T with $\boldsymbol{p}^T$ very close to $\boldsymbol{p}^\infty$ the same will be true at $\boldsymbol{p}^T$. But then the firm at time $T + 1$ would have chosen a price vector $\boldsymbol{p}^{T+1}$ yielding a profit greater than Π^∞, which contradicts the maximality of Π^∞. Thus at $\boldsymbol{p}^\infty$, ∇W is normal to the iso-profit surface, which is exactly the content of the first-order conditions for a constrained welfare maximum.

As by assumption there is only one point satisfying these conditions for any level of the firm's equilibrium profits, $\boldsymbol{p}^\infty$ must be unique.[6] ■

The proposition argues that, when the regulated firm faces a constraint as in (3), the only long-run equilibrium is such that the firm chooses the price vector that maximizes social welfare, given that the firm obtains that amount of profits in equilibrium.

The basic reason for our result is the following. Prices satisfying (2) simply come as the result of the maximization of social welfare given a constraint on the minimum profit level. The same price vector can also be obtained as the solution to the dual problem of maximizing firm's profits under a constraint of a minimum level of welfare. The constraint in (3) simply acts as a linear approximation of the constraint on the welfare when this is fixed at the level $W(\boldsymbol{p}^{t-1})$. In the two-goods example of Figure 1, in any period the constraint is defined by a line tangent to the iso-welfare contour at the prices set in the previous period. Because of the convexity of W, this constraint thus requires the firm to set prices where the social welfare is no smaller than in the previous period.

An intuitive illustration of the reasons of convergence to optimal prices goes as follows. In any period of time, the profit maximizing monopolist chooses its optimal price vector $\boldsymbol{p}^t$ such that the upper contour set

---

[6]The Appendix reformulates this proposition and provides an alternate proof when the uniqueness assumption of the solution to (2) is relaxed.

$\Pi(\boldsymbol{p}^t)$ is tangent to the GPC constraint in (3). Recall that the GPC is by construction the slope of the welfare function at $\boldsymbol{p}^{t-1}$ prices. Then, if $\boldsymbol{p}^t$ is different than $\boldsymbol{p}^{t-1}$, the GPC at $t+1$ will be different from the one at time t. If, on the other hand, $\boldsymbol{p}^t$ is equal to $\boldsymbol{p}^{t-1}$, it implies that the profit function and the welfare function are tangent to each other at $\boldsymbol{p}^t$ and are both also tangent to the GPC at time t. This is exactly what the constrained welfare maximization requires. Together with the fact that the GPC will not move in the following period, this ensures that $\boldsymbol{p}^t$ satisfies the necessary condition for a maximum.

The main novelty of this result is that it provides a description of the properties of price cap regulatory schemes under a very general hypothesis on the structure of the preferences of the society. Our result then gives a generalization of those in the existing literature that have shown that Laspeyres-type price cap regulation guarantees the convergence to Ramsey prices. However, Ramsey prices are optimal as long as the welfare function is strictly utilitarian, that is, when the welfare function is a simple sum of the individuals' welfare. Optimality characteristics of Laspeyres-type price cap regulation are then dependent on the society's preferences being unconcerned with the distribution of welfare across individuals.

Our result shows that the Generalized Price Cap is able to guarantee a long-run equilibrium with optimal prices for almost *any* welfare function. The only restriction on the welfare function that is needed for our result is that the welfare function is quasi-convex. Then, for almost any type of social preferences, the only long-run equilibrium from the application of the GPC entails the firm setting the price vector that also delivers the highest social welfare, given the profit obtained by the firm in equilibrium. The properties of Laspeyres-type price cap regulation with respect to Ramsey prices come simply as a special case of our more general result for the case of utilitarian welfare function when the consumers have quasi-linear preferences. Indeed, when the welfare function is strictly utilitarian and consumers have quasi-linear preferences, $\partial W/\partial p_m = -q_m$, for all $m = 1, \ldots, M$, and the GPC simply takes the form of a Laspeyres-type price cap.

Note also that the GPC is fully consistent with the hypothesis typical in the economic of regulation on the information available to the regulator. As already noted, a regulatory contract based on the GPC implies that, in each period, the regulator simply knows the welfare function and the price vector chosen by the firm in the previous period.

However, the convergence to optimal prices under the GPC depends crucially on the initial conditions and, in particular, on prices and profits. Initial prices play a crucial role in allowing the regulatory mechanism to work as they determine the level of profits at the beginning of the regulatory process. On the one hand, the firm's initial profit needs to be higher than its reservation level. This guarantees that the firm is willing to participate in the first period. But this also guarantees that the firm is willing to participate in any following period. This is because profits are weakly increasing over time. Indeed, as the regulatory constraint in (3) is a weak inequality, in any period the firm can at least obtain the same profits as in the previous period by setting prices identical to those chosen in the previous period. Then, by revealed preference, if it chooses different prices, it must obtain profits higher than in the previous period. On the other hand, if initial profits are higher than $\bar{\Pi}$, which is higher than the level of profits the firm earns in the long-run equilibrium, even a myopic firm would not change the price vector initially set.[7]

Furthermore, the initial level of prices determines the prices charged in the long-run equilibrium and the corresponding levels of profit and welfare.[8] Indeed, we have already noted that, assuming regularity of the maximization problem in (1), there exists *one* (and only one) price vector that satisfies the necessary conditions for a welfare maximum for *any* level of profits allowed to the firm. Our results show that the sequence of price vectors chosen by the regulated firm under the GPC converges to *one of the price vectors* that satisfy these conditions. However, the GPC does not have any built-in mechanism that is able to control or predict, at the beginning of the regulatory process, the price vector prevailing in the long-run equilibrium among the many price vectors that satisfy the necessary conditions for a welfare maximum, one for each admissible level of $\bar{\Pi}$. The long-run equilibrium price vector depends primarily on the initial prices vector and on demand and cost conditions. Similarly, these conditions determine the long-run equilibrium profits and welfare.[9] Since we are assuming that the regulator does not have information on the demand schedules and on the firm's cost function, the regulator cannot anticipate

[7] The relationship between initial conditions and the long-run equilibrium has been a rather neglected topic in the literature on price cap regulation. Armstrong and Vickers (1991) use a static model to investigate the welfare consequences of allowing third-degree price discrimination by a pure monopolist subject to average price regulation. They show that if the regulatory constraint is based on a price index with nondiscriminatory quantity weights, price discrimination increases social welfare. On the other hand, if weights are based on current quantities (or equivalently, under average revenue regulation), price discrimination increases the monopolist's profit but decreases consumer surplus.

[8] The only attempt (to the best of our knowledge) to investigate the relationship between the prices charged by a price capped firm at the beginning of the regulatory contract and the prices charged by the same firm in the long-run equilibrium is due to De Fraja and Iozzi (2000). They show that, given two initial price vectors, an initial price vector that is preferable in the current period may not be preferable along the path of convergence to the Ramsey prices. In other words, prices leading to higher welfare in the short run may turn out to lead to lower welfare in the long run.

[9] This is not the case, for instance, of the regulatory mechanism proposed by Vogelsang and Finsinger (1979). They propose a price constraint that, in term of Figure 1, changes slope in any period of time according to the prices chosen by the firm in the previous period. However, the intercept of their constraint is reduced in any period by the rate of profits obtained by the firm in the previous period. They show that this constraint is able to force the firm in the long-run equilibrium to charge Ramsey prices and to obtain zero profits.

the price vector that will prevail in the long-run equilibrium, and, at the same time, it has no control over the long-run equilibrium levels of profits and welfare.

3. The GPC and Distributionally Weighted Utilitarian Preferences

This section provides an illustrative application of the properties of the Generalized Price Cap. While in the previous section we derived our results under very general conditions, we assume here a well-specified form of both consumers' and society's preferences. This allows us to specify the Generalized Price Cap in a format that is much more suitable for practical application by regulators.

3.1 The Setup

In this section, we assume that demand originates from N consumers with quasi-linear preferences. Indirect utility to the nth consumer (where $n = 1, \ldots, N$) is given by $V^n(\boldsymbol{p}, y^n) = v^n(\boldsymbol{p}) + y^n$, where y^n is the consumer's income, which we take to be constant over time for all individuals. Denoting by $q_m^n(\boldsymbol{p})$ the demand of the mth good by the nth consumer (where $n = 1, \ldots, N$ and $m = 0, \ldots, M$), aggregate demand in each market is denoted by $q_m(\boldsymbol{p}) \equiv \sum_{n=1}^{N} q_m^n(\boldsymbol{p})$. Aggregate demand functions are assumed to have the same properties as detailed in Section 2.1.

With quasi-linear preferences, the consumer's surplus is an exact measure of the welfare of the consumer (see, e.g., Bös 1981). Denoting by S^n such measure, we have

$$S^n(\boldsymbol{p}) = \sum_{m=1}^{M} \int_{p_m} q_m^n(\boldsymbol{p})\, dp. \tag{5}$$

From Roy's identity in the case of quasi-linear preferences

$$q_m^n(p_m) = -\frac{\partial S^n(\boldsymbol{p})}{\partial p_m} \quad \text{for } \forall\, m = 1, \ldots, M. \tag{6}$$

Following Feldstein (1972a, 1972b, 1972c) and the optimal taxation literature (see, e.g., Myles 1995), we assume that the preferences of the society can be described by a distributionally weighted additive welfare function. Formally,

$$W(\boldsymbol{p}; \boldsymbol{y}) = \sum_{n} S^n(\boldsymbol{p}) \cdot u'(y^n) \tag{7}$$

where $\boldsymbol{y}$ is the N-dimensional vector of consumers' income, y^n is the income of the nth consumer, and $u'(y^n)$ is the marginal social utility of a small increase in his or her income. We assume $u' > 0$ and $u'' \leq 0$.

This social welfare function says that the welfare of the society is given by the weighted sum of the individuals' welfare, where the weights are given by the marginal social utility of income for each individual. Under the above standard assumptions on the shape of $u(\cdot)$, (7) implies that society values more an increase in utility by a low-income consumer than an equal welfare increase for a high-income individual.

From the convexity in prices of the indirect utility function in the case of quasi-linear preferences and from the nature of $u(\cdot)$, it follows that $W(\boldsymbol{p}; \boldsymbol{y})$ is itself a convex function of prices, being a positive linear combination of convex functions.

Let

$$R_m = \frac{\sum_{n=1}^{N} q_m^n(\boldsymbol{p}) \cdot u'(y^n)}{\sum_{n=1}^{N} q_m^n(\boldsymbol{p})} \quad \text{for any } m = 1, \ldots, M. \tag{8}$$

R_m is the distributional characteristic of the good m. It is a weighted average of the marginal social utilities, where each consumer's marginal social utility is weighted by that consumer's consumption of good m. The conventional welfare assumption that $u'(y)$ declines as y increases implies that the value of R_m will be greater for a necessity than for a luxury.[10]

3.2 The Full Information Benchmark

Having set to $\bar{\Pi}$ the minimum level of profits that can be obtained by the monopolist in any period of time, optimal (second-best) prices are given by the solution to the following maximization problem:

$$\begin{aligned} &\max_{p} \sum_{n} S^n(\boldsymbol{p}) \cdot u'(y^n) \\ &\text{s.t. } \Pi(\boldsymbol{p}) \geq \bar{\Pi}. \end{aligned} \tag{9}$$

Assume now that a solution to this problem exists and it is unique. Its features are described in the following proposition.

[10]Luxuries are goods that take a larger share of the budget of consumers with higher income and vice versa for necessities.

PROPOSITION 3: *Let $\hat{\mathbf{p}}$ be the price vector that solves the problem (9). Then, $\hat{\mathbf{p}}$ is implicitly given by the $M + 1$ conditions*

$$\begin{cases} \Pi(\hat{\mathbf{p}}) = \bar{\Pi} \\ \sum_n \left.\frac{\partial S^n}{\partial p_m}\right|_{p_m=\hat{p}_m} u'(y^n) + \mu \left.\frac{\partial \Pi}{\partial p_m}\right|_{p_m=\hat{p}_m} = 0 \end{cases} \tag{10}$$

where μ is the Lagrange multiplier. In particular, when η_k denotes the own-price elasticity of good k and $\partial q_m/\partial p_k = 0$ for any m, $k = 1,\ldots,N$ and $m \neq k$, $\hat{\mathbf{p}}$ satisfies

$$\frac{\dfrac{\hat{p}_m - c_m}{\hat{p}_m}}{\dfrac{\hat{p}_k - c_k}{\hat{p}_k}} = \frac{\eta_k}{\eta_m}\,\frac{(R_m - \mu)}{(R_k - \mu)} \quad \text{for any } m, k = 1,\ldots,M,\ m \neq k \tag{11}$$

Proof: See Feldstein (1972a).

For simplicity, we discuss the properties of Feldstein prices referring to the case of two goods, m and k, with independent demands. In equations (11), the traditional Ramsey inverse-elasticity rule is adjusted by the factor $(R_m - \mu)/(R_k - \mu)$. Whenever the distributional characteristics are irrelevant, R_m is equal to R_k, and the usual Ramsey ratio of optimal "tax rates" results. By looking at (8) we see that this is the case only if "*1) the marginal social utility of income is the same for all households, or 2) the relative quantities purchased of the two goods are the same for all households, or 3) some extremely improbable balancing of differences in quantities and social utilities occurs*" (Feldstein 1972a, p. 34).

More interesting, however, is the case of the distributional characteristics of the two goods being different. Take the case of R_m being less than R_k, where the consumption of good k is relatively more concentrated in low-income consumers. Then, the markup for good k is clearly lower in the case of Feldstein prices than in the case of Ramsey prices. As this is the good more consumed by low-income consumers, this implies that their welfare is higher than under Ramsey prices. The ratio $(R_m - \mu)/(R_k - \mu)$ then acts as an equity adjustment reducing the adverse distributional effects of the inverse-elasticity rule.

3.3 The GPC and Feldstein Prices

We study the case when the regulator proposes to the firm a regulatory contract that entails that, at any time t from 1 to infinity, the firm is constrained to choose prices that satisfy the following constraint:

$$\sum_m p_m^t \cdot \tilde{q}_m^{t-1} \leq \sum_m p_m^{t-1} \cdot \tilde{q}_m^{t-1}, \tag{12}$$

where $\tilde{q}_m^{t-1} \equiv R_m^{t-1} q_m^{t-1}$, or equivalently, using (8),

$$\tilde{q}_m = \sum_{n=1}^{N} q_m^n(\boldsymbol{p}) \cdot u'(y^n). \tag{13}$$

In words, $\tilde{q}_m^{t-1}$ is an adjusted measure of the aggregate consumption of good m at time $t - 1$, where the quantities consumed by each individual are adjusted using the marginal social utility of income of that individual.

Upon acceptance of the regulatory contract, in each period t the regulated firm faces the problem of choosing prices such that

$$\begin{aligned} &\max_{p^t} \sum_m p_m^t \cdot q_m^t - c(\boldsymbol{q}^t(\boldsymbol{p}^t)) \\ &\text{s.t.} \sum_m p_m^t \cdot \tilde{q}_m^{t-1} \leq \sum_m p_m^{t-1} \cdot \tilde{q}_m^{t-1} \quad \text{for any } t = 1, \ldots, \infty. \end{aligned} \tag{14}$$

Note that for the regulator to check that the firm is complying with the contract, only limited information is needed. As with traditional price cap regulation, together with the knowledge of current prices set by the regulated firm, the regulator only needs to know market data relative to the previous period. In particular, in order to evaluate the $\tilde{q}_m^{t-1}$'s, the regulator needs to know the vector $\boldsymbol{y}$ of consumers' income—assumed to be constant over time, the function $u(y)$ expressing the social utility of income, and the quantities purchased by each consumer.[11]

The following proposition illustrates the properties of this regulatory contract.

PROPOSITION 4: *Let social welfare be given by (7) and let the price cap constraint take the form (12). Then (i) social welfare is monotonically nondecreasing in time; (ii) the sequence of price vectors $\{\boldsymbol{p}^t\}$ chosen by a regulated firm as the solution to its constrained maximization problem (14) converges to a unique vector that satisfies the first-order conditions for a constrained maximum of the welfare function (7).*

[11] It has to be noted that, unlike the observed quantities sold in the previous time period on which Laspeyres-type price cap regulation is based, these $\tilde{\boldsymbol{q}}$'s may leave room for discussion and wasteful rent seeking between firm and regulator. However, the experience of the administration of the new price cap formula adopted by Oftel in 1997 (whose details are discussed in Section 4) shows that it is possible to overcome these additional difficulties, provided that there are appropriate instruments to grant accuracy and reliability of the market data supplied by the firm to the regulator.

Proof: For any value of the index $m = 1, \ldots, M$, we have

$$\begin{aligned}
-\tilde{q}_m^{t-1} &= -R_m^{t-1} q_m(\boldsymbol{p}^{t-1}) \\
&= -R_m^{t-1} \sum_n q_m^n(\boldsymbol{p}^{t-1}) \\
&= \sum_n -q_m^n(\boldsymbol{p}^{t-1}) \cdot u'(y^n) \quad \text{by (8)} \\
&= \sum_n \frac{\partial S^n}{\partial p_m}\bigg|_{p^{t-1}} \cdot u'(y^n) \quad \text{by (6)} \\
&= \frac{\partial W}{\partial p_m}\bigg|_{p^{t-1}} \quad \text{by (7).}
\end{aligned}$$

Then, the constraint in (12) is identical to the one in (3). Moreover, $W(\cdot)$ in (7) has the properties required in Section 2.1, so Propositions 1 and 2 apply. ■

This proposition indicates that, when the preferences for the society can be described by a distributionally weighted welfare function, the price cap (12) guarantees the convergence of the prices set by the regulated firm to the optimal prices. The intuition for this result is straightforward. It is indeed sufficient to note from (13) that the adjusted quantities $\tilde{\boldsymbol{q}}$ are the derivatives of the social welfare function given in (7). Hence, the price cap constraint adopted here in (12) is just a special case of the more general constraint in (3), and the discussion provided there applies.

It is interesting to compare the incentive provided to the regulated firm by (12) with those under traditional Laspeyres-type price cap regulation. Both constraints can be reformulated as an upper limit on the weighted average of price changes for the regulated firm, that is,

$$\sum_m \frac{p_m^t}{p_m^{t-1}} w_m^{t-1} \leq 1. \tag{15}$$

In the case of the Laspeyres-type constraint, w_m^{t-1} equals the revenue share at time $t-1$ for good m (i.e., $w_m^{t-1} = p_m^{t-1} q_m^{t-1} / \sum_k p_k^{t-1} q_k^{t-1}$). For the specific form of the GPC we discuss here, it is easy to show that (15) is equivalent to (12) when $w_m^{t-1} = p_m^{t-1} \tilde{q}_m^{t-1} / \sum_k p_k^{t-1} \tilde{q}_k^{t-1}$, that is, when the revenue share is evaluated using the adjusted quantities $\tilde{\boldsymbol{q}}$.

Consider now the case of a good (say, good j) that is mainly consumed by low-income consumers. By the definition of $\tilde{\boldsymbol{q}}$ in (13), it is easy to see that $p_j^{t-1} \tilde{q}_j^{t-1} / \sum_k p_k^{t-1} \tilde{q}_k^{t-1} > p_j^{t-1} q_j^{t-1} / \sum_k p_k^{t-1} q_k^{t-1}$. This means that the weight used in the GPC is larger than the weight that would be used in a traditional Laspeyres-type price cap formula. Hence, any increase (decrease) in the price of this good increases (decreases) the average price level more than it would do were the weights evaluated using the unadjusted quantities.

In general, this illustrates how the generalized price cap in (12) places a constraint on the prices of the goods consumed mainly by low-income consumers that is tighter than under the traditional price cap. Hence, under (12), the regulated firm finds it optimal to set these prices lower than under the traditional price cap.

4. Final Remarks

This paper has presented a generalized form of the price cap constraint that has been shown to have very desirable properties in terms of allocative efficiency for almost any form of the welfare function.

However, some of the critical points in the previous literature on price cap regulation still remain unsolved. The first such critical point has already been mentioned in the paper and is related to the fact that the actual long-run equilibrium profits and welfare are out of the control of the regulator. This problem, typical also of traditional Laspeyres-type price cap regulation (Brennan 1989), can in principle be solved if in any period of time the constraint is tightened by the rate of profit obtained by the firm in the previous period (Vogelsang and Finsinger 1979). In $RPI - X$ regulation, this may be obtained by interpreting the X factor as being equal to the previous period rate of profit (Bradley and Price 1988). Under these conditions, the regulatory algorithm is equivalent to the one proposed by Vogelsang and Finsinger (1979), which guarantees Ramsey prices and zero profit in the long-run equilibrium. However, this theoretical result comes at the expense of more restrictive assumption on the firm's cost function.

The second critical point has to do with the assumption of myopic profit maximization on the firm's side. As Sappington (1980) first pointed out for the Vogelsang and Finsinger (1979) mechanism, dynamic price adjustment schemes may be vulnerable to strategic behavior when the firm cares about future profits. However, although there may actually be strategic problems before convergence, Vogelsang (1989) shows that the convergence properties of the regulatory mechanism in Vogelsang and Finsinger (1979) hold also when the regulated firm is assumed to maximize the discounted stream of profits.

To conclude this paper, we want to show the usefulness of our results both for regulatory practice and for the analysis of regulatory policies. We do so by using our results to interpret and evaluate some recent changes in the regulatory instruments that have occurred in the United Kingdom.

In 1997, Oftel decided to modify the price cap formula that was used since 1984 to regulate the prices set by British Telecom for domestic customers. While the previous formula was basically a traditional Laspeyres-type price cap formula, in this new formula the weights are the shares of total revenues accruing to the regulated firm only from those consumers who are in the first eight deciles of total expenditure in telecommunications services.

The motivation put forward by Oftel for this change was the recognition of the fact that the price reductions undertaken by British Telecom in the last 15 years have primarily benefited the business and high-expenditure residential users, with very little advantage accruing to low-consumption residential users. As already mentioned in the introduction of this paper, the main reason for this has been the competitive pressures faced by BT in the business and high-consumption market.

Formally, indicating by $\breve{q}_m$ the quantity of good m purchased by consumers who are in the first eight deciles of total expenditure in telecommunications services, the price cap formula adopted by Oftel takes the following form:

$$\sum_m \frac{p_m^t}{p_m^{t-1}} \frac{p_m^{t-1} \breve{q}_m^{t-1}}{\sum_k p_k^{t-1} \breve{q}_k^{t-1}} \leq 1 \tag{16}$$

By comparing $\breve{q}_m$'s in (16) and $\tilde{q}_m$'s in (12), it is easy to see that the price cap formula proposed by Oftel is an instance of our Generalized Price Cap for the case of a social welfare function given by $W(\boldsymbol{p}, \boldsymbol{y}) = \sum_n S^n(\boldsymbol{p}) \cdot a(y^n)$ where, denoting by $\bar{y}$ the highest income in the first eight deciles of households' income, $a(y^n) = 1$ when $y^n \leq \bar{y}$ and $a(y^n) = 0$ when $y^n > \bar{y}$.[12]

The theoretical framework derived in this paper allows us to evaluate the properties of this price cap formula adopted by Oftel. In particular, we can conclude that the adoption of such a price cap formula allows Oftel to pursue the distributional objectives that were stated by Oftel itself. Indeed, this new price cap formula implies that a stricter control is placed on the prices of the goods that make up a large share of the typical bill of low-consumption customers. To the extent that the goods subject to price regulation are normal goods, this provision can be seen as part of a regulator's strategy to protect the interests of those low-consumption consumers who have not benefited from previous price reductions. Moreover, we can argue that this price cap formula maintains the long-run optimality properties of price cap regulation, although it takes into account some distributional objectives. In other words, by adopting this new price cap formula, Oftel is able to pursue its distributional objectives in a socially optimal manner (at least in a long-term perspective).

[12]Strictly speaking, the similarity between the two formulae requires also that the expenditure in telecommunication services increases with income for all consumers.

Appendix

This Appendix reformulates Proposition 2 and provides an alternate proof when the uniqueness assumption of the solution for any admissible level of $\bar{\Pi}$ is relaxed.

Under this milder set of assumptions, we have the following proposition.

PROPOSITION 2a: *Let $\{\boldsymbol{p}^t\}$ be the sequence of prices chosen by a regulated firm as the solution to (4). $\{\boldsymbol{p}^t\}$ has convergent subsequences, and the limits of any such subsequences must all satisfy the first-order condition for a constrained welfare maximum. If Π is strictly convex in prices, then the entire sequence $\{\boldsymbol{p}^t\}$ has a unique such limit.*

Proof: Everything except the uniqueness is exactly as in the proof of Proposition 2. For uniqueness, let $\boldsymbol{p}^\infty$ and $\boldsymbol{p}^{\infty\prime}$ be limits of the two subsequences of $\{\boldsymbol{p}^t\}$. They must both have the same limiting profits Π^∞, and each one must lie in the sequence of constraint half-spaces approaching the other. Since the iso-profit surface $\Sigma^\infty = \Pi^{-1}(\Pi^\infty)$ is convex and the constraint half-spaces at the limit points are tangent to Σ^∞, it follows that the constraint half-spaces at $\boldsymbol{p}^\infty$ and $\boldsymbol{p}^{\infty\prime}$ must coincide and the line segment from $\boldsymbol{p}^\infty$ to $\boldsymbol{p}^{\infty\prime}$ must lie on Σ^∞. Hence were Π to be strictly concave, we would have to have $\boldsymbol{p}^\infty = \boldsymbol{p}^{\infty\prime}$. ■

References

ARMSTRONG, M., and J. VICKERS (1991) Welfare effects of price discrimination by a regulated monopolist, *Rand Journal of Economics* **22**, 571–580.

BÖS, D. (1981) *Economic Theory of Public Enterprise.* Berlin: Springer-Verlag.

BRADLEY, I., and C. PRICE (1988) The economic regulation of private industries by price constraints, *Journal of Industrial Economics* **37**, 99–106.

BRENNAN, T. J. (1989) Regulating by capping prices, *Journal of Regulatory Economics* **1**, 133–147.

BURN, P., I. CRAWFORD, and A. DILNOT (1995) Regulation and redistribution in utilities, *Fiscal Studies* **16**, 1–22.

DE FRAJA, G., and A. IOZZI (2000) Short term and long term effects of price cap regulation, Discussion Papers in economics 00/61, University of York, Department of Economics and Related Studies.

DEPARTMENT OF TRADE AND INDUSTRY (1998) *A Fair Deal to Consumers: Modernising the Framework for Utility Regulation.* London: HMSO.

FELDSTEIN, M. S. (1972a) Distributional equity and the optimal structure of public prices, *American Economic Review* **62**, 32–36 (corrected version of p. 33, footnote 7: *American Economic Review* **62**, 763).

FELDSTEIN, M. S. (1972b) The pricing of public intermediate goods, *Journal of Public Economics* **1**, 45–72.

FELDSTEIN, M. S. (1972c) Equity and efficiency in public sector pricing: The optimal two-part tariff, *Quarterly Journal of Economics* **86**, 175–187.

GIULIETTI, M., and C. WADDAMS PRICE (2000) *Incentive Regulation and Efficient Pricing: Empirical Evidence*, Centre for Management under Regulation Research Paper series 00/2, University of Warwick, Warwick Business School.

HANCOCK, R., and C. WADDAMS PRICE (1995) Competition in the British domestic gas market: efficiency and equity, *Fiscal Studies* **16**, 81–105.

MYLES, G. D. (1995) *Public Economics.* Cambridge: Cambridge University Press.

OFFER and OFGAS (1998) *The Social Dimension: Action Plan: Offer and Ofgas Proposals.* Birmingham: Offer.

OFGEM (1999a) *Social Action Plan: A Framework Document.* London: Ofgem.

OFGEM (1999b) *Prepayment Meters: A Consultation Document.* London: Ofgem.

OFGEM (2000) *The Social Action Plan.* London: Ofgem.

OFTEL (1997) *Pricing of Telecommunications Services from 1997. Oftel's Proposals for Price Control and Fair Trading. A Consultative Document.* London: Oftel.

SAPPINGTON, D. (1980) Strategic firm behavior under a dynamic regulatory adjustment process, *Bell Journal of Economics* **11,** 360–372.

VICKERS, J. (1997) Regulation, competition, and the structure of prices, *Oxford Review of Economic Policy* **13**, 15–26.

VOGELSANG, I. (1989) Price cap regulations of telecommunications services: A long run approach; in *Deregulation and Diversification of Utilities*, M. A. CREW, ed. Boston: Kluwer Academic Publishers.

VOGELSANG, I., and J. FINSINGER (1979) A regulatory adjustment process for optimal pricing by multiproduct monopoly firms, *Bell Journal of Economics* **10**, 157–171.

WADDAMS PRICE, C. (forthcoming) Gas: Regulating for social needs; in *Regulating Utilities: New Issues, New Solutions*, C. ROBINSON, ed. Cheltenham: Edward Elgar.

WADDAMS PRICE, C., and R. HANCOCK (1998) Distributional effects of liberalising UK residential utility markets, *Fiscal Studies* **19**, 295–319.