Tradable Quotas Taxation and Market Power *

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Abstract

We investigate how *corrective* taxation can improve the efficiency properties of tradable quotas systems affected by market power. Indeed, we show that, when there is a dominant firm in the tradable quota market, the regulator can set an *ad hoc* taxation on firms' traded quotas that restores cost effectiveness without driving the dominant firm's net demand to zero. Achieving cost effectiveness with market power and quotas taxation implies some costs in terms of tax revenue that, however, can be justified by the corresponding reduction of compliance costs. Moreover, we see that there may be cases where all firms result to be better off after the implementation of corrective taxation.

JEL numbers: Q58, H23. **Keywords**: tradable quotas markets, market power, tradable quotas taxation.

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1 Introduction

In recent years environmental policies have been characterized by a remarkable increase in the adoption of market instruments, with price signals to regulated agents arising from emissions quantity restrictions coupled with trading schemes (Hepburn, 2006). Several types of market instruments have been put in place. Markets for tradable pollution permits, for instance, have been established to control SO2 emissions and other air pollutants in the US, as well as to cut CO2 emissions in the EU; further, the development of an international permits market for CO2 emissions has been one of the cornerstones of the flexibility mechanisms under the Kyoto Protocol. Markets for tradable certificates have been introduced also to stimulate investments in energy efficiency and in electricity generation from renewable energy sources. The functioning of these tradable quotas (TQ) systems has been investigated extensively by the literature starting from the seminal article by Montgomery (1972) as, in some cases, they have the potential to attain environmental policy targets cost-effectively, i.e. at the minimum aggregate cost. The property of cost effectiveness, however, relies upon the somehow controversial hypothesis that TQ are traded in perfectly competitive markets. When the assumption of perfect competition is relaxed, some players can exploit their market power by decreasing TQ supply/demand, leading to larger total abatement costs (Hahn, 1984; Westskog, 1996). Even though the presence of market power is empirically debated in the practice of emissions trading (Tietenberg, 2006) and its relevance should be probably assessed case by case (Sturn, 2008), the ability to manipulate emissions' price has been recognized to be a potential problem in the case of a hypothetical Kyoto-like international emissions trading system (Alvarez and Andrè, 2015), as well as a source of concern in local or nationwide carbon markets. This is testified, for instance, by the different ways in which pilot carbon trading schemes introduced in China are trying to prevent, or at least reduce, market power (Zhang, 2015), or by the attention devoted to the emergence of strategic behaviors in other TQ systems, such as the Scandinavian market for renewable energy certificates

(Amundsen and Bergman, 2012).

The analysis of the effects of market power on the economic performance of TQ systems is also a thought provoking research question, as it is shown by the large theoretical literature that has followed the seminal article by Hahn (1984) (see, for instance, Disegni Eshel, 2005; Hagem and Westkog, 2009; Montero, 2009; Godal and Meland, 2010; Hintermann, 2011; Liski and Montero, 2011; and Haita, 2014). In some cases, authors provide policy suggestions to address inefficiencies that might arise when TQ markets are not perfectly competitive. Hahn (1984), for instance, suggests that a possible way to eliminate market power in TQ systems is through an ad hoc, cost-effective initial allocation of TQ. However, there are situations where the regulator cannot control the initial allocation of TQ to each emission source, or it could be unaware of the presence of market power when the initial distribution of quotas is realized, while being able to observe and regulate it only ex-post (Hagem and Westskog, 2009).

In this paper we put forward an alternative proposal to the efficient allocation of TQ discussed by Hanh (1984). Namely, we investigate the possibility of restoring cost effectiveness through an *ad hoc* differentiation of prices faced by each firm in the TQ market. We show that the task of differentiating prices can be assigned to a system of taxes and rebates that would allow the regulator to tackle market power and achieve cost effectiveness. Indeed, we derive the conditions required by an optimal tax/rebate rule to restore cost effectiveness in TQ markets where one dominant firm has the possibility to affect the equilibrium price. For the sake of simplicity, we focus on a theoretical model dealing with emissions trading, but results could be easily replicated in an alternative model where TQ are either energy saving certificates or renewable energy certificates[1].

Specifically, we consider I firms emitting pollution. There are two types of firms, namely, a dominant firm and $I-1$ firms belonging to a competitive fringe.[2]

[1] Supplementary material showing how the model presented in this paper can be interpreted in terms of energy saving certificates and renewable energy certificates, is available at http://ediliovalentini.jimdo.com/research.

[2] Focusing on a setting *à la* Hahn (1984), with a single dominant firm, does not affect the generality of our conclusions (we thank an anonymous reviewer for drowing our attention on

Each firm optimally chooses its level of emissions, given its initial endowment of quotas; we assume that a system of firm-specific taxes (rebates) can be applied to revenues (costs) arising from quotas selling (buying) behavior. Each firm decides whether to be a net seller or buyer of quotas by comparing its cost of increasing/reducing emissions and the price of quotas, which is exogenous when the firm is price taker and endogenous when the firm is a dominant firm.

We find that an optimal corrective taxation implies a tax (rebate) rate on the net selling (buying) dominant firm lower (higher) than the tax and rebate rate which is applied to the other firms. Such difference between the rates applied to the dominant firm and to other firms brings about that restoring cost effectiveness comes at a cost in terms of additional public expenditure. As this expenditure is a net transfer from taxpayers to the TQ market, it can be justified as long as the benefit of restoring cost effectiveness is larger than the deadweight loss of the required tax revenue. Moreover, we identify a case where all firms are better off when an ad hoc corrective tax rule is implemented.

This is not the first paper dealing with TQ taxation. Fischer (2006), for instance, investigates the interaction between multinational taxation and abatement in an international emissions trading scenario where the equilibrium permits price is exogenous, while Yale (2008) examines under what circumstances income taxation interferes with cap-and-trade environmental regulation. Both Fischer (2006) and Yale (2008) deal with a comprehensive corporate income taxation which taxes both profits (net of abatement costs) and permits' revenues/costs by the same tax rate. Costantini et al. (2013), instead, isolate the specific impact of permits taxation in an international emissions trading market where no other taxes are taken into account. In this way they elicit the impact of permits taxation within an emissions trading scheme that would perform in a cost effective way without this type of taxation[3]. None of these papers, how-

this point). Indeed, our main results can be easily replicated in an alternative framework where more than one firm feature market power. Such framework is also discussed in the supplementary material available at http://ediliovalentini.jimdo.com/research.

[3] Another paper in this stream of literature is Kane (2009) who provides a descriptive analysis of the different fiscal treatments affecting the permits trading markets.

ever, considers market power or the possibility that TQ taxation can be used as corrective regulatory tool.

Our paper is also related to the literature on environmental policy design under market power (see, among others, Gersbach and Requate, 2004 and Fischer, 2011) and to those papers suggesting ways to restore cost effectiveness in an TQ system featuring market power. In particular, Hagem and Westskog (2009) suggest a mechanism restoring cost effectiveness by making allocation in one period dependent on the market price of permits observed in previous period(s). However, such mechanism does not work when the TQ system prevents the regulator from manipulating the allocation of quotas, a circumstance that would not hamper the taxing system proposed in this paper.

The rest of the paper is organized as follows: Section 2 presents the theoretical model; Section 3 illustrates the properties of the proposed corrective taxation scheme restoring cost effectiveness; Section 4 addresses its distributional consequences, and, finally, Section 5 concludes.

2 The model

We adopt a framework *à la* Hahn (1984), and assume a market featuring I firms. Each firm $i \in I$ minimizes net emissions' cost $c_i(x_i) + p(x_i - e_i)$, where x_i and e_i are, respectively, the amount of pollution emitted by firm i and the initial endowment of quotas which is exogenously allocated to firm i, $c_i(x_i)$ is the (gross) cost of pollution (with $c_i' < 0$ and $c_i'' \geq 0$), $p(x_i - e_i)$ is the cost (revenue) of buying (selling) quotas and p is the equilibrium price.

As in Hahn (1984), all firms are price takers except one, the dominant firm, labeled as d. The remaining $I - 1$ firms belong to a competitive fringe F, and they are labeled as f. In a standard two stage game, the dominant firm sets emission quantities *(first stage)* before the price takers firms clear the market *(second stage)*.

Specifically, given the quotas price which arises from the after-trade market clearing condition $\sum_{i=1}^{I} x_i = \sum_{i=1}^{I} e_i = E$, in the second stage each firm

$f \in F$ chooses the level of emissions minimizing the net emission cost. E labels the (exogenous) aggregate emissions cap. The first order condition of this minimization problem, in the absence of taxation, is as follows[4]:

$$c'_f(\hat{x}_f) + p = 0, \tag{1}$$

where $\hat{x}_f$ is the equilibrium emissions level of firm f. Note that, if all firms are price takers, then condition (1) for all firms implies that the exogenous environmental target is achieved at minimum costs.

In the first stage, when the dominant firm decides its optimal levels of emissions, it anticipates how the fringe, and consequently the equilibrium price of quotas, will react to its choice; the first order condition of the dominant firm's minimization problems is:

$$c'_d(\hat{x}_d) + p + \frac{\partial p}{\partial x_d}(\hat{x}_d - e_d) = 0 \tag{2}$$

where $\hat{x}_d$ is the equilibrium emissions level of firm d, and $\frac{\partial p}{\partial x_d}(\hat{x}_d - e_d)$, (i.e. the marginal effect on quotas price of polluting decisions of firm d times the size of firm d in the market) captures the degree of market power enjoyed by the firm d. Note that a cost effective solution can be achieved if the dominant firm receives an amount of quotas equal to the emissions' level that it would choose in equilibrium (Hahn, 1984), i.e. if $\hat{x}_d = e_d$.

3 Cost effectiveness and quotas taxation

Differently from Hahn (1984) we define a corrective taxation that allows for cost effectiveness also when the dominant firm trades in the quotas market.

Specifically, assume that a tax rate $t_i \in [0, 1)$ is applied on quotas revenues/costs. Therefore, for any firm $i \in I$, the minimization problem becomes

$$\min_{x_i} c_i(x_i) + p(1 - t_i)(x_i - e_i). \tag{3}$$

According to (3), $|pt_i(x_i - e_i)|$ is the absolute value of the tax payed by firms when they are net sellers (i.e. when $x_i < e_i$) while, in the case of net buyers

[4] We assume that the relevant second order conditions are always satisfied.

(i.e. when $x_i > e_i$), it represents either (the absolute value of) a subsidy to the firm or a tax credit (a rebate) on its profit tax. Under this specification the first order conditions for the fringe become:

$$c'_f(x^*_f) + p(1 - t_f) = 0, \tag{4}$$

where x^*_f is the equilibrium emissions level of firm f when it also accounts for quotas taxation. Condition (4) implicitly defines the demand function by fringe firms; also, by totally differentiating (4) we can conclude that:

$$\frac{dx_f}{dp} = -\frac{(1 - t_f)}{c''_f(x^*_f(p, t_f))} < 0$$

for $f \in F$. Additionally, equilibrium on the TQ market implies $x_d + \sum_{f \in F} x_f = E$. Differentiating, we can see how the equilibrium price is affected by the dominant firm's emissions choice:

$$\frac{\partial p}{\partial x_d} = -\frac{1}{\sum_{f \in F} \frac{dx_f}{dp}} = \sum_{f \in F} \frac{c''_f(x^*_f(p, t_f))}{(1 - t_f)} > 0.$$

Moving to the first order condition of the dominant firm, we get:

$$c'_d(x^*_d) + p(1 - t_d) + \frac{\partial p}{\partial x_d}(1 - t_d)(x^*_d - e_d) = 0. \tag{5}$$

where x^*_d is the equilibrium emissions level of firm d when it also accounts for quotas taxation. The following proposition defines a set of conditions for cost effectiveness under market power and quotas taxation.

Proposition 1 *Tradable quotas taxation can restore cost effectiveness without driving the net demand of the dominant firm to 0, if:*

(a) all firms f belonging to the competitive fringe are subject to the same tax/rebate rate;

(b) the dominant firm d is subject to a tax/rebate rate which is

(b.1) lower than the rate applied to competitive fringe firms, when the dominant firm is a net seller,

(b.2) greater than the rate applied to competitive fringe firms, when the dominant firm is a net buyer.

Proof.

The cost effectiveness condition requires that $-c_i'(x_i^*) = -c_j'(x_j^*)$ for any $i, j \in I$. Such condition, together with (4) and (5), implies that:

1. $t_i = t_j$ for any $i, j \in F$, with $i \neq j$,

2. $p(t_d - t_f) = \frac{\partial p}{\partial x_d}(1 - t_d)(x_d^* - e_d)$ for any $f \in F$.

Specifically, point 1 comes from equalizing the marginal abatement costs in (4) for any pair of firms belonging to the competitive fringe F and it is a formal restatement of enunciate (a). Point 2 comes from equalizing the marginal abatement costs between the dominant firm and any firm belonging to the competitive fringe as they are defined in in (5) and (4), respectively. Point 2 implies that $t_d \gtreqless t_f$ as long as $x_d^* \gtreqless e_d$ as it is enunciated in (b). To conclude the proof and show that cost effectiveness does not require that the dominant firm's net demand equals 0, notice that for any $(x_d^* - e_d) \neq 0$ there always exist some price p and a couple of rates t_d and t_f such that the condition reported in point 2 is respected. ■

The above analysis suggests that in a TQ system with market power cost effectiveness can be restored without the need of reallocating emission targets across firms. As a matter of fact, the tax rate structure suggested in Proposition 1 affects the trading incentive and brings about a final allocation of quotas counteracting the impact of market power. Such final allocation, however, is not directly commanded by the regulator, as in Hahn (1984), since it is obtained by means of price-type economic incentives. The condition defined in point 2 of the proof characterizes the optimal level of t_d for any possible level of t_f. Therefore, when t_f represents the profit tax rate which is already in place and applied to the revenues/losses that fringe firms obtain/suffer from selling/buying quotas, we can identify the level of t_d that would optimally distort the choice of the dominant firm and restore the cost-effective outcome[5].

4 Tax revenue and distributional implications

The equivalence between quantity and price instruments requires a number of demanding assumptions (Hepburn, 2006). Even in the case of perfect information, an important difference between quantity and price instruments is due to the fact that the former does not imply any variation in the public budget (apart from the implementation costs), while the latter may imply an increase (in the case of taxes) or a decrease (in the case of subsidies) in the public budget. The tax structure analyzed in this paper implies a public revenue from quotas sellers that needs to be compared with the public expenditure aimed at refunding quotas buyers. The following Corollary, which comes from condition (b) of Proposition 1, tells us that there is a cost in terms of public budget when we use quotas taxation to achieve cost effectiveness under market power.

Corollary 1 *In a tradable quotas market affected by market power, taxation restoring cost effectiveness implies that the revenue raised from net sellers is smaller than the revenue lost from net buyers.*

The revenue generated from the emissions trading taxation is $R = \sum_i^I p_t t_i (e_i - x_i)$, where p_t denotes the equilibrium price under market power and corrective taxation. Given the market clearing condition $\sum_i^I (e_i - x_i) = 0$, when the quotas market is perfectly competitive, Proposition 1 requires that $t_i = t_j$ for any $i, j \in I$ implying $R = 0$, i.e. that the total tax payed by net sellers is exactly equal to the total rebate refunded to net buyers. Differently, as we introduce some degree of market power, Proposition 1 tells us that we need to differentiate

[5] Notice that it is also possible to satisfy the conditions from Proposition 1 by setting $t_f = 0$ and a positive (negative) t_d if the dominant firm is a net buyer (seller). This would, however, imply that the dominant firm receives a rebate or an explicit subsidy under any possible market outcome. Also, this would imply relaxing our assumption that $t_i \in [0, 1)$ for all i. We thank an anonymous referee for driving our attention on this point.

the tax rates between the dominant firm, d, and firms f in the fringe F. Specifically, Proposition 1 implies $t_d > t_f$, when the dominant firm is a net buyer, and $t_d < t_f$, when the dominant firm is a net seller. Consequently, the presence of a dominant firm brings about a loss in public revenue with respect to the case without market power, i.e. $R < 0$. Indeed, we can have two cases:

- the dominant firm is a net buyer (so that the net positions of the fringe firms sum up to a net selling position): in this case, $t_d > t_f$, and the rebate accruing to the dominant firm is larger than the revenue raised by taxing quotas sold by fringe firms.
- the dominant firm is a net seller (so that the net positions of the fringe firms sum up to a net buying position): in this case, $t_d < t_f$ and the rebate accruing to fringe firms is larger than the revenue raised by taxing quotas sold by the dominant firm.

Finally, the variations in the relative quotas price faced by the fringe and by the dominant firm affect the net emissions costs, bringing about a redistribution of the total abatement costs across firms. This point is addressed by the following proposition.

Proposition 2 *There exists (at least) a specification of the model leading to Proposition 1 such that the derived corrective taxation scheme implies a redistribution of net emissions cost which is beneficial for all firms.*

Proof. See the Appendix ■

Propositions 2 tells us that there may be cases in which restoring cost effectiveness through tradable quotas taxation generates net gains for all involved firms, being them dominant or not. The importance of these cases relies on the fact that, when they occur, we can expect no hold up from firms regulated according to Proposition 1. On the contrary, restoring cost effectiveness by an ex-post re-allocation of permits would necessarily imply that some firms are worse off with respect to the initial allocation. Of course, as it is already clarified in Corollary 1, this comes at a cost in terms of aggregate public revenue.

5 Discussion and Concluding Remarks

This paper provides a new perspective on the use of corrective taxation to deal with market power inefficiencies in TQ markets. In a very simple setting, we have introduced the possibility of affecting the prices faced by each firm in the TQ market through a corrective taxation which taxes the revenues generated by selling TQ and subsidizes the costs of buying TQ. Moreover, we have derived conditions under which tax rates guarantee that the impact of market power on total compliance costs is neutralized, by showing their effects on tax revenues. The implementation of the corrective taxation implies a loss of public revenue, which should be compensated through other types of taxation. Nevertheless, the revenue loss may be justified by the gains related to have cost effectiveness restored, at least under non (or little) distortionary public revenues collection. Furthermore, we show that restoring cost effectiveness does not necessarily require the need to drive the net demand of quotas of the dominant firm to zero, and it may be beneficial for all regulated firms.

A first question arising from our analysis concerns the effective relevance of our results for policy. The answer rests on the consideration that, in real life, there are relevant cases where reallocating quotas across sources is severely limited or impossible. In these cases, the possibility for regulators to affect the initial distribution in order to neutralize market power is very limited. Our mechanism may be seen as a viable option when an efficient distribution of quotas across regulated firms is not possible or it is too costly to be performed.

A second natural issue raised by our analysis is the informative requirement of our corrective taxation scheme. Indeed, the implementation of cost effective tax rates would imply a perfect knowledge of the cost structure of regulated firms. This limit is also common to other papers addressing mechanisms to restore cost effectiveness in TQ systems under market power (Hahn, 1984, Hagem and Westskog, 2009). Our theoretical conclusions, however, are useful in directing regulatory authorities in the presence of market power. Proposition 1 indeed suggests that the observation of the reactivity of the price to changes in emissions, together with the equilibrium price and net position on the market (which are easily available) could be used to infer the direction of the needed tax rate correction, or (seeing it the other way round) could be informative on the potential role of existing tax rates differentials in reducing the impact of market power on cost effectiveness. For instance, observing a relatively small demand elasticity and a net selling position by the dominant firm, the regulator could infer the need to reduce its tax rate.

The very simple structure of our setting allows us to derive neat results, and leaves room for additional research. A first direction of research is related to the explicit inclusion of output market considerations, which are at the moment left out of the analysis; this would imply an extension of our model along the lines of Sartzetakis (1997), Disegni Eshel (2005) and Hintermann (2011), among others. Another promising extension is linked to the second point made above, and concerns the design of realistic implementation tools to apply our corrective taxation approach to existing tradable quotas markets, for example through a trial and error process that could lead to a reduction of total compliance costs. Finally, a comprehensive welfare analysis could shed further light on the public finance implications of introducing the corrective tax rule derived in this paper.

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Appendix

For the aim of this proof we focus on a simplified case, featuring the dominant firm, d and one representative price taker firm, f. Further, we assume that quotas endowments are $e_f = 1 - \alpha$, and $e_d = \alpha$, and the cost functions are

$$c_i(x_i) = \frac{1}{2}\left(1 - x_i\right)^2$$

for any $i = f, d$. In order to have a binding cap on emissions we assume that $0 < \alpha < 1$ (emissions are lower than Business As Usual - BAU - for the dominant firm), so that also $0 < 1 - \alpha < 1$ (the same condition holds for the fringe). As a result, total quotas endowment is lower than overall BAU emissions. From (4) we get

$$x_f = 1 - p\,(1 - t_f)\,;$$

given the equilibrium on the market, i.e. $x_f + x_d = 1$, the equilibrium price as a function of the dominant firm's emissions is:

$$p_t = \frac{x_d}{1 - t_f}. \tag{6}$$

By substituting (6) back into the dominant firm's first order condition (5) we can derive the dominant firm's emissions level

$$x_d^* = \frac{\alpha\,(1 - t_d) + 1 - t_f}{3 - t_f - 2t_d}. \tag{7}$$

As a result, the equilibrium price and the corresponding fringe emissions are:

$$p^* = \frac{(1 - t_d)\,\alpha + (1 - t_f)}{(1 - t_f)\,(3 - t_f - 2t_d)}$$

and

$$x_f^* = \frac{(1 - t_d)\,(1 - \alpha) + (1 - t_d)}{3 - t_f - 2t_d}.$$

According to the conditions required by Proposition 1, the dominant firm's tax rate must be set in such a way to satisfy

$$p^*(t_d - t_f) = \frac{1 - t_d}{1 - t_f}(x_d^* - e_d)$$

that, under the current specification, implies

$$t_d^* = \frac{1 - 2\alpha + t_f}{2\,(1 - \alpha)} < 1. \tag{8}$$

Notice also that $t_d^* > 0$ always holds when $\alpha < \frac{1}{2}$, as $t_f > 0$; on the other hand, we have to assume $t_f > 2\alpha - 1$ when the dominant firm is a net quotas seller under perfect competition (i.e. if $\alpha > \frac{1}{2}$).

If the tax rate is set according to (8) then:

$$p_t^* = \frac{1}{2\left(1 - t_f\right)}$$

while the emissions of the dominant firm and the fringe are, respectively:

$$x_{d\ t}^* = \frac{1}{2}$$

and

$$x_{f\ t}^* = \frac{1}{2}$$

that are both equal to the emissions that would be chosen under perfect competition (without taxation).

Under market power without corrective taxation we simply set $t_d = 0$ and $t_f = 0$, so that p^* becomes

$$p_m^* = \frac{1}{3}\left(\alpha + 1\right),$$

and the optimal level of emissions chosen by the fringe and the dominant firm are, respectively,

$$x_{f\ m}^* = \frac{1}{3}\left(2 - \alpha\right)$$

and

$$x_{d\ m}^* = \frac{1}{3}\left(\alpha + 1\right).$$

Comparing each firm's costs under market power and corrective taxation with those arising under market power without corrective taxation, we get

$$c_f(x_{f\ t}^*) + p_t^*(1 - t_f)(x_{f\ t}^* - e_f) - \left(c_f(x_{f\ m}^*) + p_m^*(x_{f\ m}^* - e_f)\right) = -\frac{5}{72}\left(2\alpha - 1\right)^2 < 0,$$

and

$$c_d(x_{d\ t}^*) + p_t^*\left(1 - t_d^*\right)\left(x_{d\ t}^* - e_d\right) - \left(c_d(x_{d\ m}^*) + p_m^*(x_{d\ m}^* - e_d)\right) = \frac{1}{24}\left(\alpha + 2\right)\frac{\left(2\alpha - 1\right)^2}{\alpha - 1} < 0$$

implying that both the fringe and the dominant firm are better off when corrective taxation is adopted.