



## splm: Spatial Panel Data Models in R

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### Abstract

**splm** is an R package for the estimation and testing of various spatial panel data specifications. We consider the implementation of both maximum likelihood and generalized moments estimators in the context of fixed as well as random effects spatial panel data models. This paper is a general description of **splm** and all functionalities are illustrated using a well-known example taken from Munnell (1990) with productivity data on 48 US states observed over 17 years. We perform comparisons with other available software; and, when this is not possible, Monte Carlo results support our original implementation.

*Keywords:* spatial panel, maximum likelihood, GM, LM tests, spatial Hausman test, R.

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## 1. Introduction

The analysis of spatial panel data is a field of econometrics that is experiencing increased methodological progress. Recent contributions include, among others: Anselin, Le Gallo, and Jayet (2008), Kapoor, Kelejian, and Prucha (2007), Baltagi, Song, Jung, and Koh (2007b), Baltagi, Song, and Koh (2003), Baltagi and Liu (2008), Baltagi, Egger, and Pfaffermayr (2007a), Baltagi, Egger, and Pfaffermayr (2009), Debarsy and Ertur (2010), Elhorst (2003), Elhorst and Freret (2009), Elhorst (2008), Elhorst (2009), Elhorst (2010), Elhorst, Piras, and Arbia (2010), Lee and Yu (2010a), Lee and Yu (2010c), Lee and Yu (2010d), Lee and Yu (2010b), Mutl (2006), Mutl and Pfaffermayr (2011), Pesaran and Tosetti (2011), Yu and Lee (2010), Yu, de Jong, and Lee (2008). Empirical applications are hindered by the lack of readily available software. Although there are packages to estimate cross-sectional spatial models in R (R Development Core Team 2012, see e.g., Bivand 2001, 2002, 2006; Bivand and Gebhardt 2000; Bivand and Portnov 2004; Piras 2010), MATLAB (The MathWorks, Inc. 2010, see e.g., LeSage 1999; LeSage and Pace 2009) and Stata (StataCorp. 2007, see e.g., Drukker, Peng, Prucha, and Raciborski 2012, 2011a; Drukker, Prucha, and Raciborski 2011c,b), procedures for estimating spatial panel data models are sparse. Notable exceptions include the MATLAB functions available from Elhorst (2011) and the Stata code supplementing Kapoor *et al.* (2007).

The R package **splm** – available from the Comprehensive R Archive Network at <http://CRAN.R-project.org/package=splm> – fills this gap by providing a comprehensive and consistent tool for the estimation of various spatial panel data models. The R environment is ideal for its development because of the vast infrastructure already in place for analyzing spatial data.

The panel literature has recently considered panel regression models with spatially autocorrelated disturbances, both in the context of fixed (FE) as well as random effects (RE) specifications. In an error components setting, Baltagi *et al.* (2003) introduce a model (also considered in Anselin 1988) where the idiosyncratic errors are spatially autocorrelated, while the individual effects are not. The variance matrix of such a model is complicated and the inverse computationally demanding. Kapoor *et al.* (2007) consider a model where spatial correlation in both the individual and error components share the same spatial parameter; and, therefore, the expression of the variance matrix is simpler and its inverse computationally easier. **splm** takes into consideration both specifications and several methods for the estimation of the regression coefficients.

The present paper describes the maximum likelihood implementation of both models (i.e., with individual effects that are/are not spatially autocorrelated). We consider fixed as well as random effects models in the context of a general spatial Cliff-Ord type model that includes a spatially lagged dependent variable and a spatially autocorrelated error term.

Additionally, **splm** features generalized moments estimators of a Cliff-Ord type model where individual effects are spatially autocorrelated. Again, random as well as fixed effects models are implemented. When other implementations were available, the estimates obtained by our implementation were tested against results available from other software. As an example, the maximum likelihood estimation of the fixed effects and random effects models were tested against the MATLAB routines made available by Elhorst (2011). For all other estimation procedures we performed Monte Carlo simulations to verify the properties of our estimator. Results are presented in Section 8.

Among other testing procedures, we also implement the joint, marginal and conditional specification (zero-restriction) Lagrange multiplier tests for individual effects and spatial correlation introduced by Baltagi *et al.* (2003).

Section 2 describes the data structure. In Section 3 we discuss the definition of classes and methods. The description of a general spatial panel regression model follows in Section 4 along with the treatment of two different specifications for the innovations of the model. Section 5 is devoted to the maximum likelihood (ML) implementation. In particular, Section 5.1 discusses and illustrates spatial random effects (RE) models, while Section 5.2 deals with the estimation of fixed effects (FE) models. Section 6 describes the implementation of the generalized moments estimators. As before, spatial RE models are discussed first in Section 6.1. Section 6.2 present the estimation theory and the generalized moments (GM) implementation of fixed effects models. Section 7 describes the implementation of various testing procedures and Section 8 discusses the numerical checks. Conclusions and indications for future developments conclude the paper.

## 2. Data structures

Panel data refer to a cross section of observations (individuals, groups, countries, regions) repeated over several time periods. When the number of cross sectional observations is con-

stant across time periods the panel is said to be balanced. The present paper only focuses on such balanced panels. In a spatial panel setting, the observations are associated with a particular position in space. Data can be observed either at point locations (e.g., housing data) or aggregated over regular or irregular areas (e.g., countries, regions, states, counties). The structure of the interactions between each pair of spatial units is represented by means of a spatial weights matrix.

The spatial weights matrix  $W$  is a  $N \times N$  positive matrix.<sup>1</sup> Observations appear both in rows and columns. Hence, the non-zero elements of the matrix indicate whether two locations are neighbors. As a consequence, the element  $w_{ij}$  indicates the intensity of the relationship between cross sectional units  $i$  and  $j$ . By convention, the diagonal elements  $w_{ii}$  are all set to zero to exclude self-neighbors. The weights matrix is generally used in row standardized form.

A possible source of confusion when developing ad-hoc routines stems from the different notation that characterizes spatial panel data models compared to traditional panel data models. On one hand, panel data are generally ordered first by cross-section and then by time period (i.e., with time being the “fast” index). On the other hand, spatial panel data are stacked first by time period and then by cross-section. In **splm**, this is treated transparently for the user. The internal ordering of the estimation functions is usually (but not always) the spatial panel data one. Nonetheless, data can be supplied according to the conventions implemented in the **plm** package for panel data econometrics (Croissant and Millo 2008). Three possibilities are available:

- a `data.frame` whose first two variables are the individual and time indexes. The `index` argument should be left to the default value (i.e., `NULL`)
- a `data.frame` and a character vector indicating the indexes variables
- an object of the class `pdata.frame`

`pdata.frames` are special objects created to deal with panel data. They are part of a general infrastructure made available in **plm** and meant to handle (serial) lag and difference operations. The methods available in **splm** are geared towards static panels; nonetheless, defining data as a `pdata.frame` might simplify the calculation of (time) lags of the regressors.<sup>2</sup>

The spatial weights matrix  $W$  can be a `matrix` object (with the estimators performing a minimal check for dimension compatibility) or a `listw` object from the class defined in **spdep** (Bivand 2011).<sup>3</sup> The class is an efficient format and has the advantage of being well established in the R environment. Functionalities for switching between the two formats are available as functions `listw2mat` and `mat2listw` from the **spdep** package.

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<sup>1</sup>The spatial weights matrix may or may not be symmetric. When it is standardized, it is generally not symmetric. **splm** can deal with all types of matrices. However, some of the methods for the calculation of the Jacobian are only used with symmetric weights. We will elaborate more on this later.

<sup>2</sup>It should be made clear that the inclusion of time lags would potentially lead to incorrect results for a dynamic model estimated with the procedures currently available. However, future improvements may include dynamic panel data models in which case `pdata.frame` objects would be extremely useful.

<sup>3</sup>Some of the functions internally transform the object of class `listw` into a sparse `Matrix` making use of code from the **Matrix** package (Bates and Mächler 2012).

### 3. Classes and methods for spatial panel models

The two main goals of **splm** are estimation and testing of spatial panel data models. On the one hand, the information provided in the output of the test procedures is similar to an object of class **htest**; and, hence, produces a similar output report. On the other hand, spatial panel models require different structures and methods from the classes available in **plm**. By and large, this is because spatial panel models involve the estimation of extra coefficients (e.g., the coefficient for the spatial lag term in the fixed effects spatial lag model or the error correlation coefficient and the variance components in the random effects specifications).

The new class **splm** inherits the general structure of **lm** objects. The **splm** object is a list of various elements including: the estimated **coefficients**, the vector of **residuals** and **fitted.values**, the most recent **call** and a **model** element containing the data employed in the estimation. As it is common for most models that are estimated by maximum likelihood, **splm** also comprises a **logLik** component with the value of the log-likelihood at the parameter optimum. This can be easily extracted and reused for testing or model selection purposes.

Some elements from **lm** objects have been excluded though. These omissions are partly due to the nature of the estimation process (which does not use, for instance, the “qr” decomposition). Specific elements have been added to accommodate for spatial and covariance parameters. In addition to the usual **vcov** element giving the coefficients’ variance covariance matrix, the element **vcov.errcomp** contains the covariance matrix of the estimated error covariance coefficients.

A new **class** is defined for the summaries of **splm** objects. Consistent with **lm** and **plm** objects, the method provides diagnostic tables for the elements of **splm** objects. **print** methods are also available with a minimal description of the model object (including **call**, **coefficients** and **covariance** parameters). Additionally, extractor methods have been defined for a few relevant elements of model objects. Along with the standard **coef**, **residuals**, and **vcov**, extractor methods are provided for the covariance matrices of the estimated spatial autoregressive coefficient and covariance components.

The availability of these extractors is consistent with the general modeling framework of the R project and favors the interoperability of **splm** objects with generic diagnostics based on Wald tests. In particular we refer to the functions **waldtest** (for joint zero-restrictions) in **lmtest** (Zeileis and Hothorn 2002) and **linearHypothesis** (for generic linear restrictions) in **car** (Fox and Weisberg 2010).

Finally, an extractor method for fixed effects and a **summary** method for displaying them are also available.

Throughout the paper, all functionalities are illustrated using the well-known Munnell (1990) data set on public capital productivity in 48 US states observed over 17 years (available in R in the **Ecdat** package, Croissant 2011). A binary contiguity spatial weights matrix for the US states is included in the package.

```
R> data("Produc", package = "Ecdat")
R> data("usaww")
```

Munnell (1990) specifies a Cobb-Douglas production function that relates the gross social product (**gsp**) of a given state to the input of public capital (**pcap**), private capital (**pc**), labor (**emp**) and state unemployment rate (**unemp**) added to capture business cycle effects. The model formula is defined once and includes a constant term:

```
R> fm <- log(gsp) ~ log(pcap) + log(pc) + log(emp) + unemp
```

We also transform the weights matrix into a `listw` object using infrastructure from the `spdep` package:

```
R> library("spdep")
R> usalw <- mat2listw(usaww)
```

## 4. Spatial panel data models

Spatial panel data models capture spatial interactions across spatial units and over time. There is an extensive literature on both static as well as dynamic models.<sup>4</sup> We start from a general static panel model that includes a spatial lag of the dependent variable and spatial autoregressive disturbances:

$$y = \lambda(I_T \otimes W_N)y + X\beta + u \quad (1)$$

where  $y$  is an  $NT \times 1$  vector of observations on the dependent variable,  $X$  is a  $NT \times k$  matrix of observations on the non-stochastic exogenous regressors,  $I_T$  an identity matrix of dimension  $T$ ,  $W_N$  is the  $N \times N$  spatial weights matrix of known constants whose diagonal elements are set to zero, and  $\lambda$  the corresponding spatial parameter. The disturbance vector is the sum of two terms

$$u = (\iota_T \otimes I_N)\mu + \varepsilon \quad (2)$$

where  $\iota_T$  is a  $T \times 1$  vector of ones,  $I_N$  an  $N \times N$  identity matrix,  $\mu$  is a vector of time-invariant individual specific effects (not spatially autocorrelated), and  $\varepsilon$  a vector of spatially autocorrelated innovations that follow a spatial autoregressive process of the form

$$\varepsilon = \rho(I_T \otimes W_N)\varepsilon + \nu \quad (3)$$

with  $\rho$  ( $|\rho| < 1$ ) as the spatial autoregressive parameter,  $W_N$  the spatial weights matrix,  $\nu_{it} \sim IID(0, \sigma_\nu^2)$  and  $\varepsilon_{it} \sim IID(0, \sigma_\varepsilon^2)$ .<sup>5</sup>

As in the classical panel data literature, the individual effects can be treated as fixed or random. In a random effects model, one is implicitly assuming that the unobserved individual effects are uncorrelated with the other explanatory variables in the model. In this case,  $\mu_i \sim IID(0, \sigma_\mu^2)$ , and the error term can be rewritten as:

$$\varepsilon = (I_T \otimes B_N^{-1})\nu \quad (4)$$

where  $B_N = (I_N - \rho W_N)$ . As a consequence, the error term becomes

$$u = (\iota_T \otimes I_N)\mu + (I_T \otimes B_N^{-1})\nu \quad (5)$$

<sup>4</sup>In our discussion, as well as in our implementation, we concentrate on static models only and leave the dynamic case as a possible extension for future research.

<sup>5</sup>Note that the spatial weights matrices in the regression equation and the error term can differ in many of our implementations. However, in our discussion of the models they are assumed to be the same for simplicity. It is also assumed that  $I_N - \rho W_N$  is non-singular where  $I_N$  is an identity matrix of dimension  $N$ .

and the variance-covariance matrix for  $\varepsilon$  is

$$\Omega_u = \sigma_\mu^2(\iota_T \iota_T^\top \otimes I_N) + \sigma_\nu^2[I_T \otimes (B_N^\top B_N)^{-1}] \quad (6)$$

In deriving several Lagrange multiplier (LM) tests, Baltagi *et al.* (2003) consider a panel data regression model that is a special case of the model presented above in that it does not include a spatial lag of the dependent variable. Elhorst (2003, 2009) defines a taxonomy for spatial panel data models both under the fixed and the random effects assumptions. Following the typical distinction made in cross-sectional models, Elhorst (2003, 2009) defines the fixed as well as the random effects panel data versions of the spatial error and spatial lag models. However, he does not consider a model including both the spatial lag of the dependent variable and a spatially autocorrelated error term. Therefore, the models reviewed in Elhorst (2003, 2009) can also be seen as a special case of this more general specification.

A second specification for the disturbances is considered in Kapoor *et al.* (2007). They assume that spatial correlation applies to both the individual effects and the remainder error components. Although the two data generating processes look similar, they do imply different spatial spillover mechanisms governed by a different structure of the implied variance covariance matrix. In this case, the disturbance term follows a first order spatial autoregressive process of the form:

$$u = \rho(I_T \otimes W_N)u + \varepsilon \quad (7)$$

where  $W_N$  is the spatial weights matrix and  $\rho$  the corresponding spatial autoregressive parameter. To further allow for the innovations to be correlated over time, the innovations vector in Equation 7 follows an error component structure

$$\varepsilon = (\iota_T \otimes I_N)\mu + \nu \quad (8)$$

where  $\mu$  is the vector of cross-sectional specific effects,  $\nu$  a vector of innovations that vary both over cross-sectional units and time periods,  $\iota_T$  is a vector of ones and  $I_N$  an  $N \times N$  identity matrix. In deriving a Hausman test for a Cliff and Ord spatial panel data model, Mutl and Pfaffermayr (2011) consider the model presented above and discuss instrumental variables estimation under both the fixed and the random effects specifications. They extend the work of Kapoor *et al.* (2007) who did not include a spatially lagged dependent variable in the regression equation. Under the random effects assumption that the individual effects are independent of the model regressors, one can rewrite Equation 7 as

$$u = [I_T \otimes (I_N - \rho W_N)^{-1}] \varepsilon \quad (9)$$

It follows that the variance-covariance matrix of  $u$  is

$$\Omega_u = [I_T \otimes (I_N - \rho W_N)^{-1}] \Omega_\varepsilon [I_T \otimes (I_N - \rho W_N^\top)^{-1}] \quad (10)$$

where  $\Omega_\varepsilon = \sigma_\nu^2 Q_0 + \sigma_1^2 Q_1$ , with  $\sigma_1^2 = \sigma_\nu^2 + T\sigma_\mu^2$ ,  $Q_0 = \left(I_T - \frac{J_T}{T}\right) \otimes I_N$ ,  $Q_1 = \frac{J_T}{T} \otimes I_N$  and  $J_T = \iota_T \iota_T^\top$ , is the typical variance-covariance matrix of a one-way error component model adapted to the different ordering of the data.

As it should be clear from the above discussion, these two panel models differ in terms of their variance matrices. The variance matrix in Equation 6 is more complicated than the one in Equation 10, and, therefore, its inverse is more difficult to calculate. In the present paper,

we consider the implementation of both error term specifications. For the first specification, we implement maximum likelihood estimation of the random as well as the fixed effects models. For the second (simpler) specification, we implement both maximum likelihood and instrumental variables estimation under the random as well as the fixed effects assumption. The next section is devoted to the discussion of the ML implementation of the two models and Section 6 to the GM implementation of the second error specification.

## 5. ML implementation

Both random and fixed effects models are implemented within the same software framework. `spm1` is the general wrapper function and the argument `model` controls the specification. In accordance with the syntax in `plm`, `model` takes up the value "within" for fixed effects, "random" for random effects, and "pooling" for no effects. The spatial structure is specified by combining the logical arguments `lag` (that, if true, adds a spatial autoregressive term in the dependent variable) and `spatial.error`. This last argument takes three possible values: "b" ("Baltagi") for the specification in Equation 3, "kkp" ("Kapoor, Kelejian and Prucha") for the specification in Equation 7, and "none" for no spatial error correlation.

### 5.1. Random effects model

For a model with spatially autocorrelated error components, ordinary least squares (OLS) is inefficient even when  $\sigma_\mu^2 = 0$ . Analogously, OLS on a random effects model (even without spatial components) is also inefficient. An alternative (i.e., more efficient) way of estimating the model is via maximum likelihood. In the present section we discuss the estimation approach of the full specification, i.e., the one with a spatial lag, random effects and spatial correlation of the form specified in Equation 3.

Scaling the error covariance matrix by the idiosyncratic error variance  $\sigma_\varepsilon^2$ , and denoting  $\phi = \sigma_\mu^2/\sigma_\varepsilon^2$ ,  $\bar{J}_T = J_T/T$ ,  $E_T = I_T - \bar{J}_T$  and  $A_N = (I_N - \lambda W_N)$ , the expressions for the scaled error covariance matrix  $\Sigma$ , its inverse  $\Sigma^{-1}$ , and its determinant  $|\Sigma|$  can be written respectively as

$$\begin{aligned}\Sigma &= \phi(J_T \otimes I_N) + I_T \otimes (B^\top B)^{-1} \\ \Sigma^{-1} &= \bar{J}_T \otimes ((T\phi I_N + (B^\top B)^{-1})^{-1} + E_T \otimes B^\top B) \\ |\Sigma| &= |T\phi I_N + (B^\top B)^{-1}| |(B^\top B)^{-1}|^{T-1}.\end{aligned}$$

Substituting into the general formula given in Anselin (1988, Ch. 6), one can derive the expression of the likelihood:

$$\begin{aligned}L(\beta, \sigma_\varepsilon^2, \phi, \lambda, \rho) &= -\frac{NT}{2} 2\pi - \frac{NT}{2} \ln \sigma_\nu^2 + T \ln |A| \\ &\quad - \frac{1}{2} \ln |T\phi I_N + (B^\top B)^{-1}| \\ &\quad + (T-1) \ln |B| - \frac{1}{2\sigma_\nu^2} u^\top \Sigma^{-1} u\end{aligned}$$

We implement an iterative procedure to obtain the maximum likelihood estimates. Starting from initial values for  $\lambda$ ,  $\rho$  and  $\phi$ , we obtain estimates for  $\beta$  and  $\sigma_\nu^2$  from the first order conditions:

$$\begin{aligned}\beta &= (X^\top \Sigma^{-1} X)^{-1} X^\top \Sigma^{-1} A y \\ \sigma_\nu^2 &= (A y - X \beta)^\top \Sigma^{-1} (A y - X \beta) / NT.\end{aligned}$$

The likelihood can be concentrated and maximized with respect to  $\lambda$ ,  $\rho$  and  $\phi$ . The estimated values of  $\lambda$ ,  $\rho$  and  $\phi$  are in turn used to update the expression for  $A$  and  $\Sigma^{-1}$ . These steps are then repeated until a convergence criterion is met. In other words, for a specific  $\Sigma$  the estimation can be operationalized by a two step iterative procedure that alternates between generalized least squares (GLS, for  $\beta$  and  $\sigma_v^2$ ) and concentrated likelihood (for the remaining parameters) until convergence.<sup>6</sup> From an implementation point of view there are (at least) a couple of different ways to proceed. First of all, we decided to include the GLS step within the objective function to be maximized (i.e., the function to be used as an argument to the optimizer). In other words, the GLS step is part of the optimization process of the likelihood.<sup>7</sup> We obtain standard errors for  $\beta$  from GLS, and we employ a numerical Hessian to perform statistical inference on the error components.<sup>8</sup>

### *Illustration*

ML estimation of spatial panel random effects models is performed by `spml` with the argument `model` set to `"random"`. The arguments `lag` and `spatial.error` allow the estimation of all combinations of a spatial lag with the different specifications for the error term. The same specifications but without random effects can be estimated by setting the `model` to `"pooling"`. It should be noted that the `effects` argument can only be set to `"individual"` in the random effects context, and it will turn out to be more useful when discussing fixed effects models.

As for other specific parameters, we provide two ways to set the initial values of the parameters managed through the optional argument `initval`.<sup>9</sup> The first option is to specify a numeric vector of initial values. As an alternative, when `initval` is set to `"estimate"` the initial values are retrieved from the estimation of nested specifications. As an example, when estimating the full model, the initial value for the spatial correlation parameter is taken to be the estimated  $\rho$  from a panel regression with spatially correlated errors. Analogously, the initial value of  $\lambda$  is the estimated spatial autocorrelation coefficient from the spatial autoregressive model; and, finally, an initial value for  $\phi$  is obtained by estimating a random effects model.

Assuming that both the spatial lag and the spatial error are defined according to the same weights matrix, Munnell's data lead to the following results for the most general model:

```
R> sararremod <- spml(formula = fm, data = Produc, index = NULL,
+   listw = usalw, model = "random", lag = TRUE, spatial.error = "b")
R> summary(sararremod)
```

Spatial panel random effects ML model

Call:

```
spml(formula = fm, data = Produc, index = NULL, listw = usalw,
      model = "random", lag = TRUE, spatial.error = "b")
```

<sup>6</sup> Note that these steps remain valid when the model to be estimated is one of the nested specifications where, for example, one of the spatial coefficients is restricted to zero.

<sup>7</sup> There are many optimizers available under R. Our final choice was to use `nlmminb`. While leading to similar values for the estimated parameters, it proved to be faster than other optimizers.

<sup>8</sup> The numerical Hessian is implemented in the function `fdHess` available from `nlme`. The Hessian is evaluated at the ML parameter values using finite differences.

<sup>9</sup> If none of the two options is specified, the optimization will start at zero.



Residuals:

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
-0.2480	-0.0411	0.0123	0.0191	0.0726	0.4840

Error variance parameters:

	Estimate	Std. Error	t-value	Pr(> t )
phi	7.530808	1.743935	4.3183	1.572e-05 ***
rho	0.536835	0.034481	15.5690	< 2.2e-16 ***

Spatial autoregressive coefficient:

	Estimate	Std. Error	t-value	Pr(> t )
lambda	0.0018174	0.0058998	0.3081	0.758

Coefficients:

	Estimate	Std. Error	t-value	Pr(> t )
(Intercept)	2.3736012	0.1394745	17.0182	< 2.2e-16 ***
log(pcap)	0.0425013	0.0222146	1.9132	0.055721 .
log(pc)	0.2415077	0.0202971	11.8987	< 2.2e-16 ***
log(emp)	0.7419074	0.0244212	30.3797	< 2.2e-16 ***
unemp	-0.0034560	0.0010605	-3.2589	0.001119 **

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Using the same function, but changing the argument `spatial.error = "kkp"` and the `lag = FALSE`, results for the [Kapoor et al. \(2007\)](#) model are obtained:

```
R> semremod <- spml(formula = fm, data = Produc, index = NULL,
+   listw = usalw, model = "random", lag = FALSE, spatial.error = "kkp")
R> summary(semremod)
```

Spatial panel random effects ML model

Call:

```
spml(formula = fm, data = Produc, index = NULL, listw = usalw,
      model = "random", lag = FALSE, spatial.error = "kkp")
```

Residuals:

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
-0.27000	-0.06430	-0.01120	-0.00448	0.04890	0.46900

Error variance parameters:

	Estimate	Std. Error	t-value	Pr(> t )
phi	6.624775	1.548063	4.2794	1.874e-05 ***
rho	0.526465	0.033344	15.7891	< 2.2e-16 ***

Coefficients:

```

                Estimate Std. Error t-value Pr(>|t|)
(Intercept)  2.3246707  0.1415894 16.4184 < 2.2e-16 ***
log(pcap)    0.0445475  0.0220377  2.0214 0.0432362 *
log(pc)      0.2461124  0.0211341 11.6453 < 2.2e-16 ***
log(emp)     0.7426319  0.0254663 29.1614 < 2.2e-16 ***
unemp       -0.0036045  0.0010637 -3.3887 0.0007022 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

Consistent with the conventions in the R environment, the `summary` method prints a short description of the model, the most recent call, a summary of the residuals and the table of estimated coefficients. The `splm` specific part of the output (printed between the summary of the residuals and the table of the estimated coefficients) reports the estimated error components and, if any, the spatial autoregressive coefficient  $\lambda$  along with standard errors from the numerical Hessian.

## 5.2. Fixed effects model

For large  $N$ , consistent estimation of the individual fixed effects is not possible because of the incidental parameter problem. Elhorst (2003) has pointed out that when the interest is primarily in the regression parameters vector  $\beta$  an extension of the fixed effects model to a spatial context may still be appropriate. Elhorst (2003) only considers the spatial lag and error models separately but not the specification that includes both a spatially autocorrelated error term and a spatial lag of the dependent variable.

A fixed effect spatial lag model can be written in stacked form as

$$y = \lambda(I_T \otimes W_N)y + (\iota_T \otimes I_N)\mu + X\beta + \varepsilon \quad (11)$$

where  $\lambda$  is the spatial autoregressive coefficient,  $W_N$  a non-stochastic spatial weights matrix,  $\iota_T$  a column vector of ones of dimension  $T$ ,  $I_N$  an  $N \times N$  identity matrix and  $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$ .<sup>10</sup> The general estimation theory for maximum likelihood resembles the cross-sectional case. The presence of the spatial lag introduces a form of endogeneity that violates the assumption of standard regression models (i.e., the regressors are uncorrelated with the error term). Elhorst (2003) suggests transforming the variables in Equation 11 by eliminating the time invariant individual effects and use these transformed variables to maximize the likelihood function. The transformation is obtained by subtracting the average for each cross-section over time. As a consequence, the fixed effects and the constant term (as well as other variables that do not vary over time) are wiped out from the model. Formally, the transformation can be written as

$$y^* = \lambda(I_T \otimes W_N)y^* + X^*\beta + \varepsilon^* \quad (12)$$

where  $y^* = Q_0y$ ,  $X^* = Q_0X$ ,  $\varepsilon^* = Q_0\varepsilon$  and  $Q_0$  was defined in Section 4. The log-likelihood function of Equation 11 is:

$$L = -\frac{NT}{2} \ln(2\pi\sigma_\varepsilon^2) + T \ln |I_N - \lambda W_N| - \frac{NT}{2\sigma_\varepsilon^2} e^\top e \quad (13)$$

<sup>10</sup>For simplicity we only discuss the one-way error component model. However, the function `splm` also allows the estimation of a two-way error component model.

where  $e = y - \lambda(I_T \otimes W_N)y - X\beta$  and  $\ln |I_N - \lambda W_N|$  is the Jacobian determinant.<sup>11</sup> Elhorst (2009) suggests a concentrated likelihood approach for maximizing Equation 13. The estimation procedure is substantially analogous to the one employed in the cross-sectional case. After the transformation, two auxiliary regressions of  $y^*$  and  $(I_N \otimes W_N)y^*$  on  $X^*$  are performed. The corresponding residuals (say  $e_0^*$  and  $e_1^*$ ) are combined to obtain the concentrated likelihood:

$$L = C + T \ln |I_N - \lambda W_N| - \frac{NT}{2} \ln[(e_0^* - \lambda e_1^*)^\top (e_0^* - \lambda e_1^*)] \quad (14)$$

with  $C$  a constant not depending on  $\lambda$ . A numerical optimization procedure is needed to obtain the value of  $\lambda$  that maximizes Equation 14. Finally, estimates for  $\beta$  and  $\sigma_\varepsilon^2$  are obtained from the first order conditions of the likelihood function by replacing  $\lambda$  with its estimated value from the ML. Analogous to the cross sectional model, the estimator for  $\beta$  can also be seen as the generalized least square estimator of a linear regression model with disturbance variance matrix  $\sigma_\varepsilon^2 Q_0$ .<sup>12</sup> Statistical inference on the parameters of the model can be based on the expression for the asymptotic variance covariance matrix derived in Elhorst (2009) and Elhorst and Freret (2009):  $\text{AsyVar}(\beta, \lambda, \sigma_\varepsilon^2) =$

$$\begin{bmatrix} \frac{1}{\sigma_\varepsilon^2} X^{*\top} X^* & \frac{1}{\sigma_\varepsilon^2} X^{*\top} (I_T \otimes \widetilde{W}) X^* \beta & \\ \frac{1}{\sigma_\varepsilon^2} \beta^\top X^{*\top} (I_T \otimes \widetilde{W}^\top \widetilde{W}) X^* \beta + T \text{tr}(\widetilde{W} \widetilde{W} + \widetilde{W}' \widetilde{W}) & & \\ & \frac{T}{\sigma_\varepsilon^2} \text{tr}(\widetilde{W}) & \frac{NT}{2\sigma_\varepsilon^4} \end{bmatrix}^{-1} \quad (15)$$

where  $\widetilde{W} = W(I_N - \lambda W)^{-1}$  and the missing elements that cannot be filled in by symmetry are zeros. The computational burden involved in the calculation of the asymptotic standard error of the spatial parameter can be very costly for large problem dimensions (mainly because of the inverse of the  $N \times N$  matrix involved in the computation). The block of the coefficient covariance matrix relative to the parameter vector  $\beta$  does not present any particular computational difficulties. Fixed effects can be recovered by

$$\mu_i = \frac{1}{T} \sum_{t=1}^T (y_{it} - \lambda \sum_{j=1}^N w_{ij} y_{jt} - x_{it} \beta) \quad (16)$$

Averaging across all observations one can also recover the intercept under the restriction that the individual effects sum to zero (see also Baltagi 2008, p. 13).

A fixed effects spatial error model can be written as

$$\begin{aligned} y &= (\iota_T \otimes I_N) \mu + X\beta + u \\ u &= \rho(I_T \otimes W_N)u + \varepsilon \end{aligned} \quad (17)$$

<sup>11</sup> Sometimes the likelihood is expressed in terms of the log Jacobian  $\sum_i \ln(1 - \lambda \omega_i)$  where  $\omega_i$  are the eigenvalues of the spatial weights matrix. The default method to compute the Jacobian is based on the eigenvalues decomposition using the functions `eigenw`. In line with the changes and improvements recently made in `spdep` (Bivand 2010), other methods are available, including the use of sparse matrices, and the Chebyshev and Monte Carlo approximations (LeSage and Pace 2009).

<sup>12</sup> Anselin *et al.* (2008) point out that various aspects of the fixed effects spatial lag model deserve further investigation. The main issue relates to the properties of  $Q_0$ . By definition  $Q_0$  is singular and therefore  $|Q_0|$  does not exist. While this is not a problem in the non-spatial case, the log-likelihood for the spatial model should be based on multivariate normality of the error term. Hence because of the the properties of  $Q_0$ , the joint unconditional likelihood becomes degenerate. Although theoretically relevant, these considerations should not be an issue in practice. To cope with this, Lee and Yu (2010d) suggest using a different transformation based on the orthonormal matrix of  $Q_0$ .

where  $\rho$  is the spatial autocorrelation coefficient and  $\varepsilon$  is a well-behaved error term.

The estimation strategy for the cross-sectional spatial error model can be easily extended to the panel context. Again a concentrated likelihood approach can be taken but an iterative procedure is needed to estimate the parameters of the spatial error model. The general idea is to iterate between ML and generalized least squares (GLS) until a convergence criterion is met. The model is transformed according to Equation 12, to eliminate fixed effects. More formally, the log-likelihood function for model Equation 17 can be written as:

$$L = -\frac{NT}{2} \ln(2\pi\sigma_\varepsilon^2) + T \ln |B_N| - \frac{1}{2\sigma_\varepsilon^2} e^\top [I_T \otimes (B_N^\top B_N)] e \quad (18)$$

with  $e = y - X\beta$  and  $B_N = (I_N - \rho W)$ .

Given  $\rho$ , estimators for  $\beta$  and  $\sigma_\varepsilon^2$  are derived from the first order conditions as

$$\beta = [X^\top (I_T \otimes B_N^\top B_N) X]^{-1} X^\top (I_T \otimes B_N^\top B_N) y \quad (19)$$

and

$$\sigma_\varepsilon^2 = \frac{e(\rho)^\top e(\rho)}{NT} \quad (20)$$

where the notation indicates the explicit dependence of the residuals on  $\rho$ . By substituting Equation 19 and Equation 20 back into Equation 18, the concentrated log-likelihood function can be derived as:

$$L = C - \frac{NT}{2} \ln[e(\rho)^\top e(\rho)] + T \ln |B_N| \quad (21)$$

where  $C$  is a constant not depending on  $\rho$  and  $B_N$  was defined above. The estimation procedure can be summarized as follows. Estimated OLS residuals (of the transformed model) can be used to obtain an initial estimate of  $\rho$ . The initial estimate of  $\rho$  can in turn be used to compute a (spatial) feasible GLS (FGLS) estimator of the regression coefficients, the error variance and a new set of estimated GLS residuals. An iterative procedure may then be employed: the concentrated likelihood and the GLS estimators are alternately computed until convergence. The asymptotic variance covariance matrix of the parameters is (Elhorst 2009)

$$\text{AsyVar}(\beta, \rho, \sigma_\varepsilon^2) = \begin{bmatrix} \frac{1}{\sigma_\varepsilon^2} X^{*\top} X^* & & & \\ & T \text{tr}(\widetilde{W}\widetilde{W} + \widetilde{W}^\top \widetilde{W}) & & \\ & & \frac{T}{\sigma_\varepsilon^2} \text{tr}(\widetilde{W}) & \\ & & & \frac{NT}{2\sigma_\varepsilon^4} \end{bmatrix}^{-1} \quad (22)$$

where  $\widetilde{W} = W(I_N - \lambda W)^{-1}$ . Considerations made for the spatial lag case also apply here, and individual effects can be recovered by

$$\mu_i = \frac{1}{T} \sum_{t=1}^T (y_{it} - x_{it}\beta) \quad (23)$$

### *Illustration*

The ML estimation of a spatial panel fixed effects model is performed through `splm` by setting the `model` argument to `"within"`.

The `spml` function allows the estimation of a model specified in terms of both spatial effects. This can be done by combining the arguments `lag` and `spatial.error` as in the following example:

```
R> sararfemod <- spml(formula = fm, data = Produc, index = NULL,
+   listw = usalw, lag = TRUE, spatial.error = "b", model = "within",
+   effect = "individual", method = "eigen", na.action = na.fail,
+   quiet = TRUE, zero.policy = NULL, interval = NULL,
+   tol.solve = 1e-10, control = list(), legacy = FALSE)
R> summary(sararfemod)
```

Spatial panel fixed effects sarar model

Call:

```
spml(formula = fm, data = Produc, index = NULL, listw = usalw,
      model = "within", effect = "individual", lag = TRUE, spatial.error = "b",
      method = "eigen", na.action = na.fail, quiet = TRUE, zero.policy = NULL,
      interval = NULL, tol.solve = 1e-10, control = list(), legacy = FALSE)
```

Residuals:

Min.	1st Qu.	Median	3rd Qu.	Max.
-0.1340	-0.0221	-0.0032	0.0172	0.1750

Coefficients:

	Estimate	Std. Error	t-value	Pr(> t )
rho	0.4553116	0.0504043	9.0332	< 2.2e-16 ***
lambda	0.0885760	0.0300044	2.9521	0.003156 **
log(pcap)	-0.0103497	0.0252725	-0.4095	0.682156
log(pc)	0.1905781	0.0230505	8.2678	< 2.2e-16 ***
log(emp)	0.7552372	0.0277505	27.2152	< 2.2e-16 ***
unemp	-0.0030613	0.0010293	-2.9741	0.002939 **

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

As is well known, the within transformation eliminates the individual effects. Thus, from an empirical point of view, it also makes the two specifications (individuals effects are/are not spatially autocorrelated) indistinguishable. Therefore, the argument `spatial.error` can equivalently take the values `b` or `kkp`, thus leading to the estimation of the same specification. There are specific arguments to `spml` for spatial `within` models that can be passed on through the special `'...'` argument. The argument `method` sets the technique for the calculation of the determinant. The default (`"eigen"`) is to express the Jacobian in terms of the eigenvalues of the spatial weights matrix. Other available options include methods based on sparse matrices (`"spam"`, `"Matrix"` or `"LU"`), and the Chebyshev (`"Chebyshev"`) and Monte Carlo (`"MC"`) approximations.

As an example, to estimate a model with only individual fixed effects:

```
R> sarfemod <- spml(formula = fm, data = Produc, index = NULL, listw = usalw,
+   model = "within", effect = "individual", method = "eigen",
```

```
+   na.action = na.fail, quiet = TRUE, zero.policy = NULL, interval = NULL,
+   tol.solve = 1e-10, control = list(), legacy = FALSE)
R> summary(sarfemod)
```

Spatial panel fixed effects error model

Call:

```
spml(formula = fm, data = Produc, index = NULL, listw = usalw,
      model = "within", effect = "individual", method = "eigen",
      na.action = na.fail, quiet = TRUE, zero.policy = NULL, interval = NULL,
      tol.solve = 1e-10, control = list(), legacy = FALSE)
```

Residuals:

Min.	1st Qu.	Median	3rd Qu.	Max.
-0.1250	-0.0238	-0.0035	0.0171	0.1880

Coefficients:

	Estimate	Std. Error	t-value	Pr(> t )
rho	0.5574013	0.0330749	16.8527	< 2e-16 ***
log(pcap)	0.0051438	0.0250109	0.2057	0.83705
log(pc)	0.2053026	0.0231427	8.8712	< 2e-16 ***
log(emp)	0.7822540	0.0278057	28.1328	< 2e-16 ***
unemp	-0.0022317	0.0010709	-2.0839	0.03717 *

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Again, an object of class `splm` is generated, for which an appropriate summary method is available. The summary method gives information about the `call`, a summary of the residuals and the table of estimated coefficients (where `rho` is the coefficient of the spatially lagged dependent variable). Fixed effects can be extracted using the function `effects`:

```
R> eff <- effects(sarfemod)
```

The result is an object of class `effects.splm` for which `print` and `write` methods are available. The `print` method displays the type of effects (with significance levels) and the constant term. The `write` method is used to write the corresponding `matrix` to a file. The name of the file can be controlled by the argument `filename`.

Analogously, one can estimate a spatial error model with time period fixed effects as:

```
R> semfemod <- spml(formula = fm, data = Produc, listw = usalw,
+   model = "within", effect = "time", method = "eigen",
+   na.action = na.fail, quiet = TRUE, zero.policy = NULL, interval = NULL,
+   tol.solve = 1e-10, control = list(), legacy = FALSE)
R> summary(semfemod)
```

Spatial panel fixed effects error model

Call:

```
spml(formula = fm, data = Produc, listw = usalw, model = "within",
      effect = "time", method = "eigen", na.action = na.fail, quiet = TRUE,
      zero.policy = NULL, interval = NULL, tol.solve = 1e-10, control = list(),
      legacy = FALSE)
```

Residuals:

Min.	1st Qu.	Median	3rd Qu.	Max.
-0.219000	-0.064500	-0.000592	0.055400	0.317000

Coefficients:

	Estimate	Std. Error	t-value	Pr(> t )
rho	0.4962301	0.0357912	13.8646	< 2.2e-16 ***
log(pcap)	0.1432725	0.0165720	8.6455	< 2.2e-16 ***
log(pc)	0.3636539	0.0109631	33.1707	< 2.2e-16 ***
log(emp)	0.5619649	0.0143684	39.1113	< 2.2e-16 ***
unemp	-0.0078930	0.0018665	-4.2288	2.349e-05 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

As before, time period fixed effects and the intercept can be recovered as follows:

```
R> eff <- effects(semfemod)
R> eff
```

Intercept:

	Estimate	Std. Error	t-value	Pr(> t )
(Intercept)	1.412536	0.050965	27.716	< 2.2e-16 ***

Time period fixed effects:

	Estimate	Std. Error	t-value	Pr(> t )
1	-0.00515318	0.05167995	-0.0997	0.9206
2	0.00103556	0.05200686	0.0199	0.9841
3	0.01161188	0.05193737	0.2236	0.8231
4	0.02086866	0.05182860	0.4026	0.6872
5	-0.01243892	0.05194369	-0.2395	0.8107
6	-0.01638407	0.05254389	-0.3118	0.7552
7	-0.01602721	0.05238016	-0.3060	0.7596
8	-0.00817852	0.05217527	-0.1568	0.8754
9	-0.00108650	0.05184557	-0.0210	0.9833
10	-0.00714318	0.05177969	-0.1380	0.8903
11	-0.02071186	0.05204947	-0.3979	0.6907
12	-0.00791710	0.05222694	-0.1516	0.8795
13	-0.01409039	0.05284233	-0.2666	0.7897
14	0.00042906	0.05286077	0.0081	0.9935
15	0.01861529	0.05225588	0.3562	0.7217

16 0.02531034 0.05216326 0.4852 0.6275  
 17 0.03126013 0.05219134 0.5990 0.5492

## 6. GM implementation

To simplify the exposition, we follow [Kapoor \*et al.\* \(2007\)](#) and describe the estimation theory of a model that does not include the spatial lag of the dependent variable. For the estimation of the full model we refer the interested reader to [Mutl and Pfaffermayr \(2011\)](#) and [Piras \(2011\)](#).

### 6.1. Random effects model

The estimation procedure for a random effects model is a combination of the traditional panel data literature on error component models and the GM approach to spatial models. [Kapoor \*et al.\* \(2007\)](#) suggest a generalization of the generalized moment estimator suggested in [Kelejian and Prucha \(1999\)](#) for estimating the spatial autoregressive parameter ( $\rho$ ) and the two variance components of the disturbance process ( $\sigma_1^2$  and  $\sigma_\nu^2$ ). Specifically, they define three sets of GM estimators based on the following moment conditions:

$$E \begin{bmatrix} \frac{1}{N(T-1)} \varepsilon^\top Q_0 \varepsilon \\ \frac{1}{N(T-1)} \bar{\varepsilon}^\top Q_0 \bar{\varepsilon} \\ \frac{1}{N(T-1)} \bar{\varepsilon}^\top Q_0 \varepsilon \\ \frac{1}{N} \varepsilon^\top Q_1 \varepsilon \\ \frac{1}{N} \bar{\varepsilon}^\top Q_1 \bar{\varepsilon} \\ \frac{1}{N} \bar{\varepsilon}^\top Q_1 \varepsilon \end{bmatrix} = \begin{bmatrix} \sigma_\nu^2 \\ \sigma_\nu^2 \frac{1}{N} \text{tr}(W^\top W) \\ 0 \\ \sigma_1^2 \\ \sigma_1^2 \frac{1}{N} \text{tr}(W^\top W) \\ 0 \end{bmatrix} \quad (24)$$

where  $\varepsilon = u - \rho \bar{u}$ ,  $\bar{\varepsilon} = \bar{u} - \rho \bar{\bar{u}}$ ,  $\bar{u} = (I_T \otimes W_N)u$ , and  $\bar{\bar{u}} = (I_T \otimes W_N)\bar{u}$ .<sup>13</sup>

The first set of GM estimators is based only on a subset of these moment conditions (the first three equations) and assigns equal weights to each of them. This first set of estimators should therefore be intended as initial estimators.

The second set of GM estimators uses all of the moment conditions and an optimal weighting scheme. It is indeed well known from the theory of GM estimators that for asymptotic efficiency the ideal weighting matrix is the inverse of the variance covariance matrix of the sample moments at the true parameter values. [Kapoor \*et al.\* \(2007\)](#) derive this matrix under the assumption of normally distributed innovations. They point out that, although the use of such a matrix is not strictly optimal in the absence of normality, it can be viewed as a reasonable approximation of the true and more complex variance covariance matrix.

The third set of GM estimators is motivated by computational difficulties. The elements of the asymptotic variance covariance matrix of the sample moments involve a computational count of up to  $O(n^3)$ . Although one could take advantage of the particular structure of  $W$ , the computation of such a matrix can still be difficult in many cases. The third set of GM estimators uses all moment conditions but a simplified weighting scheme.

<sup>13</sup> In a random effects model without a spatial lag of the dependent variable, the OLS estimator of  $\beta$  is consistent, and thus it can be used to calculate the estimated disturbances ( $u$ ) employed in the GM procedure.



Using any of the previously defined estimators for the spatial coefficient and the variance components, a feasible GLS estimator of  $\beta$  can be defined based on a spatial Cochrane-Orcutt type transformation of the original model. However, following the classical error component literature, a convenient way of calculating the GLS estimator is to further transform the (spatially transformed) model by premultiplying it by  $I_{NT} - \theta Q_1$ , where  $\theta = 1 - \sigma_\nu/\sigma_1$ . The feasible GLS estimator is then identical to an OLS calculated on the “doubly” transformed model. Finally, small sample inference can be based on the following expression for the coefficient’s variance-covariance matrix

$$\Psi = (X^{*\top} \Omega_\varepsilon^{-1} X^*)^{-1} \quad (25)$$

where the variables  $X^*$  can be viewed as the result of a spatial Cochrane-Orcutt type transformation of the original model, and  $X^*$  and  $\Omega_\varepsilon^{-1}$  depend on the estimated values of  $\rho$ ,  $\sigma_\nu^2$  and  $\sigma_1^2$  respectively.

### *Illustration*

`spgm` is a general interface to estimate various nested specifications of the model presented in Section 4. The function also gives the possibility of including additional (other than the spatial lag) endogenous variables. To make sure that we are estimating a random effects specification, the argument `effects` should be set to `"random"`. Along with a mandatory `formula` object to describe the model, the function consists of a series of optional arguments. Among them, there are two logical vectors that control for the basic model specification: `spatial.error` and `lag`. When both arguments are `FALSE`, an endogenous variable should be specified (`endog`) along with a set of `instruments`. In this particular case, the function uses an estimation engine (`ivspml`) to perform instrumental variables estimation for panel data models. The argument `method` can be used to select among different estimators.<sup>14</sup>

When `spatial.error` is `TRUE` and `lag` is `FALSE`, the model corresponds to the one in [Kapoor et al. \(2007\)](#) and the residuals employed in the GM estimator come from an OLS regression. The argument `moments` allows to opt for one of the three sets of GM estimators. The default is to perform the initial estimator. If the argument `moments` is set to `"fullweights"`, the second estimator (i.e., the one involving the full expression of the variance covariance matrix of the moments conditions) is performed.<sup>15</sup> Finally, to obtain the third estimator the argument `moments` should be set to `"weights"`.

On Munnell’s data this would lead to:

```
R> GM_error <- spgm(formula = fm, data = Produc, listw = usaww,
+   moments = "fullweights", model = "random", spatial.error = TRUE)
R> summary(GM_error)
```

### Spatial panel random effects GM model

<sup>14</sup>Those are a within two stage least squares estimator (`"w2s1s"`), a between two stage least squares estimator (`"b2s1s"`), the GLS random effects two stage least squares estimator (`"g2s1s"`), and the error component two stages least squares (`"ec2s1s"`) of [Baltagi \(1981\)](#). These estimators are also implemented in `plm`. We only extended them to deal with the case of a spatially lagged dependent variable.

<sup>15</sup>The calculation of the trace terms in the expression of the variance covariance matrix of the sample moments uses code from the `Matrix` package.

Call:

```
spgm(formula = fm, data = Produc, listw = usaww, model = "random",
      spatial.error = TRUE, moments = "fullweights")
```

Residuals:

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
-0.26600	-0.06560	-0.00717	-0.00480	0.04850	0.45900

Estimated spatial coefficient, variance components and theta:

	Estimate
rho	0.5480458
sigma <sup>2</sup> <sub>v</sub>	0.0011228
sigma <sup>2</sup> <sub>1</sub>	0.0880980
theta	0.8871080

Coefficients:

	Estimate	Std. Error	t-value	Pr(> t )
(Intercept)	2.2273109	0.1350925	16.4873	< 2.2e-16 ***
log(pcap)	0.0540235	0.0219720	2.4587	0.013942 *
log(pc)	0.2565950	0.0209339	12.2574	< 2.2e-16 ***
log(emp)	0.7278192	0.0252306	28.8466	< 2.2e-16 ***
unemp	-0.0038108	0.0011004	-3.4631	0.000534 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

The `summary` method, after a short description of the model, prints the most recent call, a summary of the residuals and the table of estimated coefficients. The output also contains a print out of the estimated spatial coefficient, the variance components  $\sigma_v^2$  and  $\sigma_1^2$ , and  $\theta$ . One of the main advantages of the GM approach compared to ML is that the former is computationally less intensive than the latter (mostly because it does not involve the computation of Jacobian terms). The function `spgm` can deal with the estimation of very large datasets. As an example, we estimated a model with  $N = 10,000$  cross-sectional observations over  $T = 20$  time periods. Considering  $K = 11$  explanatory variables, the time to perform the second set of GM estimators was slightly more than 28 seconds on an Intel Core Duo MacBook with 4 GB of memory and a processor speed of 2.4 GHz.

When both `spatial.error` and `lag` are `TRUE` the complete model is estimated (i.e., one that has the spatial lag of the dependent variable and spatially autocorrelated residuals and individual effects). In this case, OLS residuals are no longer consistent because of the spatially lagged dependent variable and the estimation procedure should be modified accordingly (for details see [Mutul and Pfaffermayr 2011](#); [Piras 2011](#); [Baltagi and Liu 2011](#)).

A simple example using the Munnell's data would produce the following output:

```
R> GM_full <- spgm(formula = fm, data = Produc, listw = usaww, lag = TRUE,
+ moments = "fullweights", model = "random", spatial.error = TRUE)
R> summary(GM_full)
```

Spatial panel random effects GM model

Call:

```
spgm(formula = fm, data = Produc, listw = usaww, model = "random",
      lag = TRUE, spatial.error = TRUE, moments = "fullweights")
```

Residuals:

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
-0.27400	-0.06050	-0.00206	-0.00194	0.05260	0.47100

Estimated spatial coefficient, variance components and theta:

	Estimate
rho	0.3409051
sigma <sup>2</sup> <sub>v</sub>	0.0011002
sigma <sup>2</sup> <sub>1</sub>	0.0928450
theta	0.8911412

Coefficients:

	Estimate	Std. Error	t-value	Pr(> t )
lambda	0.02185030	0.01350631	1.6178	0.1057
(Intercept)	2.01866772	0.16797180	12.0179	< 2.2e-16 ***
log(pcap)	0.04668406	0.02244161	2.0802	0.0375 *
log(pc)	0.26596681	0.02036336	13.0610	< 2.2e-16 ***
log(emp)	0.72160852	0.02473123	29.1780	< 2.2e-16 ***
unemp	-0.00513207	0.00097481	-5.2647	1.404e-07 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

The first row of the table of estimated coefficients produced by the `summary` method reports the estimated coefficient of the spatially lagged dependent variable. Interestingly the results for the other coefficients are very stable when such a variable is included.

## 6.2. Fixed effects model

When the random effects assumption is questionable, one can estimate a fixed effects model instead. [Mutul and Pfaffermayr \(2011\)](#) note that under the fixed effects assumption OLS estimation of the regression equation is no longer consistent and the method of moment estimator can no longer be based on OLS residuals. They suggest to replace OLS with spatial two stage least squares within residuals ([Baltagi and Liu 2011](#)). Since in our discussion we are focusing on a model without the spatial lag of the dependent variable, a simple within estimator will produce consistent estimates of the model parameters. The first three moment conditions in [Kapoor \*et al.\* \(2007\)](#) can be reformulated in terms of these within residuals (see [Mutul and Pfaffermayr 2011](#), for details). One can then estimate the spatial parameter  $\rho$  using the GM procedure described in [Kapoor \*et al.\* \(2007\)](#) based only on these first three moments conditions. With an estimate of the spatial parameter, one can take a spatial Cochrane-Orcutt type transformation of the within transformed variables and estimate the resulting model by OLS.

*Illustration*

The function `spgm` serves as an interface also for the fixed effects once the argument `model` is set to "within". There is no need to specify the argument `moments`. The usual model specification using the Munnell's data leads to the following results:

```
R> GM_error <- spgm(formula = fm, data = Produc, lag = TRUE,
+   listw = usaww, model = "within", spatial.error = TRUE)
R> summary(GM_error)
```

Spatial panel fixed effects GM model

Call:

```
spgm(formula = fm, data = Produc, listw = usaww, model = "within",
     lag = TRUE, spatial.error = TRUE)
```

Residuals:

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
1.83	2.10	2.20	2.21	2.30	2.70

Estimated spatial coefficient, variance components and theta:

	Estimate
rho	0.3328374
sigma <sup>2</sup> <sub>v</sub>	0.0011278

Coefficients:

	Estimate	Std. Error	t-value	Pr(> t )
lambda	0.1313010	0.0245669	5.3446	9.060e-08 ***
log(pcap)	-0.0201442	0.0268540	-0.7501	0.4531718
log(pc)	0.1931190	0.0255344	7.5631	3.936e-14 ***
log(emp)	0.7304211	0.0303485	24.0678	< 2.2e-16 ***
unemp	-0.0036698	0.0010261	-3.5763	0.0003484 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Note that the results in terms of the estimated spatial coefficients are reasonably close to those obtained using the ML estimator. On the one hand, the value of  $\rho$  is 0.455 if the model is estimated by ML, and it drops to 0.333 when the model is estimated by GM. On the other hand,  $\lambda$  changes from 0.088 (ML) to 0.131 (GM). Furthermore, there is almost no difference between the ML and GM estimates for the coefficients other than the spatial ones.

## 7. Tests

In this section we describe the implementation of several LM tests for the panel data regression model with spatial error correlation derived in Baltagi *et al.* (2003). In the spirit of Mutl and Pfaffermayr (2011), we also implemented a spatial Hausman test.

### 7.1. LM tests

Since the seminal work of [Breusch and Pagan \(1980\)](#), Lagrange multiplier (LM) tests have been extensively employed to test for random effects and serial or cross-sectional correlation in panel data models. Requiring only the estimation of the restricted specification, LM tests are particularly appealing in a spatial random effects setting because of the computational difficulties related to the estimation of the full model.

[Baltagi et al. \(2003\)](#) derive joint, marginal and conditional tests for all combinations of random effects and spatial correlation for the model specification presented in Section 4. In particular, the hypotheses under consideration are:

1.  $H_0^a : \lambda = \sigma_\mu^2 = 0$  under the alternative that at least one component is not zero
2.  $H_0^b : \sigma_\mu^2 = 0$  (assuming  $\lambda = 0$ ), under the one-sided alternative that the variance component is greater than zero
3.  $H_0^c : \lambda = 0$  assuming no random effects ( $\sigma_\mu^2 = 0$ ), under the two-sided alternative that the spatial autocorrelation coefficient is different from zero
4.  $H_0^d : \lambda = 0$  assuming the possible existence of random effects ( $\sigma_\mu^2$  may or may not be zero), under the two-sided alternative that the spatial autocorrelation coefficient is different from zero
5.  $H_0^e : \sigma_\mu^2 = 0$  assuming the possible existence of spatial autocorrelation ( $\lambda$  may or may not be zero) and the one-sided alternative that the variance component is greater than zero

The joint LM test for the first hypothesis of no random effects and no spatial autocorrelation ( $H_0^a$ ) is given by:

$$LM_j = \frac{NT}{2(T-1)}G^2 + \frac{N^2T}{b}H^2 \quad (26)$$

where,  $G = \tilde{u}'(J_T \otimes I_N)\tilde{u}/\tilde{u}'\tilde{u} - 1$ ,  $H = \tilde{u}'(I_T \otimes (W + W')/2)\tilde{u}/\tilde{u}'\tilde{u}$ ,  $b = \text{tr}(W + W')^2/2$  and  $\tilde{u}$  denotes OLS residuals.

Equation 26 is also the point of departure for the derivation of the marginal LM tests used to verify  $H_0^b$  and  $H_0^c$ . The standardized version of the marginal LM test of no random effects assuming no spatial correlation is given by

$$SLM_1 = \frac{LM_1 - E(LM_1)}{\sqrt{\text{Var}(LM_1)}} \quad (27)$$

where  $LM_1$  is the square root of the first term in Equation 26. Analogously, the standardized version of the marginal LM test of no spatial autocorrelation assuming no random effects is given by

$$SLM_2 = \frac{LM_2 - E(LM_2)}{\sqrt{\text{Var}(LM_2)}}. \quad (28)$$

where  $LM_2$  is now the square root of the second term in Equation 26. Note that both Equation 27 and Equation 28 should be asymptotically normally distributed as  $N \rightarrow \infty$  (for

fixed  $T$ ) under  $H_0^b$  and  $H_0^c$  respectively.<sup>16</sup> Based on Equation 27 and Equation 28, a useful one-sided test statistic for  $H_0^a : \lambda = \sigma_\mu^2 = 0$  can be derived as:

$$LM_H = (LM_1 + LM_2)/\sqrt{2} \quad (29)$$

which is asymptotically distributed  $N(0, 1)$ . A test for the joint null hypothesis can, therefore, be based on the following decision rule:

$$\chi_m^2 = \begin{cases} LM_1^2 + LM_2^2 & \text{if } LM_1 > 0, LM_2 > 0 \\ LM_1^2 & \text{if } LM_1 > 0, LM_2 \leq 0 \\ LM_2^2 & \text{if } LM_1 \leq 0, LM_2 > 0 \\ 0 & \text{if } LM_1 \leq 0, LM_2 \leq 0 \end{cases}$$

Under the null the test statistic  $\chi_m^2$  has a mixed  $\chi^2$ -distribution given by:

$$\chi_m^2 = (1/4)\chi^2(0) + (1/2)\chi^2(1) + (1/4)\chi^2(2) \quad (30)$$

When using  $LM2$ , one is assuming that random individual effects do not exist. However, especially when the variance component is large, this may lead to incorrect inference. This is why Baltagi *et al.* (2003) derive a conditional LM test against the spatial autocorrelation coefficient being zero assuming that the variance component may or may not be zero. The expression for the test assumes the following form:

$$LM_\lambda = \frac{\hat{D}(\lambda)^2}{[(T-1) + \hat{\sigma}_\nu^4/\hat{\sigma}_1^4]b} \quad (31)$$

where,  $\hat{D}(\lambda)^2 = \frac{1}{2}\hat{u}' \left[ \frac{\hat{\sigma}_\nu^4}{\hat{\sigma}_1^4}(\bar{J}_T \otimes (W' + W)) + \frac{1}{\hat{\sigma}_\nu^4}(E_T \otimes (W' + W)) \right] \hat{u}$ . Also,  $\hat{\sigma}_1^4 = \hat{u}'(\bar{J}_T \otimes I_N)\hat{u}/N$ ,  $\hat{\sigma}_\nu^4 = \hat{u}'(E_T \otimes I_N)\hat{u}/N(T-1)$  and, contrary to previous tests that use OLS residuals, the residuals  $\hat{u}$  are ML. The comparative disadvantage of this last test is that its implementation is slightly more complicated because it is based on ML residuals. A one sided test is simply obtained by taking the square root of Equation 31. The resulting test statistic should be asymptotically distributed  $N(0, 1)$ . Similarly, when using  $LM1$  one is assuming no spatial error correlation. This assumption may lead to incorrect inferences particularly when  $\lambda$  is not very close to zero. A conditional LM test assuming the possible existence of spatial error correlation can be derived as:

$$LM_\mu = (\hat{D}_\mu)^2 \left( \frac{2\hat{\sigma}_\nu^4}{T} \right) (TN\hat{\sigma}_\nu^4 ec - N\hat{\sigma}_\nu^4 d^2 - T\hat{\sigma}_\nu^4 g^2 e + 2\hat{\sigma}_\nu^4 gh d - \hat{\sigma}_\nu^4 h^2 c)^{-1} \times (N\hat{\sigma}_\nu^4 c - \hat{\sigma}_\nu^4 g^2)$$

where,  $g = tr[(W'\hat{B} + \hat{B}'W)(\hat{B}'\hat{B})^{-1}]$ ,  $h = tr[\hat{B}'\hat{B}]$ ,  $d = tr[(W'\hat{B} + \hat{B}'W)]$ ,  $c = tr[((W'\hat{B} + \hat{B}'W)(\hat{B}'\hat{B})^{-1})^2]$  and  $e = tr[(\hat{B}'\hat{B})^2]$ . A one-sided test can be defined by taking the square root of Equation 32 based on ML residuals. The test statistic should be asymptotically distributed  $N(0, 1)$ .

### Illustration

The `bsktest` function can compute the joint, marginal or conditional tests for random effects and spatial error correlation. There are currently five options to the argument `test`, corresponding to the tests in the Baltagi *et al.* (2003): "LM1", "LM2", "LMJOINT", "CLMlambda", and

<sup>16</sup> For details on the expressions for the expected values and the variances of both tests see Baltagi *et al.* (2003).

"CLMmu". An optional logical parameter `standardize` is available to compute the standardized  $SLM_1$  and  $SLM_2$  marginal tests in place of  $LM_1$  and  $LM_2$ . In the following example we perform the standardized test of Equation 27. The alternative hypothesis is one of no random regional effects.

```
R> test1 <- bsktest(x = fm, data = Produc, listw = mat2listw(usaww),
+   test = "LM1")
R> print(class(test1))
```

```
[1] "htest"
```

```
R> test1
```

Baltagi, Song and Koh SLM1 marginal test

```
data: log(gsp) ~ log(pcap) + log(pc) + log(emp) + unemp
SLM1 = 0.083, p-value = 0.9338
alternative hypothesis: Random effects
```

The function `bsktest` returns an object of class `htest` for which a `print` method is available. The next example shows how to calculate the standardized test in Equation 28. The alternative hypothesis is one of no random regional effects.

```
R> test2 <- bsktest(x = fm, data = Produc, listw = mat2listw(usaww),
+   test = "LM2")
R> test2
```

Baltagi, Song and Koh LM2 marginal test

```
data: log(gsp) ~ log(pcap) + log(pc) + log(emp) + unemp
SLM2 = 0.0151, p-value = 0.988
alternative hypothesis: Spatial autocorrelation
```

The conditional tests  $LM_\lambda$  and  $LM_\mu$  are perhaps the most useful tests in this framework, because they test for one effect, and are robust against the other. This last example shows how to check for spatial correlation in the errors of a model that possibly incorporates random effects. This time we print the output directly:

```
R> bsktest(x = fm, data = Produc, listw = mat2listw(usaww),
+   test = "CLMlambda")
```

Baltagi, Song and Koh LM\*-lambda conditional LM test  
(assuming  $\sigma^2_\mu \geq 0$ )

```
data: log(gsp) ~ log(pcap) + log(pc) + log(emp) + unemp
LM*-lambda = 9.7157, p-value < 2.2e-16
alternative hypothesis: Spatial autocorrelation
```

## 7.2. Spatial Hausman test

The Hausman test (Hausman 1978) compares random and fixed effects estimators and tests whether or not the random effects assumption is supported by the data. Mutl and Pfaffermayr (2011) show how to extend this procedure to a spatial framework. The Hausman test statistic takes the form

$$H = NT(\hat{\theta}_{FGLS} - \hat{\theta}_W)^\top (\hat{\Sigma}_W - \hat{\Sigma}_{FGLS})^{-1} (\hat{\theta}_{FGLS} - \hat{\theta}_W) \quad (32)$$

where  $\hat{\theta}_{FGLS}$  and  $\hat{\theta}_W$  are, respectively, the spatial GLS and within estimators, and  $\hat{\Sigma}_W$  and  $\hat{\Sigma}_{FGLS}$  the corresponding estimates of the coefficients' variance covariance matrices.  $H$  is asymptotically distributed  $\chi^2$  with  $k$  degrees of freedom where  $k$  is the number of regressors in the model.

### *Illustration*

The method `sphtest` computes the spatial Hausman test described in the previous section. The argument can either be a `formula` describing the model to be estimated, or an object of class `splm`. If the argument is a `formula`, it should be specified along with three additional arguments: an object of class `listw`, a description of the model to be estimated (`spatial.model`) and the estimation method (`method`). Furthermore, if the estimation method is ML, the argument `errors` indicates which specification of the error term has to be considered.

The following example illustrates the function when the argument is a `formula`. We estimate a model without a spatial lag but with an autocorrelated error term. Since the estimation method is "GM" there is no need to specify the structure of the error term.

```
R> test1 <- sphtest(x = fm, data = Produc, listw = mat2listw(usaww),
+   spatial.model = "error", method = "GM")
R> test1
```

```
Hausman test for spatial models
```

```
data: x
chisq = 7.4824, df = 4, p-value = 0.1125
alternative hypothesis: one model is inconsistent
```

The function `sphtsest` returns an object of class `hstest` for which a `print` method is available. The next example shows that if the two models are estimated separately, the two objects of class `splm` can be given as arguments to the function.

```
R> mod1 <- spgm(formula = fm, data = Produc, listw = usaww, lag = TRUE,
+   moments = "fullweights", model = "random", spatial.error = TRUE)
R> mod2 <- spgm(formula = fm, data = Produc, listw = usaww, lag = TRUE,
+   model = "within", spatial.error = TRUE)
R> test2 <- sphtest(x = mod1, x2 = mod2)
R> test2
```



## Hausman test for spatial models

```
data: fm
chisq = 41.7396, df = 5, p-value = 6.65e-08
alternative hypothesis: one model is inconsistent
```

**7.3. Linear hypothesis testing**

Many functions for model estimation in R (e.g., `lm`, `glm` and all of the estimators in package **plm**) return objects that are compatible with generic extractor functions such as `coef` and methods such as `vcov`. In general, this is done to allow interoperability with functions calculating linear hypothesis tests such as `coeftest` from package **lmtest** (Zeileis and Hothorn 2002) for zero-restrictions, and `linearHypothesis` from package **car** (Fox and Weisberg 2010) for linear hypotheses. The model object produced by **splm** is consistent with such a framework, and, among other things, enable users to perform restriction tests on the model parameters. In the following example, a compact table of regressors' coefficient estimates is printed:

```
R> library("lmtest")
R> coeftest(sararremod)
```

z test of coefficients:

	Estimate	Std. Error	z value	Pr(> z )
(Intercept)	2.3736012	0.1394745	17.0182	< 2.2e-16 ***
log(pcap)	0.0425013	0.0222146	1.9132	0.055721 .
log(pc)	0.2415077	0.0202971	11.8987	< 2.2e-16 ***
log(emp)	0.7419074	0.0244212	30.3797	< 2.2e-16 ***
unemp	-0.0034560	0.0010605	-3.2589	0.001119 **

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Next, a test for the hypothesis that the elasticity of growth to public capital (`pcap`) and private capital (`pc`) are the same is performed:

```
R> library("car")
R> linearHypothesis(sararremod, "log(pcap) = log(pc)")
```

Linear hypothesis test

Hypothesis:  
log(pcap) - log(pc) = 0

Model 1: restricted model  
Model 2: log(gsp) ~ log(pcap) + log(pc) + log(emp) + unemp

Df	Chisq	Pr(>Chisq)
1		
2	1 38.145	6.566e-10 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

## 8. Numerical checks

The complexity of the estimation procedures implemented in the package requires that some checks be performed. We have done this in two ways. When it was possible, we compared our results against those of other available software. At the same time, we carried out a Monte Carlo simulation for the cases in which this was not possible.

### 8.1. Numerical check against other software

As a first step, we compare the estimators in the package with the MATLAB routines made available by [Elhorst \(2011\)](#). The comparison is performed on the [Baltagi and Griffin \(2001\)](#) cigarette consumption data set for 46 US states over a period of six years. The parameter estimates from the two implementations are presented along with the relevant  $t$  statistics for six model specifications: pooling, fixed, and random effects for both the spatial lag and the spatial error models.<sup>17</sup>

The first step is to load the data and the spatial weighting matrix that is then standardized and transformed in a `listw` object:<sup>18</sup>

```
R> cigar <- read.table("cigardemo.txt", header = TRUE)
R> fm <- logc ~ logp + logpn + logy
R> wcig <- as.matrix(read.table("spat-sym-us.txt"))
R> wcig <- wcig/apply(wcig, 1, sum)
R> lwcig <- mat2listw(wcig)
```

Running the procedure `demopanelcompare.m` (from [Elhorst 2011](#)) in MATLAB 7.1.0 on Windows 2000 gives the results reported in the first (coefficient) and third ( $t$  statistic) columns of [Table 1](#) (fixed effects) and [Table 2](#) (random effects). These figures are compared with the corresponding values from `splm` obtained through the following code (second and fourth column).

```
R> sarfe <- splm(formula = fm, data = cigar, listw = lwcig, lag = TRUE,
+   model = "within", effect = "individual", spatial.error = "none")
R> semfe <- splm(formula = fm, data = cigar, listw = lwcig, lag = FALSE,
+   model = "within", effect = "individual", spatial.error = "b")
R> sarre <- splm(formula = fm, data = cigar, listw = lwcig, lag = TRUE,
+   model = "random", spatial.error = "none")
R> semre <- splm(formula = fm, data = cigar, listw = lwcig, lag = FALSE,
+   model = "random", spatial.error = "b")
```

The results of the fixed effects specifications are very similar, both in terms of the parameters and the  $t$  statistics. The use of different optimization routines is a possible source of the (extremely small) numerical differences.

As explained before, the random effects estimators in `splm` optimize the original likelihood with the random effects explicitly considered in the error covariance matrix  $\Sigma$ . On the other

<sup>17</sup>Note that Elhorst's routines do not report standard errors but only  $t$  statistics and  $p$  values. For the convenience of the readers, we report  $t$  statistics instead of  $p$  values, since the latter are often very small.

<sup>18</sup>The original data from the web page of [Elhorst \(2011\)](#) are in spreadsheet format and have been saved to text before importing them into R.

		Coefficient estimate		<i>t</i> statistic	
		MATLAB	<b>splm</b>	MATLAB	<b>splm</b>
FE lag	$\lambda$	0.198966	0.198648	2.952892	2.9477
	logp	-0.608632	-0.608614	-12.653520	-12.6529
	logpn	0.233016	0.232903	3.559311	3.5575
	logy	0.294657	0.294722	7.708424	7.7099
FE error	$\rho$	0.299957	0.302676	4.263236	4.3099
	logp	-0.618106	-0.618338	-13.173769	-13.1806
	logpn	0.129409	0.128986	2.020166	2.0124
	logy	0.335804	0.335879	7.491027	7.4753

Table 1: Comparison of estimated coefficients and *t* statistics, spatial lag and spatial error models with individuals fixed effects. Elhorst's MATLAB routines as in `demopanelcompare.m` file and the `splm` function from the **splm** package, default settings (see code).

		Coefficient estimate		<i>t</i> statistic	
		MATLAB	<b>splm</b>	MATLAB	<b>splm</b>
RE lag	$\lambda$	0.183991	0.18127	2.693161	2.9243
	Const.	2.510781	2.521162	8.101490	14.8281
	logp	-0.619098	-0.618952	-11.871057	-11.8683
	logpn	0.229340	0.228368	3.287806	3.5016
	logy	0.313008	0.313567	7.650605	8.1882
RE error	$\rho$	0.311347	0.310914	4.081663	4.2105
	Const.	3.150075	3.157798	14.779637	14.8385
	logp	-0.627936	-0.629792	-12.385034	-12.4393
	logpn	0.123410	0.123491	1.793438	1.8052
	logy	0.364420	0.361601	7.629106	7.5787

Table 2: Comparison of estimated coefficients and *t* statistics, spatial lag and spatial error models with individuals random effects. Elhorst's MATLAB routines as in `demopanelcompare.m` file and the `splm` function from the **splm** package, default settings.

hand, Elhorst's routines applies the quasi-demeaning principle that is standard in non-spatial panel data estimators to eliminate the random effects. The likelihood is then optimized on the transformed data. As for the parameter variance covariance matrix, Elhorst (by default) relies on exact expressions, while the **splm** implementation uses the numerical Hessian approximation. The software approach is therefore substantially different. Given the differences in the environment and the optimizer, there is, in principle, room for larger differences than those found in the fixed effects case. However, the parameter estimates are almost identical; and only slightly larger differences are found in the *t* statistics. Almost none of these differences is relevant, with only the exception of the *t* statistic on the intercept of the spatial lag model, where the MATLAB procedure yields 8.10, and the value in **splm** is 14.83.

Finally, we perform a comparison on the pooled specification, i.e., without individual effects. Table 3 compares the results of `splm` with those of Elhorst's routines on a pooled specification, and with results in `spdep`. In fact, the pooled model can be reproduced also in `spdep` using the functions `lagsarlm` and `errorsarlm`. The user only needs to construct a block

		Coefficient estimate			<i>p</i> value/ <i>t</i> statistic		
		MATLAB	<b>splm</b>	<b>spdep</b>	MATLAB	<b>splm</b>	<b>spdep</b>
Lag	$\lambda$	0.081949	0.082250	0.08225	0.238648	0.1612	0.1629
model	Const.	1.305900	1.304801	1.304801	3.623877	4.4698	3.6211
	logp	-1.038360	-1.038347	-1.038347	-8.655961	-8.6811	-8.6559
	logpn	0.180022	0.180146	0.180146	1.429878	1.4654	1.4309
	logy	0.683524	0.683452	0.683452	9.722431	10.4750	9.7213
Error	$\rho$	0.144970	0.147554	0.14755	0.058506	0.02949	0.030924
	Const.	1.486675	1.484186	1.484186	4.758165	4.7444	4.7444
	logp	-1.060017	-1.060385	-1.060385	-8.968419	-8.9732	-8.9732
	logpn	0.150345	0.150483	0.150483	1.200308	1.2009	1.2009
	logy	0.729537	0.730092	0.730092	10.461687	10.4572	10.4572

Table 3: Comparison of estimates and diagnostics (*p* values for  $\lambda/\rho$ , *t* statistics for the remaining coefficients) for the pooled spatial lag and error models. Elhorst's MATLAB routines as in `demopanelcompare.m` file, `spml` function from the **splm** package (default settings) and `lagsarlm/errorsarlm` functions from the **spdep** package (default settings).

diagonal matrix whose diagonal elements are the spatial weighting matrix  $W$  (i.e., generating  $W_{pooled} = I_T \otimes W$ ). Since **spdep** only reports an asymptotically equivalent likelihood ratio test comparing the specification at hand with a non-spatial model but no significance test for spatial parameters, *p* values are reported for the spatial parameters instead of *t* statistics.

The code for reproducing pooled panel specifications in **splm** and **spdep** is as follows:

```
R> sarpool <- spml(formula = fm, data = cigar, listw = lwcig,
+   model = "pooling", spatial.error = "none", lag = TRUE)
R> sempool <- spml(formula = fm, data = cigar, listw = lwcig,
+   model = "pooling", spatial.error = "b", lag = FALSE)
R> pool.lwcig <- mat2listw(kronecker(diag(1, 6), listw2mat(lwcig)))
R> sarpool.2 <- lagsarlm(formula = fm, data = cigar, listw = pool.lwcig)
R> sempool.2 <- errorsarlm(formula = fm, data = cigar, listw = pool.lwcig)
```

Despite some implementation differences, the parameter estimates of **splm** and **spdep** are identical up to the sixth decimal. Those from MATLAB are also very similar. In terms of the *t* statistics (and *p* values, for the spatial parameters), **spdep** and MATLAB (both based on exact analytical covariances) show almost identical values for the  $\beta$ s. Interestingly, the values for the spatial parameters presents some differences (0.24 vs. 0.16 and 0.06 vs. 0.03). Although the covariance in **splm** is derived from a numerical Hessian, the results are very similar with those from **spdep**. The only exception is the value of the *t* statistic for the intercept, which is higher in **splm**: 4.47 against 3.62.

## 8.2. Monte Carlo simulation

Since there is no available software to estimate the general model, we also performed a (small) Monte Carlo simulation. The design is based on the two different specifications for the random effects. For the fixed effects case, the demeaning technique used in estimation removes the effects; and, therefore, the two specifications are indistinguishable.

	Estimate of $\lambda$			Estimate of $\rho$		
	$\lambda = -0.4$	$\lambda = 0.2$	$\lambda = 0.6$	$\lambda = -0.4$	$\lambda = 0.2$	$\lambda = 0.6$
$\rho = -0.4$	0.00016 (0.026)	-0.00077 (0.022)	-0.00078 (0.014)	-0.00579 (0.091)	-0.00288 (0.092)	-0.00865 (0.090)
$\rho = 0.2$	0.00028 (0.028)	-0.00121 (0.026)	-0.00163 (0.019)	-0.00521 (0.085)	-0.00637 (0.083)	-0.00549 (0.083)
$\rho = 0.6$	-0.00013 (0.030)	-0.00134 (0.033)	-0.00246 (0.028)	-0.00611 (0.057)	-0.00719 (0.058)	-0.00561 (0.060)

Table 4: ML estimation results for all combinations of spatial parameters over 2,000 simulation runs for the complete model with “Baltagi-type” random effects. Bias and RMSE (in brackets).

	Estimate of $\lambda$			Estimate of $\rho$		
	$\lambda = -0.4$	$\lambda = 0.2$	$\lambda = 0.6$	$\lambda = -0.4$	$\lambda = 0.2$	$\lambda = 0.6$
$\rho = -0.4$	-0.00007 (0.026)	-0.00056 (0.022)	-0.00055 (0.014)	-0.0043 (0.091)	-0.00304 (0.090)	-0.00483 (0.091)
$\rho = 0.2$	-0.00034 (0.028)	0.00002 (0.026)	-0.00053 (0.019)	-0.00564 (0.083)	-0.00594 (0.082)	-0.00789 (0.085)
$\rho = 0.6$	0.00004 (0.032)	-0.00085 (0.032)	-0.00066 (0.027)	-0.00423 (0.057)	-0.00416 (0.057)	-0.00683 (0.063)

Table 5: ML estimation results for all combinations of spatial parameters over 2,000 simulation runs for the complete model with fixed effects. Bias and RMSE (in brackets).

The idiosyncratic innovations are distributed as a standard Normal, and the individual effects as  $N(0, 2)$ . Along with an intercept term, we consider two regressors:  $x_1$  is sampled from a Uniform  $[-7.5, 7.5]$ ,  $x_2$  is drawn from a standard Normal.<sup>19</sup> The coefficients for the intercept as well as for the other regressors are set to 1. Our spatial layout is given by the 48 states of the continental US. The spatial weighting matrix is a simple binary contiguity one. We consider only one value for the number of time periods and set  $T = 7$ . We allow three different values for both  $\lambda$  and  $\rho$ , namely  $-0.4$ ,  $0.2$ , and  $0.6$ . For all experiments, 2,000 replications are performed.

In the following tables, we report bias and root mean-squared error (RMSE) for all the combinations of the spatial parameters. The tables on the left are relative to the estimate of  $\lambda$ , the ones on the right refer to  $\rho$ . Results are presented only for the two spatial parameters.<sup>20</sup>

Tables 4–5 are relative to the maximum likelihood estimators. In particular, Table 4 presents the results for the “Baltagi” random effects specification, Table 5 displays the results for the fixed effects, and Table 6 is devoted to the “KKP” random effects specification. Tables 7 and 8 present results from the generalized moments estimator: the random effects model is contained in Table 7, and the fixed effects model is displayed in Table 8.

When the true data generating process is assumed to be known, all estimators in the simulation show negligible bias and low root mean squared error. The results are satisfactory, especially considering our moderate sample size.

<sup>19</sup>The simulation parameters are chosen with a target  $R^2$  of 0.7.

<sup>20</sup>Results for the other parameters are available from the authors.

	Estimate of $\lambda$			Estimate of $\rho$		
	$\lambda = -0.4$	$\lambda = 0.2$	$\lambda = 0.6$	$\lambda = -0.4$	$\lambda = 0.2$	$\lambda = 0.6$
$\rho = -0.4$	-0.00028 (0.027)	-0.00053 (0.022)	-0.00043 (0.014)	-0.00516 (0.083)	-0.00934 (0.083)	-0.00857 (0.084)
$\rho = 0.2$	0.00159 (0.027)	-0.00089 (0.026)	-0.00079 (0.018)	-0.01408 (0.079)	-0.01342 (0.081)	-0.01276 (0.077)
$\rho = 0.6$	0.00143 (0.030)	0.00152 (0.034)	-0.00149 (0.027)	-0.0091 (0.054)	-0.01109 (0.055)	-0.01091 (0.060)

Table 6: ML estimation results for all combinations of spatial parameters over 2,000 simulation runs for the complete model with “KKP-type” random effects. Bias and RMSE (in brackets).

	Estimate of $\lambda$			Estimate of $\rho$		
	$\lambda = -0.4$	$\lambda = 0.2$	$\lambda = 0.6$	$\lambda = -0.4$	$\lambda = 0.2$	$\lambda = 0.6$
$\rho = -0.4$	0.00022 (0.027)	0.00021 (0.022)	0.00034 (0.014)	-0.00706 (0.093)	-0.00256 (0.093)	-0.00287 (0.095)
$\rho = 0.2$	0.00177 (0.028)	0.00078 (0.026)	0.00142 (0.019)	-0.00572 (0.081)	-0.0086 (0.081)	-0.01137 (0.085)
$\rho = 0.6$	0.00089 (0.033)	0.00316 (0.034)	0.00357 (0.028)	0.00739 (0.159)	0.0072 (0.169)	0.00281 (0.151)

Table 7: Generalized moments estimation results for all combinations of spatial parameters over 2,000 simulation runs for the complete model with “KKP-type” random effects. Bias and RMSE (in brackets).

	Estimate of $\lambda$			Estimate of $\rho$		
	$\lambda = -0.4$	$\lambda = 0.2$	$\lambda = 0.6$	$\lambda = -0.4$	$\lambda = 0.2$	$\lambda = 0.6$
$\rho = -0.4$	-0.00007 (0.026)	-0.00010 (0.022)	-0.00047 (0.014)	0.00271 (0.095)	-0.00272 (0.095)	0.00155 (0.094)
$\rho = 0.2$	0.00088 (0.028)	0.00014 (0.026)	-0.00004 (0.019)	-0.00909 (0.081)	-0.01093 (0.082)	-0.00673 (0.081)
$\rho = 0.6$	0.00126 (0.032)	0.00147 (0.034)	0.00147 (0.028)	-0.01098 (0.062)	-0.00938 (0.062)	-0.01446 (0.064)

Table 8: Generalized moments estimation results for all combinations of spatial parameters over 2,000 simulation runs for the complete model with fixed effects. Bias and RMSE (in brackets).

### 8.3. Numerical check of covariances

In the previous section we performed a numerical check of the precision of the estimates. We now focus on the estimation of the coefficients covariance, and, hence, on the reliability of inference.

We report the empirical 5% rejection rate for the  $z$  test of significance for both the spatial autoregressive parameter  $\lambda$  and the spatial autocorrelation coefficient  $\rho$ , for all combinations of the two. The results can be interpreted as a measure of the empirical power (for nonzero parameter values), and of the empirical size (for zero parameter values). The magnitude of

RE (B) ML, test $\lambda = 0$				RE (B) ML, test of $\rho = 0$			
	$\lambda = -0.2$	$\lambda = 0$	$\lambda = 0.2$		$\lambda = -0.2$	$\lambda = 0$	$\lambda = 0.2$
$\rho = -0.2$	1.000	0.056	1.000	$\rho = -0.2$	0.628	0.610	0.627
$\rho = 0$	1.000	0.069	1.000	$\rho = 0$	0.065	0.068	0.060
$\rho = 0.2$	1.000	0.055	1.000	$\rho = 0.2$	0.663	0.662	0.663

  

RE (KKP) ML, test $\lambda = 0$				RE (KKP) ML, test of $\rho = 0$			
	$\lambda = -0.2$	$\lambda = 0$	$\lambda = 0.2$		$\lambda = -0.2$	$\lambda = 0$	$\lambda = 0.2$
$\rho = -0.2$	1.000	0.052	1.000	$\rho = -0.2$	0.680	0.708	0.686
$\rho = 0$	0.999	0.065	1.000	$\rho = 0$	0.059	0.067	0.056
$\rho = 0.2$	1.000	0.059	1.000	$\rho = 0.2$	0.696	0.684	0.686

  

FE ML, test $\lambda = 0$				FE ML, test of $\rho = 0$			
	$\lambda = -0.2$	$\lambda = 0$	$\lambda = 0.2$		$\lambda = -0.2$	$\lambda = 0$	$\lambda = 0.2$
$\rho = -0.2$	1.000	0.076	1.000	$\rho = -0.2$	0.676	0.674	0.670
$\rho = 0$	1.000	0.099	1.000	$\rho = 0$	0.083	0.087	0.080
$\rho = 0.2$	1.000	0.067	1.000	$\rho = 0.2$	0.709	0.725	0.708

  

RE GM, test $\lambda = 0$			
	$\lambda = -0.2$	$\lambda = 0$	$\lambda = 0.2$
$\rho = -0.2$	1.000	0.056	1.000
$\rho = 0$	1.000	0.058	1.000
$\rho = 0.2$	1.000	0.048	1.000

  

FE GM, test $\lambda = 0$			
	$\lambda = -0.2$	$\lambda = 0$	$\lambda = 0.2$
$\rho = -0.2$	1.000	0.061	1.000
$\rho = 0$	1.000	0.050	1.000
$\rho = 0.2$	1.000	0.056	1.000

Table 9: Evaluation of covariance estimates: low correlation scenario. Empirical 5% rejection rates of significance  $z$  tests for  $\lambda$  (left column) and  $\rho$  (right column) for all combinations of spatial parameters in  $(-0.2, 0, 0.2)$ . Rejection rates are a measure of empirical size where the parameter is set to zero, of empirical power elsewhere.

the deviation from the nominal size is assessed using the outcome of the test as a sample from a binomial variate with probability equal to the nominal size and number of draws equal to the sample size so that the standard error is  $\sqrt{0.05 \times 0.95/2000}$  and the 95% confidence interval is approximately  $0.04 - 0.06$ .

Table 9 reports the empirical rejection rates for all nine combinations of  $\lambda = (-0.2, 0, 0.2)$  and  $\rho = (-0.2, 0, 0.2)$ .<sup>21</sup> The low levels of spatial correlation are chosen in order to check power against weak spatial dependence.

In Table 10 we report the same measures for a more clear-cut situation where the values of the

<sup>21</sup> In both scenarios, in one simulation run of the combination  $\lambda = 0, \rho = 0$  both the RE (B) and the FE maximum likelihood estimators returned a singular matrix error: both these results were discarded and in these two cases the report is based on 1999 out of 2000 runs.

RE (B) ML, test $\lambda = 0$				RE (B) ML, test of $\rho = 0$			
	$\lambda = -0.6$	$\lambda = 0$	$\lambda = 0.6$		$\lambda = -0.6$	$\lambda = 0$	$\lambda = 0.6$
$\rho = -0.6$	1.000	0.056	1.000	$\rho = -0.6$	1.000	1.000	1.000
$\rho = 0$	1.000	0.069	1.000	$\rho = 0$	0.068	0.068	0.058
$\rho = 0.6$	1.000	0.051	1.000	$\rho = 0.6$	1.000	1.000	1.000

  

RE (KKP) ML, test $\lambda = 0$				RE (KKP) ML, test of $\rho = 0$			
	$\lambda = -0.6$	$\lambda = 0$	$\lambda = 0.6$		$\lambda = -0.6$	$\lambda = 0$	$\lambda = 0.6$
$\rho = -0.6$	1.000	0.051	1.000	$\rho = -0.6$	1.000	1.000	1.000
$\rho = 0$	1.000	0.065	1.000	$\rho = 0$	0.058	0.067	0.054
$\rho = 0.6$	1.000	0.067	1.000	$\rho = 0.6$	1.000	1.000	1.000

  

FE ML, test $\lambda = 0$				FE ML, test of $\rho = 0$			
	$\lambda = -0.6$	$\lambda = 0$	$\lambda = 0.6$		$\lambda = -0.6$	$\lambda = 0$	$\lambda = 0.6$
$\rho = -0.6$	1.000	0.077	1.000	$\rho = -0.6$	1.000	1.000	1.000
$\rho = 0$	1.000	0.099	0.997	$\rho = 0$	0.082	0.087	0.078
$\rho = 0.6$	1.000	0.065	0.989	$\rho = 0.6$	1.000	1.000	1.000

  

RE GM, test $\lambda = 0$			
	$\lambda = -0.6$	$\lambda = 0$	$\lambda = 0.6$
$\rho = -0.6$	1.000	0.056	1.000
$\rho = 0$	1.000	0.058	1.000
$\rho = 0.6$	1.000	0.058	1.000

  

FE GM, test $\lambda = 0$			
	$\lambda = -0.6$	$\lambda = 0$	$\lambda = 0.6$
$\rho = -0.6$	1.000	0.060	1.000
$\rho = 0$	1.000	0.050	1.000
$\rho = 0.6$	1.000	0.061	1.000

Table 10: Evaluation of covariance estimates: moderate correlation scenario. Empirical 5% rejection rates of significance  $z$  tests for  $\lambda$  (left column) and  $\rho$  (right column) for all combinations of spatial parameters in  $(-0.6, 0, 0.6)$ . Rejection rates are a measure of empirical size where the parameter is set to zero, of empirical power elsewhere.

spatial parameters are chosen over combinations of  $-0.6, 0, 0.6$ . This simulation is designed to assess power against substantial spatial correlation.

Empirical size is generally reasonable in both scenarios, with the partial exception of FE models (discussed below). As for the rest, the “significance region”  $(0.4 - 0.6)$  contains most of the results. In both maximum likelihood RE cases, slight overrejection happens mostly in the rather overparameterized case when our SARAR specification is applied to the DGP where both  $\lambda = \rho = 0$ ; yet, the rejection rate never reaches 7%. The rejection rate is also consistently satisfactory for all combinations of  $\lambda$  and  $\rho$  for the GM estimators, especially when the small size of the test sample has been taken into account.

Test power is also reasonably good, especially for the test on  $\lambda$ , where even in the low spatial correlation scenario the covariance estimates are precise enough to always tell its presence in



any of the models considered. The test on  $\rho$  is not applicable to the GM models, which do not allow estimating the covariance of this parameter. As for the ML estimators, in the low spatial correlation scenario, estimates of the power of the test are all close to 70%. When the level of spatial correlation is higher, the power approaches one.

The FE case deserves special attention. Some of the rejection rates (8%,) are well above the upper limit of the “significance region”. This result might at first be considered a sign of lack of precision of the numerical Hessian approximation used in the FE SARAR estimator. However, the same evidence is confirmed for the FE SAR and SEM estimators, which implement the analytical expression of the parameter covariance. We believe that this result might be due to the serial correlation induced in the residuals by the demeaning transformation. We performed simulation experiments that seem to confirm our hypothesis. From those experiments, the results indicate that the overrejecting behavior is more serious for small  $T$  (i.e.,  $T = 3$ ), with rejection rates over 12%, and disappears when  $T$  is large (i.e.,  $T = 25$ ). This seems therefore a methodological issue that merits further research, rather than a software implementation problem.<sup>22</sup> Considering that in fixed effects models the distinction between the Baltagi and KKP specifications vanishes, in light of the above result it is advisable to use the `spgm` function for estimating panels of short-to-moderate time dimension.

Covariance estimation through numerical Hessians proved reliable and precise, also when compared with analytical expressions.<sup>23</sup> One final issue relates to the emergence of negative estimates for the parameters’ variance. This occurs almost exclusively when the data generating process is one of the nested specifications (i.e., the value of the spatial parameter is zero), with frequencies of around 3%. Generalized moments estimators, on the other hand, are completely free from this problem.

## 9. Conclusions

The analysis of spatial panel data is a sub-field of econometrics that has lately been experiencing increased methodological progress. Applied applications however are hindered by the lack of readily available software. The R environment is ideal for its development because of the vast infrastructure already in place for analyzing spatial data.

**splm** is a new package for the estimation and diagnostic testing of various spatial panel models. Supported estimation techniques include ML as well as GM. Lagrange multiplier tests along with a spatial version of the Hausman test are also provided.

The available techniques cover a good part of the recent developments in the spatial panel data literature, providing easy access to estimation and tests procedures not yet available in any commercial software. Some of the functionalities in **splm** are also available as **MATLAB** or **Stata** code, but this is the first attempt to provide a comprehensive tool within an organized statistical programming environment.

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<sup>22</sup>The fact that time-demeaning of serially uncorrelated residuals induces serial correlation with a coefficient of  $-1/(T - 1)$  is documented in Wooldridge (2002, p. 270, Equation 10.52). The Monte Carlo simulations on which these considerations are based are available in the supplementary files. This problem was first recognized by Lee and Yu (2010c). They show that the direct approach will produce inconsistent estimates of some of the parameters (including the variance). As already mentioned in Footnote 12, they suggest an alternative transformation which corrects for this bias. In a future release of **splm** we plan to add this feature.

<sup>23</sup>Simulation results regarding the comparison between numerical and analytical Hessian-based covariance estimation are available in the supplementary files.

Whenever possible, the package is consistent with the standard conventions of the R environment and in particular it borrows functionalities from **spdep** and **plm**. A new class had to be defined for spatial panel model objects, along with methods for providing the standards expected by the average R user. We also achieved interoperability with generic functions, e.g., those available in other packages such as **car** or **lmtest**.

The main developments in the foreseeable future should be directed toward the inclusion of new methodologies (e.g., Lee and Yu 2010a,c; Pesaran and Tosetti 2011, among others). Furthermore, we plan to extend the package and open it to the development of dynamic spatial panel data models. Different approaches to the implementation of the VC matrix of the estimators are also on our research agenda.

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