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A measure of well-being across the Italian urban areas: an Integrated DEA-Entropy Approach

Eugenia Nissi- Annalina Sarra
Department of Economics- Viale Pindaro, 42 -65127 Pescara (Italy)

nissi@unich.it- asarra@unich.it

Abstract:

In recent years, there has been an increasing proliferation of initiatives focusing on the concept of quality of life and well-being. At the centre of these studies there is the recognizing that the GDP offers only a partial perspective of factors affecting people's lives. Following this line of the research, this paper is aimed at computing the well-being efficiencies of a sample of Italian Province capital cities, using a methodological approach that combines Data Envelopment Analysis (DEA) with Shannon's entropy formula. To avoid subjectivity in choosing a representative set of variables that proxy the phenomenon under study, we rely on the theoretical framework adopted by the Italian National Institute of Statistics (ISTAT) within the Equitable and Sustainable Well-being (BES) project. The dashboard of indicators included in the analysis are related to the Ur-BES initiative, promoted by ISTAT to implement the BES framework at cities level. In a first step of the analysis, an immediate focus on separate dimensions of urban well-being is obtained by summarizing the plurality of available indicators through the building of composite indices. Next, the adopted integrated DEA-Shannon entropy approach has permitted to increase the discriminatory power of DEA procedure and attain a more reliable profiling of Italian Province capital cities well-being efficiencies. The results show a marked duality between the Northern and Southern cities, highlighting important differences in many aspects of human and ecosystem well-being.

Keywords: well-being, composite indicators, Data Envelopment Analysis (DEA), Shannon's entropy, benchmarking, Italy.

1.Introduction

Today, over half of the world's population lives in urban areas and the urban population is increasing by 2% annually (UNDESA 2014). Thus, more than two billion people are expected to be added to urban populations over the next three decades.

With the rapid growth of urban centres, research in understanding the complex linkages between urbanization, environmental change and well-being is ever more attracting a lot of attention (Bai et al. 2012).

Urban and community quality of life and well-being has become central to policy in most European Union (EU) countries, as documented in many scientific publications (Banai and Rapino 2009; Insch and Florek 2008; Sirgy and Cornwell 2002; Smith et al. 1997).

Urban areas are extremely complex, multifaceted and dynamic environments, in which a large number of factors, mainly environmental, social, cultural and economic, have an impact on individual and societal well-being and pose challenge for maintaining and improving their levels (Bai and Imura 2000; McMichael 2000).

On one hand, the socio-economic concentration of activities in urban areas makes them important generators of development, growth, innovation and poverty reduction. Likewise, cities also experience negative aspects. Natural resources depletion, pollution, health hazards, inequalities in opportunities, income and access to services, are just a few of the factors that prevent human wellbeing and environmental quality.

The rationale for well-being assessment at urban scale essentially lies in its intersection with public policy. By directly measuring the well-being of its citizens, policy makers may better understand the local needs and gain insights on programs that are not effective as well as on local advantages and their unique qualities. In such way, decision makers are constantly informed about the social progress achieved in the area in which they live and operate.

Furthermore, since higher level of well-being have been shown to have a positive impact on a number of conditions, which include improvements in physical and mental health, social and environmental behaviours, productivity (Howell 2007; OECD 2011), promoting well-being is becoming a serious concern of governmental policy.

In a word, to track and trace cities well-being, become an effective municipality tool to guide policy formulation and inform how cities are governed and regulated.

However, the evaluation of well-being at urban level poses some conceptual and operative challenges. A more disaggregated approach should select high-quality indicators and be representative of the specific features of each local community. Also, ideally, well-being indicators at urban level should be available over long periods, sensitive to changes and characterized by a high level of comparability.

In recent decades, many initiatives around the world have been undertaken, to improve the measurement of well-being at sub-national levels, by integrating different source of data (OECD 2015) and growing attention about the limitations of GDP as a simple and intuitive measure in guiding the progress of societies.

Also in Italy, there is an rising interest in measuring the well-being at local level, that is in regions and municipal entities (Mguni and Caistor-Arendar 2013).

Well-known Italian surveys are those promoted by IlSole24ore newspaper and Italia Oggi, which, on a yearly basis, elaborate a ranking of quality of life in the Italian provinces. Another important campaign "Sbilanciamoci" computes the Regional Quality Development index (QUARS) to assess the quality of life for the Italian regions.

A new contribution to the measurement of local well-being in Italy, comes from the Italian National Institute of Statistics (ISTAT), which, in conjunction with the National Council for Economy and Labour (CNEL), has given rise to a project, named "Equitable and Sustainable Well-Being" (whose Italian acronym, used hereafter, is BES), with the final aim of both developing a collective definition of progress in the Italian society and producing a shared set of indicators of the most relevant economic, social and environmental domains.

At the centre of these initiatives for the development of well-being indicators “beyond GDP”, there is a conceptualization of well-being as a multidimensional phenomenon.

In parallel with the national experience, ISTAT is testing, through the projects "BES of the Provinces" and "Ur-BES of metropolitan cities", the implementation of BES framework at local level.

The current article is aimed at constructing an aggregate index of urban well-being for the Italian Province capital cities. To avoid subjectivity in choosing a representative set of variables that proxy the phenomenon under study, we rely on the theoretical framework adopted by ISTAT within the BES project (see Cnel- ISTAT 2012), which, in turn, is based on the conceptual model published by OECD (Hall et al., 2010), as detailed in the next section.

With the purpose of representing most of the dimensions of well-being on which the BES framework is built, but forced by data availability and coverage, we use 44 variables that approximate the most important spheres of Equitable and Sustainable well-being at urban level. To evaluate the relative efficiency of Italian Province capital cities in promoting well-being, we go through two different steps of analysis. Firstly, for each domain, we summarize the socio-economic indicators by the Mazziotta -Pareto's method of penalties (Mazziotta and Pareto 2007; 2016). Next, the efficiency of that combination of indicators is facilitated by Data Envelopment Analysis (Charnes et al. 1978).

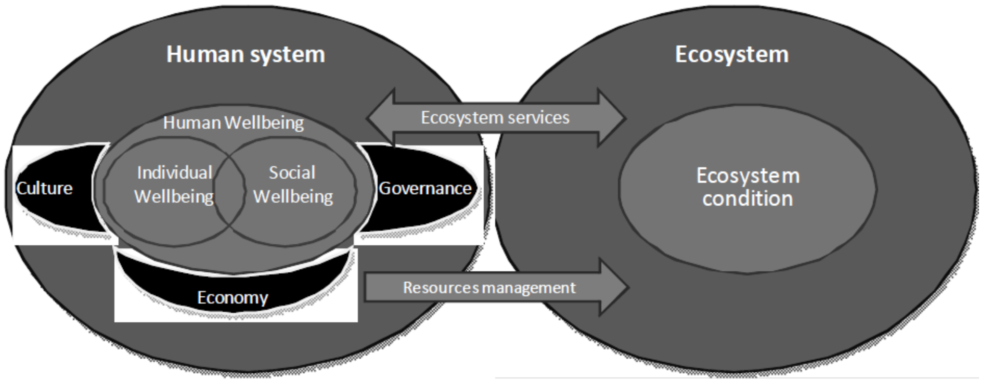
DEA methodology enables to aggregate information in a sensible manner, because it provides a built-in method of data standardization, since decisional units are ranked from zero to one, according to their level of efficiency, with weights generated endogenously from the data.

In order to capture generalized evaluations that are no longer determined by inherent input-output relations, and thus adapt the efficiency assessment to the well-being context, we use a unitary input DEA model, with entities defined only by outputs (Lovell et al. 1995; Cherchye et al. 2007).
In addition, in this paper, we follow a DEA-Shannon's entropy integrated approach to improve the discriminatory power of DEA method and construct a composite efficiency index. We apply this methodology to the data extracted from ISTAT databases and related to the UrBES statistics (ISTAT 2015), for monitoring equitable and sustainable well-being in the Italian Province capital cities. The paper is structured as follows: Section 2 is devoted to illustrate the conceptual framework on which is based our work. In Section 3, we review DEA methodology and its role in constructing composite indicators. Next, we discuss the limited discriminatory power of classic DEA models and introduce the Shannon-DEA procedure. Section 4 presents data used in this paper and summarizes the main results whereas some final remarks are given in Section 5.

2. The theoretical framework: the Equitable and Sustainable well-being

The estimation of the well-being that exists within a given society is a rather complex task since it is a multifaceted concept, being the outcome of compound interactions between a number of elements (Costanza et al. 2009).
When starting a process of measuring well-being, the main challenge is to define the domains and the indicators that can be valuable at the scale considered. In other words, a conceptual framework is required to have a reference structure for understanding well-being and what the dimensions and components of this concept are. A conceptual framework should also clarify the linkages among the various components and establish comprehensible guidelines for their operationalization (Hall et al. 2010). This implies that the basic structure chosen should present a list of indicators together with the suggested scales of measurement and a description of the relevant measuring tools.
Over the last three decades, there have been many initiatives, promoted by an expert group or as a result of public debates, to develop frameworks for progress and well-being at the community, national and international scale. Each of them uses its own conceptual approach as well as its own set of statistical measures.
In particular, new impulse to the research aimed at improving data and indicators which integrate the GDP has been added by the United Nations Development Programme (UNDP), the European Commission "GDP and beyond" (European Commission 2009), the OECD "Better Life" Initiatives, launched by the OECD in 2011 (OECD 2011) and the results of the so-called Stiglitz–Sen–Fitoussi report (2009).
The latest literature on this topic assesses critically the idea that economic growth is always synonymous with improved well-being and underlines the necessity to go beyond the GDP for the measurement of social development (Michalos 2008; Costanza et al. 2009; Fleurbaey 2009; Stiglitz et al. 2009; UNDP 2010; Larraz Iribas and Pavia 2010; OECD 2011).
Following these recommendations, most of the proposed well-being frameworks, used in social studies, measure that concept along different areas of life, such as income and material living conditions, health, education, employment, social relations, security, state of environment etc.
As already discussed in Rapley (2003), a range of factors have been identified to describe both the subjective and objective conditions of well-being and quality of life.
In this paper, we rely on the theoretical framework adopted by ISTAT within the BES project, which has adjusted for the Italian context the conceptual model published by OECD (Hall et al. 2010).
The framework under review, displayed in Figure 1, considers that societies rely upon two systems: the Human System and the Ecosystem, which represent the goals to achieve, that is the Human well-being and the Ecosystem well-being. In the theoretical framework depicted below, particular relevant are the relations between the Human System and Ecosystem, which are represented both in terms of "Resource management" and "Ecosystem services".

Figure .1 Framework of the progress of the societies



Source: Hall et al. 2010

Resources management characterizes the effects, in terms of resource depletion and pollution, of the human system on the ecosystem. The two systems are linked by ecosystem services in both directions. The ecosystem promotes the human system by means of positives services like food, clean water; likewise it can also do hurt through earthquakes and inundations. The human system may also provide positive services to the ecosystem (or its capacity for supporting life) through providing food and water for wild animals in times of privation.

Within this framework, Human well-being is the key domain since it embraces the core human ends that societies pursue. The Human well-being can be understood by looking at a number of attributes. Some features are specific to each person (health, education, etc.) and can be clustered together as attributes of “individual well-being”. Other attributes, are shared with other people (family or neighbourhood) or reflect the relations between them (e.g. the extent and quality of relationships with others), or how a society is peaceful, resilient, cohesive. All of these factors can be clustered together as “social well-being”.

It follows that Human well-being can be viewed as an equilibrium between individual well-being and societal well-being.

Human well-being requires some supportive pillars, such as culture, governance and economy. These elements are functional to achieve Human well-being and, accordingly, they are deemed intermediate objectives. On the other hand, the ecosystem has only one domain, namely the ecosystem condition, which represents the well-being of the ecosystem.

Putting together all these elements, one can speculate that there is progress of a society, or societal progress, when an improvement occurs in human well-being and the ecosystem condition.

Extremely important for this definition, is to recognize that the present well-being has to be able to improve over time and has to be related to the progress of future generations, introducing, in such way, an inter-generational sustainability dimension, often absent in other frameworks. Furthermore, well-being cannot be assessed without considering a intra-generational perspective. Thus, well-being has to be equitable among different social groups and among generations. As a result, the framework proposed by Hall (2010), delineates progress as an increase in equitable and sustainable well-being.

It is worth noting that the framework displayed in Figure 1 is not immediately operational, since, for each broad domain of that conceptual model, more precise dimensions need to be defined. In this respect, a list of dimensions, covering human well-being and ecosystem condition and a set of intermediate goals, covering economy, culture, and governance, is provided in Hall et al. (2010).

In general, we can observe that OECD conceptual model is compatible with Sen's capabilities approach (Sen 1993), whilst it does not associate progress to an increase in individuals’ evaluations of happiness/life satisfaction.

A broad comparison between the proposed framework and those adopted by some initiatives established to measure well-being and societal progress can be found in the annex of Hall's cited

paper. Here, it is sufficient to say that there is a considerable degree of overlap in how different initiatives view progress and well-being. However the presence of distortions in how some dimensions fit within the OECD framework, avoids to find a complete correspondence.
To summarize, the framework at issue encompasses most of the alternative frameworks proposed so far and it is broad enough to be adaptable to different cultures and societies.
The OECD framework has, indeed, represented the starting point to develop a conceptual structure for measuring the societal progress in Italy. In that country, an important contribution in this field comes from the Italian National Institute of Statistics. In 2010, ISTAT and the National Council for Economics and Labour (CNEL) launched an inter-institutional project, named "Equitable and Sustainable well-being" (Benessere Equo e Sostenibile -BES), with the final aim of developing a collective definition of progress in the Italian society and producing a shared set of indicators of the most relevant economic, social and environmental domains.
In line with the OECD perspective, the framework on which BES is built relies on the assumptions that well-being can be
regarded as a combination of individual and social components, and that there is progress of well-being when it is equitable and sustainable.
An important part of BES initiative is represented by the Ur-BES project to measure well-being at urban level. It is focused to build tools that allows administrators and local communities to analyse in detail the local well-being of their territories. The Ur-BES project aims at emphasising the well-being disparities among cities and points out single or multiple lacks of balance and relative deprivations of urban areas compared to district, regional or national parameters.
At such level of analysis, researchers have to deal with the trade-off between the need for synthesis and the loss of information (i.e. variability). Besides, some crucial issues concerning the dataset, such as the different structures and dimensions of the units of analysis, have to be kept in debit account, to get reliable and robust results for policy making.
To account for the complexity of well-being concept at urban level, the BES framework articulates the concept of well-being in 11 areas: Health, Education and training, Working and life balance, Social relationships, Economic wellbeing, Safety, Quality of services, Landscape and cultural heritage, Environment, Politics and institutions, Research and development (ISTAT 2015).
The dashboard of indicators includes 64 variables, viewed as a combination of individual and social well-being, selected through a participatory process, in which, all sectors of the society, in particular the civil society, have been involved in expressing their preferences.
The process of choosing the key indicators has been driven by the ability of selected variables in measuring the improvement or worsening of significant aspects of cities well-being and their accuracy in guaranteeing temporal and territorial comparisons.
It is worth noting that BES project has promoted the development of statistical indicators for what is considered relevant for the progress of a country, by adopting a formative measurement model (Diamantopoulos et al. 2008). This implies that any change in the formative indicators leads to a variation of both the latent variable and its operative definition. Additionally, following the Diamantopoulos' approach, the internal consistency of the formative indicators is of minimum relevance since two non-correlated indicators can be both significant for the same construct.

3. Methodology

In the previous section, we saw how the conceptualization of well-being as a multidimensional phenomenon, poses, on theoretical grounds, the challenge of defining which factors are relevant for this construct.
From the empirical point of view, a key problem is, instead, represented by the choice of a proper methodology to aggregate the subset of indicators, in order to obtain a reliable measure of the aspects affecting people's well-being (Gonzalez et al. 2011).
The most substantial issue is related to the arbitrary choices for weighting the sub-indicators.

An overview of difficulties arising in the construction of composite indicators is provided by Nardo et al. (2008), who stress how the subjectivity in determining the weights directly affects the quality and reliability of the resulting index.
Facing this issue, researchers have chosen different approaches. At first sight, the simplest way to weight sub-indicators is to impose equal influence to the different components. The alternative of opting for an equal weight scheme may be appropriate in the absence of an underlying theoretical framework, or when there is insufficient knowledge of the casual relationship between the variables (OECD 2008).
Instead of assigning arbitrarily given weights, another possible solution relies on the summary of experts' judgment. To this end, there are a variety of ways to summarize their opinions, even if it is becoming increasingly popular to resort to multi-criteria methods, which are methods that essentially involve different decision alternatives, to be evaluated on the basis of conflicting objectives. One of the most employed is the Analytical Hierarchic Process (AHP), which is based on an ordinal pair-wise comparison of attributes (Saaty 1980; 2001).
Besides, different statistical models, such as principal components analysis or factor analysis could be used to group individual indicators (Nicoletti et al. 2000). These multivariate statistical techniques account for the highest variation in the data set, replacing the original variables with the smallest possible number of factors that reflect the underlying "statistical" dimension of the data set.
Another methodological stream avoids the possible arbitrariness of equal weighting, by deriving weights directly from the data. Among data-oriented weighting methods, there are the Data Envelopment Analysis (Charnes et al. 1978) and the Benefit of Doubt (BoD) approach (OECD 2008; Nardo et al. 2008; Cherchye et al. 2007).
In the following sections, we first introduce the DEA technique and its applications to composite indicators through the BoD approach, then we will describe the model used in this paper.
Next, we discuss the limited discriminatory power of classic DEA models and focus on the discrimination improvement using Shannon's entropy and on the way to get a more comprehensive measure for the study at hand.

3.1 Data Envelopment Analysis (DEA)

Data Envelopment Analysis (DEA) is a widely used technique, originally developed to estimate the efficiencies of Decision Making Units (DMUs) within production contexts, characterised by multiple outputs and inputs (Charnes et al. 1978).
DEA allows to aggregate multiple inputs and outputs of the units under study into a relative efficiency score. More specifically, DEA compares the resources used (inputs) and the quantities produced (outputs) of a DMU to the levels of other units, and the result is the construction of an efficient frontier, establishing a dichotomous classification between efficient and inefficient units, with the DMUs lying on the frontier are efficient (unitary score), the other are inefficient (score of less than unity).
There is a wide variety of DEA models for assessing these units. A taxonomy and a general model framework can be found in Cook and Seiford (2009).
Over the past three decades, the scope of DEA has broadened considerably, with successfully applications to many different types of entities engaged in a wide variety of activities.
The properties of DEA as a powerful aggregation tool have been exploited for the first time to evaluate the quality of life by Hashimoto and Ishikawa (1993). After the abovementioned pioneer work, there have been other experiences involving the application of DEA to social indicators (see, among others, Hashimoto and Kodama 1997; Despotis 2005a; Murias et al. 2006; Somarriba and Pena 2009).
In the absence of specific knowledge about the true weights, the DEA approach, by allowing specific weights, helps to overcome the limitations of a fixed weighting schema, that, as known, can depreciate some units while favouring others.

In literature, it has been underlined the theoretical resemblance between that problem and the one of constructing composite indicators, in which quantitative sub-indicators are available but exact knowledge of weights is not (Cherchye et al. 2007).
The application of DEA to the field of the composite indicators is known as the Benefit of Doubt (BoD), originally proposed by Melyn and Moesen (1991) to evaluate macroeconomic performance.
On the same line with DEA, the BoD approach also retrieves information on the appropriate weighting schema from data themselves, providing unit-specific weights.
The core idea of BoD method is that a good relative performance of a unit in a particular dimension signifies that this units considers that dimension as relatively important. Equally, a poor performance indicates that a unit attaches less relative importance to that dimension.
It can be easily verified that the BoD model is like a DEA model with a "dummy inputs" equal for all units and the indicators as outputs (Cherchye et al. 2007).
In this paper, aimed at assessing the production of human well-being and ecosystem well-being at urban level, we do not have the classic production model but we can only rely on secondary variables, obtained as rates or combinations of primary variables. It follows that the production process can be conceptualized by considering each city as a “firm”, which uses government resources to produce well-being outputs, such as better education, improvement of health status, greater access to labour markets, reduction of environmental pollution and so on.
As a result, the relative performance of cities has to be evaluated with reference to the outputs they produce or the services they provide, without considering the resources they consume in the process. Thus, a DEA model with only outputs would be appropriate and, in order to avoid the inconsistencies that arise in a model without inputs, a single unitary input has to be included as in Koopmans (1951), Lovell et al. (1995), Despotis (2005b) and Cherchye et al. (2007). According to Koopmans (1951), this unitary input can be interpreted as a “helmsman” underlying every unit, who attempts to guide the units towards outputs maximization.
For the purpose of our work, we make use of the approach proposed by Lovell and Pastor (1999), in which the CCR and BCC models are equivalent. By adopting the output orientation, the linearized unitary input DEA-model is expressed by the following linear programming:

$$\max h_0 \tag{1}$$

$$\text{s.t.} \quad \sum_{k=1}^{n} \lambda_k y_{jk} \geq h_0 \; y_{jo} \qquad \forall j \tag{2}$$

$$\sum_{k=1}^{n} \lambda_k \leq 1 \tag{3}$$

$$\lambda_k \geq 0, \qquad \forall k \tag{4}$$

In equations (1)-(4), h_0, denotes the inverse of efficiency of the DMU under analysis (DMUo), y_{jk} is the jh output $(j = 1,..,s)$ of the DMU_k (k=1,...,n) and λ_k is the individual contribution of each DMU in the formation of DMUo's target.

3.2 DEA-Shannon entropy integrated approach

Over the last decades DEA has generated a good deal of attention, with applications in many real life studies, particularly in the areas of operations research and management science.
The strength of DEA lies mainly in its simplicity in handling multiple inputs and multiple outputs, without requiring any specific assumption on the functional relationships between them, and in its flexibility in choosing the weights. Additionally, the DEA procedure can be easily adapted to measure environmental and social aspects (i.e. quality of life, personal welfare) by including indicators that

are not expressed in monetary terms and changing the objective function in the standard model in order to recognize the change in focus.
However, the same characteristics that make DEA a powerful tool can also create some drawbacks.
An important limitation of the basic DEA models regards their inabilities to generate useful ranking across DMU_s because more than one of them might be scored as 100% efficient.
The revision of literature shows that many methods have been proposed to improve the discrimination power of the traditional DEA methods. Readers can refer to Adler and Yazhemsky (2010) to have a brief overview of these methods and their performance.
Promising approaches to alleviate the weak discrimination capability of classic DEA models are the cross-efficiency evaluation technique (Doyle and Green 1995; Green et al. 1996; Sexton et al. 1986; Wang and Chin 2010; 2011; Anderson et al. 2002) and the super-efficiency model (Andersen and Petersen 1993; Chen 2005; Lee et al. 2011; Chen et al. 2013).
Unlike standard DEA models, in cross evaluation the core idea is a peer evaluation rather than to operate in a pure self-evaluation mode. Cross-efficiency in DEA requires to evaluate each DMU_s according to the optimal weighting scheme of other $DMU_{s.}$ The results arising from these comparisons are used to obtain a cross-efficiency matrix, in which the diagonal members show the DEA efficiency scores of the DMUs and the off-diagonal cells give the cross-efficiency scores. Aggregation of the cross-efficiencies is then used to rank the DMUs.
Super efficiency DEA model was also introduced as another technique in literature to overcome the discriminatory power problems. Super-efficiency is often computed in two phases. The first phase is aimed to build the efficient frontier using a standard DEA model. In the next phase is provided a complete ranking of the efficient units, namely of the units that have identical efficiency scores equal to one in the basic DEA model. The super efficiency DEA model is obtained when a DMU under evaluation is excluded from the reference set. This only affects the efficiency scores of the extreme efficient DMUs allowing discrimination among efficient units. In this case, these DMU_s can obtain an efficiency score greater than one, i.e. super- efficiency. Instead, the exclusion of the inefficient DMU_s does not alter the efficient frontier, leaving their efficiency scores unchanged.
To improve discrimination in DEA, the present study exploits the idea of combining the traditional DEA methods and Shannon's entropy theory, first developed by Soleimani-Damaneh and Zarepisheh (2009).
As shown by different scholars, (Bian and Yang 2010; Jayaraman and Srinivasan 2014; Xie et al. 2014, lo Storto 2016) this integrated approach provides a methodology to combine the efficiency scores of different DEA models as well as a convenient way of creating a comprehensive ranking for all DMUs.
The motivation for the use of the Shannon's entropy based DEA mostly lies in its better performance respect to the simple DEA method in distinguishing two (or more) efficient DMUs.
Additionally, Xie et al. (2014) demonstrated via a numerical example that the integrated DEA-Entropy Approach has some advantages in ranking DMU_s as compared to the super-efficiency model.
Furthermore, instead of the traditional average set of weights as in cross-efficiency analysis, the Shannon's entropy based DEA procedure retrieves weights by integrating different DEA models into evaluation simultaneously.
In the current study, the efficiency of the Italian Provinces capital cities in providing well-being to their citizens is computed by specifying a DEA model with a single unitary input for all possible different combination of outputs, as detailed in the next section.
The combined DEA-Entropy procedure makes use of the assessing of the examined DMUs through a set of different models, say:

$$M = \{ M_{1,} M_2, \dots , M_k\}$$

The efficiency scores are presented in the matrix form:

$$\begin{pmatrix} E_{11} & E_{12} & \dots & E_{1k} \\ E_{21} & E_{22} & \dots & E_{2k} \\ E_{n1} & E_{n2} & \dots & E_{nk} \end{pmatrix}$$

in which n is the number of DMU$_s$ and k is the number of different DEA models performed. Subsequently, to achieve a more balance ranking of DMU$_s$, the efficiency scores of various DEA models have been combined using Shannon's entropy method and the degree of importance of each of the considered models calculated via some established steps (Soleimani-damaneh and Zarepisheh 2009), as detailed below.

Step 1: Normalization of the efficiency matrix E by recalculating the individual efficiencies as:

$$e_{jk} = \frac{E_{jk}}{\sum_{j=1}^{n} E_{jk}} \ , \qquad k = 1,2, \dots , K$$

Step 2: Computation of the Shannon's entropy index for each DEA model

$$f_k = -(\ln n)^{-1} \sum_{j=1}^{n} e_{jk} \ \ln(e_{jk}) \, , k = 1,2, \dots , K$$

Step 3: Calculation of the diversification degree of M_k as $d_k = 1 - f_k, \quad k = 1,2 \dots , K$. It is worth noting that the higher d_k, the greater the discrimination power of the DEA model M_k. As a result, this measure can be used to rate the importance of model M_k.

Step 4: Evaluation of the degree of importance of model M_k by calculating the weights
$W_k = \frac{d_k}{\sum_{k=1}^{K} d_k}, k = 1,2, \dots , K$ such that $\sum_{k=1}^{K} W_k = 1$

Step 5: Computation of a comprehensive efficiency score as:

$$\theta_j = \sum_{k=1}^{K} W_k E_{jk} \ , \qquad j = 1,2, \dots , n$$

If $\theta_j = 1$ then DMU$_j$ $(j = 1,2, \dots , n)$ is comprehensive DEA efficient.
It is clear to see that when some DMU$_s$ are efficient under one subset of outputs but inefficient under another subset, their comprehensive well-being score would be inefficient. In other terms, a DMU will be comprehensively efficient in providing well-being to its citizens, if and only if it is efficient under all subset variable models. This method can be used with either constant or variable returns to scale, or with either an input or an output orientation.

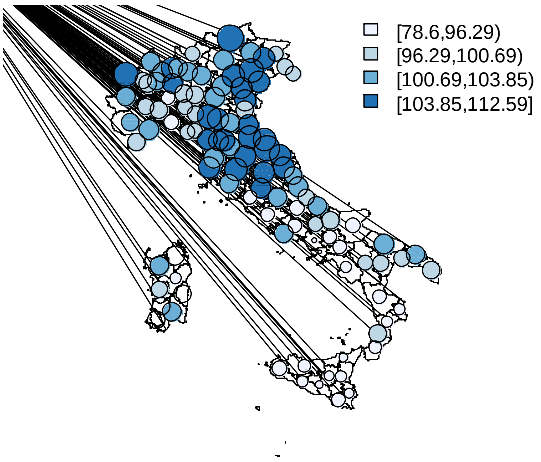
4. Benchmarking of equitable and sustainable well-being in the Italian urban areas

The methodology described in the previous section is employed to perform a benchmarking study, aimed at ranking and comparing the Italian urban well-being efficiencies. The case study considers the Italian Province capital cities as units of analysis and employs the Ur-BES Report available data (ISTAT 2015), which refers to 64 particular indicators, belonging to the dimensions identified by BES (Cnel-ISTAT 2012). The framework of Ur-BES appraises well-being in Italian Province capital cities by a great deal of variables, belonging to eleven different dimensions, given that the Ur-BES report does not take into account the variables describing the dimension related to subjective well-being. Unfortunately, at the current stage, Ur-BES cannot be considered a perfect model yet, as it does not measure satisfactorily all BES domains.
Thus, owing to the data unavailability and missing values, our analysis is restricted to eight domains, as listed in Table 1, while the sample size is limited to 103 Province capital cities.

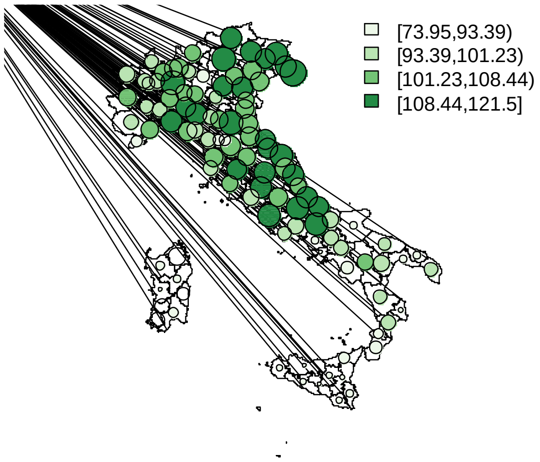
Table 1. Dimensions and variables

Dimension	Indicator	Polarity
Health	Life expectancy at birth (male) (year 2011)	+
	Life expectancy at birth (female) (year 2011)	+
	Infant mortality rate (year 2011)	-
	Mortality rate for road accidents (15-34 years old) (year 2011)	-
	Age-standardised cancer mortality rate (19-64 years old) (year 2011)	-
	Age-standardised mortality rate for dementia and related illnesses (people aged 65 and over) (year 2011)	-
Education and training	Participation primary school (year 2011)	+
	Participation in upper secondary education (year 2011)	+
	Participation in tertiary education (19-25years old) (year 2011)	+
	Early leavers from education and training (year 2011)	-
	Young people who do not work and do not study (year 2011)	-
	Level of literacy (year 2011)	+
	Level of numeracy (year 2011)	+
Work and life balance	Employment rate of people 20-64 years old (year 2011)	+
	Non-participation rate (15-74 years old) (year 2011)	-
	Incidence rate of fatal occupational injuries or injuries leading to permanent disability (year 2011)	-
	Employment rate of women with and without children(year 2011)	+
Economic well-being	Per capita adjusted disposable income (year 2011)	+
	Distribution of IRPEF incomes (year 2011)	+
	Quality of dwellings (year 2011)	+
	Number of people in workless households (year 2011)	-
	Households with suffering bank debts (year 2011)	-
Social relationships	Volunteers in no-profit organizations (per 100 residents aged 14+) (year 2011)	+
	No-profit organizations (year 2011)	+
	Social cooperatives (year 2011)	+
	Number of paid workers in local units of social cooperatives (year 2011)	+
Security	Homicide rate (year 2011)	-
	Burglaries (year 2011)	-
	Pickpocketing (year 2011)	-
	Robberies(year 2011)	-
Landscape and cultural heritage	Presence of historic rural landscapes (year 2011)	+
	Conservation of historic urban fabric (year 2011)	+
	Libraries (year 2011)	+
	Museums (year 2011)	+
	Visitors of libraries (year 2011)	+
	Visitors of museums and similar institutions (year 2011)	+
Environment	Drinkable water supplied every day per capita (year 2011)	+
	Exceeding of the daily limit for the protection of human health for PM10 (Maximum number) (year 2011)	-
	Urban parks and gardens (year 2011)	+
	Protected Natural Areas (year 2011)	+
	Urban green areas (year 2011)	+
	District heating (year 2011)	-
	Noise pollution (year 2011)	-
	Cars with Euro-4 standard (year 2011)	+

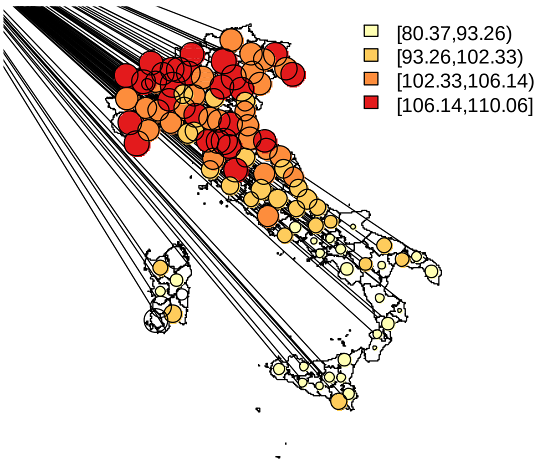
More specifically, in our work, we do not consider the pillars related to contextual domains (Politics and institutions, Research and Innovation and Quality of services), which have an impact on different areas of individual well-being and such that transverse and hard to measure.
Albeit we are aware that the elementary indicators are not transposable and negligible, because each one is explicitly designed to capture a specific aspect of the construct's domain, we believe that the current study can, anyway, provide a useful basis for better understand local well-being drivers and disparities across the Italian Province capital cities.
In the first stage of analysis, the available plural indicators, able to measure the urban well-being in each domain, have been synthesized through the building of a composite index.
In general, the procedure of summarizing a complex phenomenon into a single number, is a delicate task, which involves the choice of individual indicators, their normalization, in order to transform indicators into pure, dimensionless numbers, and the choice of an appropriate aggregation method.
In this last regard, a number of possible aggregation strategies can be found in literature, ranging from simple mathematical formulas, such as the mean-min function (Casadio et al. 2012) to complex procedures, such as the Multicriteria Analysis (Munda and Nardo 2009). Recently, a new and non-compensatory composite index, denoted as Mazziotta-Pareto Index (MPI) has been proposed, as a combined measure of a set of non-substitutable indicators (Mazziotta and Pareto, 2007; Mazziotta et al. 2010). The MPI requires that all indicators have all the same importance and a compensation among them is not allowed. This index is also known as the Method of Penalty Coefficient of Variation and consists of an arithmetic mean adjusted by a function of variability.
The underlying ratio is that of penalizing the geographical areas showing unbalanced values of the indicators.
For each given dimension of UrBES, the synthetic indices have been obtained through a variant of the Mazziotta-Pareto index, known as Adjusted Mazziotta-Pareto Index (AMPI) (Mazziotta and Pareto 2016). More details of the method are given in the Appendix section.
In the last column of Table 1, we displayed the sign (polarity) with each indicator contributes to the building of the AMPI for each dimension of well-being considered in this study.
Figure 2 allows a visual inspection of composite indicators by displaying a map of Italy in which the Italian Province capital cities are grouped in four classes (quartiles) according to the value of each AMPI. For each domain darker colours refer to higher values of the synthetic index, a lighter colour indicates a lower performance while the size of bubble is proportional to the associated data. Our results show diverse patterns for the different dimensions of well-being, highlighting the persistence of disparities between the Southern Italian cities and the ones of Centre-North in important quality of life aspects. Looking at the synthetic indicator of the education dimension of well-being, we observe that Province capital cities of the South are characterized by a substantial disparity in the dimension of people's well-being related to the Education and Training.
It is worth noting that education influences many important aspect of people's lives (Michalos 2008), since it not only enters the definition of a good life according to the human development concept but also for its instrumental relevance for fostering innovation, labour productivity and increasing income per capita.



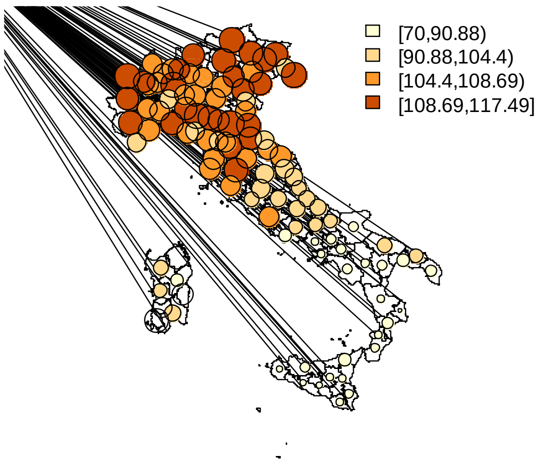
Health



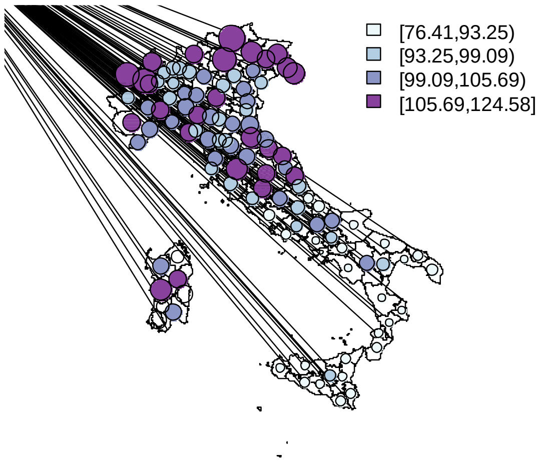
Education and training



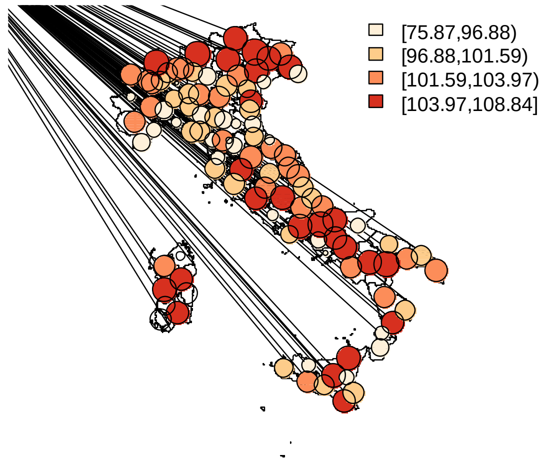
Work and life balance



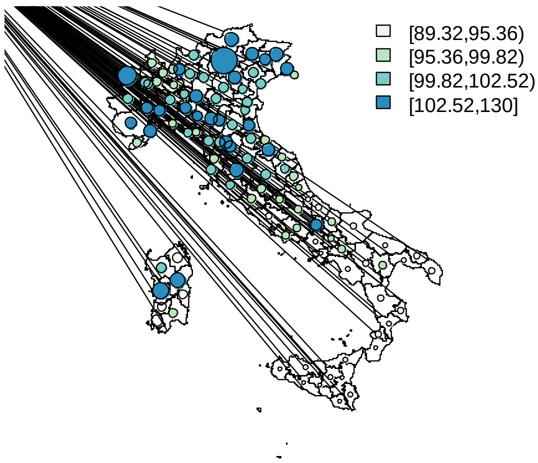
Economic well-being



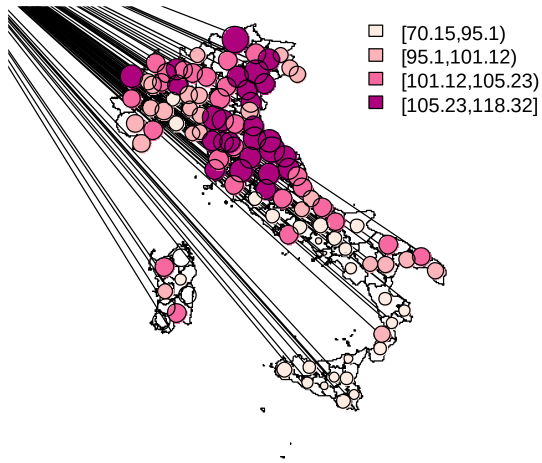
Social relationships



Security



Landscape and cultural heritage



Environment

Figure 2. Italian Province capital cities grouped in quartiles according to the composite indicator of each well-being domain (Source: our elaboration on ISTAT data)

From Figure 2 also emerges a clear gap for the employment domain, which plays a crucial role in defining well-being, both from the perspective of the opportunity for individuals to fulfil their job aspirations and from the perspective of earnings people must have to satisfy needs, personal ambitions and desires. According to the level and distribution of the corresponding synthetic index, the North and Central urban areas of Italy obtain scores that are higher than the Southern Province capital cities, revealing a clear advantage in the labour market condition. Since the pioneering initiative of the UNDP Human Development Index, the relationship between health and well-being is documented in many studies (see, among others, Howell et al. 2007). In our context, the values assumed by the synthetic health index, show that the gap North-South of Italy is less pronounced. Best performing are mostly the Central Province capital cities of Toscana, Umbria, Marche and Emilia-Romagna. Significant divergences still characterize the social relationships and landscape and cultural heritage domains: Northern and Central principal cities are the best performers. As for the dimensions of personal security, economic well-being and environment, the divide North-South of Italy is less manifest. Paying attention to the security index, we observe differentiated urban performances, not reproducing the recurrent divide from North and South of Italy. In fact, we find in the highest positions both Northern and Mezzogiorno Province capital cities.

According to the values of the environmental index, calculated on the basis of data which monitor the themes linked to air, noise, green areas, the maximum values are reached by cities of Northern regions but environment seems to be a major concern also in Southern cities, mainly located in Basilicata, Calabria, Sicilia and Sardegna. After constructing synthetic indices for 8 different dimensions of well-being, merging 44 variables, we employ these partial composite indicators as outputs in a unitary input DEA-model.

In the second step of the analysis, we proceed with the evaluation of the relative efficiency of the Italian Province capital cities in producing equitable and sustainable well-being, using the combined DEA Shannon's Entropy method.

To illustrate the utility of the proposed approach in our particular context, we list in Table 2 the efficiency scores of the original DEA model, namely prior of applying the Shannon-DEA procedure. Note in this Table, that we have 22 (21%) DEA efficient Italian Province capital cities. These results inform us about the weak discriminatory power of DEA in evaluating the relative efficiency of DMUs in producing equitable and sustainable well-being.

Tab.2 : Efficiency scores of basic DEA model

Province Capital cities	**Eff**	**Province Capital cities**	**Eff**
Torino	1	Bergamo	0.962
Vercelli	0.950	Brescia	0.986
Novara	0.961	Pavia	0.977
Cuneo	1	Cremona	0.975
Asti	0.992	Mantova	0.971
Alessandria	0.970	Lecco	0.994
Biella	1	Lodi	0.996
Verbano-Cusio-Ossola	1	Bolzano / Bozen	1
Valle d'Aosta / Vallée d'Aoste	1	Trento	1
Imperia	0.981	Verona	0.999
Savona	0.967	Vicenza	0.993
Genova	0.978	Belluno	1
La Spezia	0.955	Treviso	0.961
Varese	0.976	Venezia	0.981
Como	0.994	Padova	1
Sondrio	1	Rovigo	0.962
Milano	1	Udine	0.990

(continued)

Province Capital cities	Eff	Province Capital cities	Eff
Gorizia	0.978	Teramo	0.963
Trieste	1	Pescara	0.979
Pordenone	0.986	Chieti	0.975
Piacenza	0.994	Campobasso	0.985
Parma	0.990	Isernia	0.984
Reggio nell'Emilia	1	Caserta	0.962
Modena	0.963	Benevento	0.975
Bologna	1	Napoli	0.931
Ferrara	0.984	Avellino	0.980
Ravenna	0.997	Salerno	0.962
Forlì-Cesena	1	Foggia	0.915
Rimini	0.984	Bari	0.958
Massa-Carrara	0.955	Taranto	0.951
Lucca	0.996	Brindisi	0.966
Pistoia	0.992	Lecce	0.961
Firenze	1	Potenza	0.973
Livorno	0.965	Matera	0.999
Pisa	0.988	Cosenza	1
Arezzo	0.988	Catanzaro	0.975
Siena	1	Reggio di Calabria	0.936
Grosseto	0.970	Crotone	0.960
Prato	1	Vibo Valentia	0.942
Perugia	1	Trapani	0.949
Terni	0.980	Palermo	0.934
Pesaro e Urbino	0.999	Messina	0.991
Ancona	1	Agrigento	0.989
Macerata	1	Caltanissetta	0.928
Ascoli Piceno	0.996	Enna	0.962
Viterbo	0.967	Catania	0.942
Rieti	0.983	Ragusa	0.956
Roma	0.980	Siracusa	0.936
Latina	0.968	Sassari	0.942
Frosinone	0.980	Nuoro	0.971
L'Aquila	0.965	Cagliari	0.969
		Oristano	0.981

From the practical point of view, the combination of the results, obtained by all different formulations of the unitary input DEA model[1], may be a reasonable way to measure DMUs performance and increase the discriminatory power of DEA.

Tab.3 : Comprehensive Efficiency scores

Province Capital cities	Eff	Rank	Province Capital cities	Eff	Rank
Torino	0.974	16	Mantova	0.912	82
Vercelli	0.903	88	Bolzano / Bozen	0.980	10
Novara	0.941	56	Trento	0.990	2
Cuneo	0.978	13	Verona	0.965	31
Asti	0.956	39	Vicenza	0.973	18
Alessandria	0.937	62	Belluno	0.990	1
Aosta	0.986	4	Treviso	0.936	63
Imperia	0.938	59	Venezia	0.949	46
Savona	0.947	49	Padova	0.979	12
Genova	0.962	34	Rovigo	0.937	60
La Spezia	0.934	66	Udine	0.967	27
Varese	0.944	52	Gorizia	0.960	35
Como	0.960	36	Trieste	0.988	3
Sondrio	0.971	22	Piacenza	0.949	47
Milano	0.983	7	Parma	0.971	21
Bergamo	0.911	83	Reggio nell'Emilia	0.966	28
Brescia	0.916	78	Modena	0.942	55
Pavia	0.952	43	Bologna	0.973	17
Cremona	0.938	58	Ferrara	0.962	33

(continued)

[1] The number of all different combinations of unitary input and output subsets from S is $K = (2^s - 1)$.

Province Capital cities	Eff	Rank	Province Capital cities	Eff	Rank
Ravenna	0.973	19	Bari	0.914	81
Forlì-Cesena	0.972	20	Taranto	0.901	90
Pesaro e Urbino	0.969	24	Brindisi	0.910	84
Ancona	0.984	6	Lecce	0.914	80
Macerata	0.966	30	Potenza	0.920	77
Ascoli Piceno	0.964	32	Matera	0.932	67
Massa-Carrara	0.932	69	Cosenza	0.921	76
Lucca	0.970	23	Catanzaro	0.926	72
Pistoia	0.966	29	Reggio di Calabria	0.881	94
Firenze	0.985	5	Trapani	0.865	99
Livorno	0.945	51	Palermo	0.861	101
Pisa	0.967	26	Messina	0.934	65
Arezzo	0.953	42	Agrigento	0.889	92
Siena	0.982	8	Caltanissetta	0.855	102
Grosseto	0.943	54	Enna	0.888	93
Perugia	0.976	15	Catania	0.866	98
Terni	0.945	50	Ragusa	0.894	91
Viterbo	0.926	71	Siracusa	0.868	95
Rieti	0.948	48	Sassari	0.907	85
Roma	0.959	38	Nuoro	0.923	74
Latina	0.902	89	Cagliari	0.921	75
Frosinone	0.924	73	Pordenone	0.960	37
Caserta	0.866	97	Isernia	0.951	45
Benevento	0.915	79	Oristano	0.934	64
Napoli	0.818	103	Biella	0.982	9
Avellino	0.932	68	Lecco	0.969	25
Salerno	0.905	86	Lodi	0.944	53
L'Aquila	0.940	57	Rimini	0.954	41
Teramo	0.928	70	Prato	0.976	14
Pescara	0.955	40	Crotone	0.868	96
Chieti	0.951	44	Vibo Valentia	0.904	87
Campobasso	0.937	61	Verbano-Cusio-Ossola	0.980	11
Foggia	0.863	100			

Each of the proposed DEA model evaluates DMUs efficiencies considering a different combination of the output variables, giving a measure of how the results are influenced by the choice of output variables. A comprehensive efficiency score and a ranking with a more discriminatory capability can be obtained from the Shannon-DEA procedure.

The results of the comprehensive measurement, displayed in Table 3, indicate that the proposed approach can be effectively applied to measure urban well-being efficiency, significantly reducing the number of fully efficient units.

A visual representation of the geographical distribution of well-being conditions in the Italian Province capital cities, is provided in Figure 3.

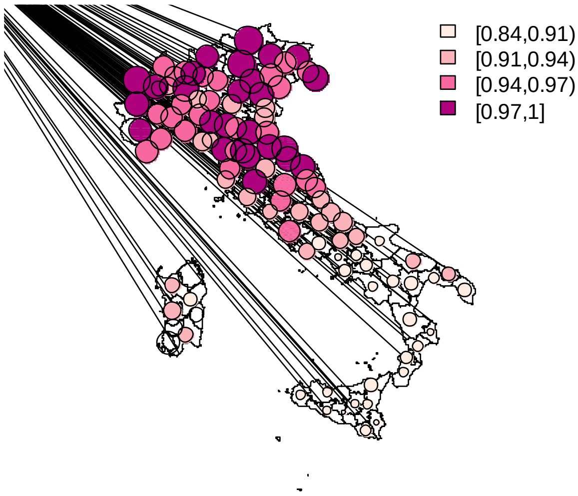


Figure 3: Comprehensive DEA scores of Well-being

According to the comprehensive efficiency index, mean well-being efficiency relative to cities is 93.8%, the maximum efficiency is 98.9% and the minimum is 81.7%. Fifty-six cities achieve a well-being efficiency score which is below the average. Among cities that are placed in the first 10 position of ranking, seven of them are located in the North of Italy (Belluno, Trento, Trieste, Aosta, Milano, Biella, Bolzano) and three in the Centre of Italy (Firenze, Ancona and Siena). Focusing on the latest 10 positions in the ranking, five cities are located in Sicily (Caltanisetta, Palermo, Trapani, Siracusa, Catania) and the remaining half in the South of Italy (Reggio Calabria, Crotone, Caserta, Foggia and Napoli).

The results of different models are compared with the ranking obtained through the model that calculates the well-being efficiency by utilizing the Shannon's entropy index. To this end, we compute as a measure of similarity between rankings the Spearman's rank correlation coefficient whereas the correlation of efficiency scores is assessed through the Pearson's index.

The Pearson and Spearman's rank correlation measurements always score less than one, ranging from -0.06 to 0.96 and from -0.05 to 0.95, respectively.

We also check the similarity of the Province capital city's relative position in the ranking of each composite indicator domain and the ranks arising from the comprehensive well-being index. The concordance and discordance that exists between rankings have been examined by using the Spearman's (ρ) and the Kendall-Tau (τ) rank correlation coefficients. As can seen from Table 4, which shows the coefficients calculated for each ranking, the results for both coefficients are similar, with the values obtained with the Spearman's rank correlation slightly higher. A major concordance is observed between the comprehensive index of well-being and the education, employment and social relationships domains.

The lowest degree of correlation (and even negative correlation) appears when measuring concordance in regard to environment and security. As a result, these indices record the most differences in regard to the rest.

These results are sensible because they stress in a certain way the importance of these basic components in the construction of an overall well-being index and are in line with the discussion that exists in the specialised literature in relation to which indicator best analyses the relationship with the social and economic progress of a region.

Tab. 4 Spearman's and the Kendall Tau rank correlation coefficients between each well-being domain and the comprehensive DEA scores

	τ	ρ
Health	0.33	0.45
Education and training	0.60	0.78
Work and life balance	0.78	0.93
Economic wellbeing	0.42	0.58
Social relationships	0.78	0.93
Security	-0.04	-0.06
Landscape and cultural heritage	0.48	0.65
Environment	0.09	0.13

Over recent years scholars have built a good deal of evidence supporting the existence of how variation in city size affects well-being and quality of life (see, among others, Berry and Okulicz-Kozaryn 2009; 2011). In this respect, Table 5 lists the scores relative to the well-being efficiency comprehensive index of Italian Province capital cities grouped by geographical area and population class. The grouping of urban areas considers tree population classes: "less than 80,000" inhabitants, "between 80,000 and 200,000" people and "more than 200,000" citizens, as reported in other studies (lo Storto 2016). In the first group of cities (less than 80,000 inhabitants) the mean efficiency well-being achieves higher scores in the North, Center and South of Italy and decreases in the cities located in the isles, mainly because of the lower efficiency scores of Caltanisetta and Trapani.

Tab. 5 Measurements of the Shannon's entropy index for cities grouped by geographical area and population size

Less than 80,000			**Between 80,000 and 200,000**			**More than 200,000**		
Cities	Eff.	Population	Cities	Eff.	Population	Cities	Eff.	Population
				North				
Imperia	0.959	42,230	Treviso	0.960	80,822	Venezia	0.961	261,555
Belluno	0.974	35,595	Alessandria	0.949	89,613	Trieste	0.977	202,346
Pavia	0.940	68,449	Ferrara	0.932	132,588	Padova	0.972	206,284
Mantova	0.932	46,593	Como	0.960	81,794	Verona	0.964	252,720
Pordenone	0.955	50,499	Ravenna	0.958	153,096	Torino	0.968	871,816
Lodi	0.926	43,285	Vicenza	0.967	111,755	Bologna	0.973	370,402
Cremona	0.940	69,839	Udine	0.967	98,246	Genova	0.953	586,162
Rovigo	0.934	50,040	Forlì	0.970	116,242	Milano	0.982	1,235,543
Vercelli	0.911	46,179	Novara	0.926	101,922			
Biella	0.967	43,855	Bergamo	0.944	115,294			
Lecco	0.965	46,628	Parma	0.963	175,536			
Asti	0.946	73,874	Reggio Emilia	0.967	162,093			
Savona	0.951	60,764	Piacenza	0.955	100,109			
Gorizia	0.949	35,186	La Spezia	0.932	92,604			
Varese	0.942	79,654	Modena	0.951	178,962			
Cuneo	0.964	54,857	Brescia	0.943	189,331			
Sondrio	0.963	21,684	Bolzano	0.992	102,214			
Verbania	0.951	30,327	Rimini	0.975	139,360			
Aosta	0.989	34,144	Trento	0.991	113,900			
mean	0.950	49,141		0.958	122,920		0.969	498,354
max	0.989	79,654		0.992	189,331		0.982	1,235,543
min	0.911	21,684		0.926	80,822		0.953	202,346
stdev	0.018	15,143		0.018	33,188		0.009	352,906

Less than 80,000			Between 80,000 and 200,000			More than 200,000		
Cities	Eff.	Population	Cities	Eff.	Population	Cities	Eff.	Population
				Center				
Macerata	0.951	42,013	Pistoia	0.949	89,154	Firenze	0.967	356,869
Frosinone	0.897	46,803	Pisa	0.946	85,901	Roma	0.948	2,611,397
Rieti	0.926	46,098	Arezzo	0.933	97,965			
Ascoli	0.947	50,081	Lucca	0.965	86,818			
Viterbo	0.906	62,947	Terni	0.947	109,295			
Massa	0.920	68,847	Pesaro	0.972	94,440			
Grosseto	0.929	78,475	Latina	0.918	117,746			
Siena	0.973	52,843	Ancona	0.971	100,696			
			Perugia	0.960	161,910			
			Livorno	0.925	156,891			
			Prato	0.971	185,153			
mean	0.931	56,013		0.951	116,906		0.957	1,484,133
max	0.973	78,475		0.972	185,153		0.967	2,611,397
min	0.897	42,013		0.918	85,901		0.948	356,869
stdev	0.023	11,946		0.018	33,171		0.009	1,127,264

Less than 80,000			Between 80,000 and 200,000			More than 200,000		
Cities	Eff.	Population	Cities	Eff.	Population	Cities	Eff.	Population
				Sud				
Benevento	0.879	61,573	Catanzaro	0.886	89,523	Taranto	0.882	200,255
Crotone	0.854	58,913	Lecce	0.896	89,492	Napoli	0.887	961,884
Chieti	0.938	51,513	Pescara	0.932	117,239	Bari	0.907	315,946
Isernia	0.915	21,957	Reggio Calabria	0.876	180,949			
Cosenza	0.895	69,502	Foggia	0.876	147,481			
Campobasso	0.913	48,798	Brindisi	0.903	88,698			
L'Aquila	0.926	67,196	Salerno	0.872	132,794			
Teramo	0.924	54,200						
Potenza	0.893	66,771						
Vibo Valentia	0.891	33,422						
Caserta	0.859	75,578						
Avellino	0.893	54,309						
Matera	0.899	59,750						
mean	0.898	55,652		0.891	120,882		0.892	492,695
max	0.938	75,578		0.932	180,949		0.907	961,884
min	0.854	21,957		0.872	88,698		0.882	200,255
stdev	0.024	14,158		0.020	62,679		0.011	335,112

Less than 80,000			Between 80,000 and 200,000			More than 200,000		
Cities	Eff.	Population	Cities	Eff.	Population	Cities	Eff.	Population
				Isles				
Enna	0.861	27,907	Siracusa	0.850	118,888	Catania	0.869	294,461
Agrigento	0.865	58,216	Cagliari	0.909	149,937	Palermo	0.875	658,078
Nuoro	0.901	36,682	Sassari	0.910	123,677	Messina	0.897	243,380
Caltanissetta	0.834	61,697						
Trapani	0.876	69,177						
Ragusa	0.883	69,832						
Oristano	0.925	31,166						
mean	0.878	50,668		0.890	130,834		0.880	398,640
max	0.925	69,832		0.910	149,937		0.897	658,078
min	0.834	27,907		0.850	118,888		0.869	243,380
stdev	0.027	16,829		0.028	13,649		0.011	171,071

According to results shown in Table 5, the mean comprehensive well-being efficiency generally increases with the size of cities. Some exceptions, in this regard, are found both for the largest cities of Southern Italy and urban contexts of Sicily and Sardegna which display a reduction in the average efficiency score of well-being. In the smaller cities ("less than 80,000 inhabitants") the mean efficiency is between 89.3% and 95.5% whereas, for the medium size cities ("between 80,000 and 200,000" inhabitants) we observe values ranging from 89.9% to 95.9%.

5. Concluding remarks

The last two decades have seen a plethora of initiatives developed to measure and track the performance of various health, economic and social aspects, such as competiveness, sustainability, well-being and quality of life, across nations and regions. These projects have been often accompanied by ranking and benchmarking studies that seek to compare intra and inter-regions performance. The process of benchmarking is essential to identify strengths and weakness for the considered areas in a comparative way, revealing which places are doing well and who has fallen behind leading places. Thus, benchmarking is a mode to set a competitive agenda and inform and guide policy formulations.

In this paper, we followed the recent inter-institutional initiative of CNEL and ISTAT who promoted the BES project for measuring the equitable and sustainable well-being in Italy, through the choice of indicators replacing GDP.

The conceptual model adopted within the BES project postulates that there is progress of a society when an improvement occurs in human well-being and the ecosystem condition.

In parallel with the national experience, ISTAT is supporting the implementation of BES framework at local level by means of the projects "BES of the Provinces" and "UrBES of metropolitan cities".

Throughout our study, a special emphasis has placed in assessing well-being at urban level. The assessment of well-being in the Italian Province capital cities is important to highlight inequalities and trends that may affect the future development model.

We investigated the relative efficiency of the Italian Province capital cities in promoting well-being by using data from the Ur-BES report (ISTAT 2015).

To this end we have gone through two different steps of analysis. Firstly, we summarized the elementary indicators included in each domain of BES framework through the Mazziotta-Pareto's method of penalties. Next, the efficiency of that combination of indicators has been facilitated by a unitary input DEA model, with entities defined only by outputs.

In our context, the classic DEA model proved to have a weak discriminatory power, identifying many DEA efficient Italian Province capital cities.
To overcome this limitation, this paper takes advantage of a robust method that implements DEA and Shannon's entropy index, to construct an aggregate measure of the Italian urban well-being in form of a comprehensive index.
The results reveal that the Shannon-DEA integrated approach enhanced the discrimination, considerably reducing the number of efficient units.
This study provided an enhanced picture of Italian Province capital cities well-being performance, disclosing economic and social disparities between the sample urban areas.
The outcome of city rankings evidences that the best standards of well-being are obtained by the Province capital cities of the Central-Northern part of Italy, whereas the lowest scores are obtained in the Southern cities and also in the urban contexts of the two major islands (Sicilia and Sardegna).
Thus, the Italian Province capital cities perform very differently and, generally, largest cities in term of population size achieve a higher well-being efficiency score.
Additional insights on the strengths and weakness of each sample areas is provided by the graphical examination of composite indicators for each domain.
To sum up, our analysis reproduces the chronic divide from North and South of Italy, with marked and persistent geographical differences for many economic and social indicators.
Understanding which areas exhibit the lowest levels along the different quality of life dimensions and in the comprehensive well-being index can contribute to the development of appropriate policy responses as well to optimize the allocation of territorial resources.
In fact, policy makers may be interested in analysing drivers of local well-being in order to focus their interventions on specific territorial areas.
Also, the assessment of well-being at urban level allows citizens to be informed on the results of the governmental action and the same time to participate with more awareness in local decisional process.
It is worth noting that UrBES is a work in progress, whose set of indicators will continue to improve thanks to already efficient collaboration between ISTAT and municipalities.
Finally, as a consequence of the data availability, our study has to be intended as a static picture of the economic, social and environmental conditions of Italian urban contexts. Accordingly, our future research efforts will be devoted to perform benchmarking analyses over time.

Appendix

Adjusted Mazziotta-Pareto Index (AMPI)

Mazziotta-Pareto Index (MPI) is a non-linear composite index method which transforms a set of individual indicators in standardized variables and summarizes them using an arithmetic mean, adjusted by a "penalty" coefficient related to the variability of each unit (Mazziotta and Pareto 2007; Mazziotta et al. 2010).
Two steps are involved in the construction of the MPI which require the normalization of individual indicators by "standardization" and the aggregation of the standardized indicators by arithmetic mean with penalty function based on "horizontal variability" (variability of standardized values for each unit). The penalty is based on the coefficient of variation and it can be added or subtracted, depending on the nature of phenomenon to be measured and hence on the direction of the individual indicators (De Muro et al. 2011).
In what follows, we describe how proceeds the construction of a variant of MPI, known as Adjusted Mazziotta-Pareto Index (AMPI) (Mazziotta and Pareto 2016).
For the AMPI it has been adopted a different procedure of data normalization to guarantee absolute comparisons over time. That data transformation requires a re-scaling of the elementary indicators

respect two *goalposts*, that is respect to a minimum and maximum, which represent the range of each indicator over the given time period.
Let $X = \{x_{ij}\}$ be the matrix with n rows (geographical units) and m columns (indicators), the normalized matrix $R = \{r_{ij}\}$ is defined through a min-max transformation.
According to the original direction of the indicator is used min-max formula (1) or (2)

$$r_{ij} = \frac{x_{ij} - Min_{xj}}{Max_{xj} - Min_{xj}} 60+70 \quad (1)$$

$$r_{ij} = \frac{Max_{xj} - x_{ij}}{Max_{xj} - Min_{xj}} 60+70 \quad (2)$$

where x_{ij} is the value of indicator j for the geographical unit i whereas Min_{xj} and Max_{xj} are the *goalposts*.
In our study, we deal with a min-max transformation in a continuous scale from 70 (minimum) to 130 (maximum).
To facilitate the interpretation of results, the "goalposts" can be chosen so that 100 represents a reference value (e.g., the average in a given year).
Let Ref_{x_j} be the reference value for the indicator i, then the goalposts are defined as $Ref_{x_j} \pm \Delta$ where $\Delta = \frac{(\sup x_j - inf\, x_j)}{2}$ and $sup\, x_j$ and $inf\ x_j$ are the minimum and maximum of indicator j across all units and all time periods considered.
The above formulas take into account the polarity of indicator, that is the sign of the relationship between the indicator and the phenomenon under study (+ if the indicator represents a positive dimension and - if the indicator represents a negative dimension).
In our case, this data transformation assures a direct reading of values in terms of well-being: higher values reflect better performance.
Let M_{ri} and S_{ri} be the media and standard deviation, respectively, of the normalized values for the *i*-th unit. The composite index is defined as:

$$AMPI_i^{+/-} = M_{ri} \pm S_{ri} cv_{ri} \quad (3)$$

where $cv_{ri} = S_{ri}/M_{ri}$ is the coefficient of variation for the *i*-th unit and the sign $\pm$ depends on the kind of phenomenon to be measured.
This approach is characterized by the employment of a product ($S_{ri} cv_{ri}$) which penalizes the units showing unbalanced values of the indicators. Thus, the AMPI can be viewed as a combination of a "average effect" (M_{ri}) and a "penalty effect" ($S_{ri} cv_{ri}$) and indicate how each indicator is located compared to the *goalposts*.

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