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Policy interest rate, loan portfolio management and bank liquidity

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Abstract

This paper analyzes how the movements of the policy interest rate affect a bank relevant variables through the changes in the composition of the loan portfolio. Using a computations approach that fully accounts for borrowers heterogeneity, we show how the variety of the bank costumers changes and how this affects the bank cash influx making it more volatile. The paper also sheds light on how the composition of loan portfolio is affected by an increase of the policy interest after it was kept at low levels: safer borrowers exit the loan portfolio first causing a gradual increase of the loan portfolio risk; the interest payment influx shrinks because more risky borrowers pay back less often. We furthermore find out that a shortening of the lending time horizon increases the volatility clustering of the bank interest payment influx.

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1. Introduction

In a number of recent economic events monetary policy actions had milder or unexpected effects with respect to those foreseen by the models used to plan them. This is probably because economic theory has not yet reached a complete understanding of the banking system role in the monetary transmission mechanism. Indeed, the impulses given by the Central Bank have to "pass through" the banking system that, according to the strand of economic literature labeled as "credit view" (see Trautwein, 2000, for a survey), significantly affect the result of policy actions. Among recent works which stress the need for a better understanding of banking sector role in the monetary transmission mechanisms, the book by Stiglitz and Greenwald (2003) is a primary example. The authors argue that: "It is precisely when monetary policy becomes of crucial importance that the traditional models fail most dramatically. Later, we will argue that the failure to understand key aspects of financial institutions and their changes lies behind some of the recent failure in macro-economic policies, including the 1991 US recession and the severe recessions and depressions in East Asia that began in 1997." (Stiglitz and Greenwald, 2003, p.4).

The link between banks behavior and monetary policy is widely analyzed in the literature. Some works tackle the topic looking at aggregate variables (see Bernanke and Gertler, 1995, for example), others focus on the relationship between monetary policy and the banks balance sheet (Kashyap and Stein, 1994; Bacchetta and Ballabriga, 2000). The effects of the introduction of capital adequacy standards on banks lending (Gambacorta and Mistrulli, 2003; McAleer et al., 2013; da Silva and Divino, 2013) and more generally on banks behavior (VanHoose, 2007; Jacques, 2008) and finally on the relationship between capital adequacy and macroeconomic fluctuations (Blum and Hellwig, 1995) have drawn particular attention in this strand of literature.

The banking sector has been one of the most important protagonist even in the recent financial turmoil[1] which motivated new studies on banks behavior following up policy actions. Some of them dwell on the relevance of banks assets quality and start talking about the "risk-taking" channel of the monetary policy (see for example Borio and Zhu, 2008; Adrian and Shin, 2009). Gambacorta (2009) surveys the theory and provides empirical evidence for this channel of monetary policy.

The theoretical analysis of the bank balance sheet decision, especially that concerning assets, is a demanding task because of the high heterogeneity in the opportunities of funds allocation. In fact, each possible borrower is one of such opportunities and thus, the bank lending activity provides the major source of heterogeneity. The mathematical approach normally used in economic modeling handles a limited number of variables so that models for banks decision making generally determine the total amount lent appointing some other variable or parameter to account for borrowers heterogeneity. An alternative modeling strategy that can fully account for heterogeneity

[1]Laeven and Valencia (2008)'s database assesses the relevance of the systemic banking crisis phenomenon. They identify 124 systemic banking crises in the period 1970-2007 in the world.

adopts the bottom up approach which is implemented by using computational techniques (Tesfatsion, 2002).

The present paper aims to improve the existing literature by using the bottom up approach to analyze the bank portfolio decision. In particular, we focus on the lending portfolio by building a model which keeps track of individual borrowers' features over time. The approach is particularly useful to understand the dynamic implication of a bank choice concerning the loan portfolio and allows for a *detailed knowledge* of the state of bank lending portfolio that any policy maker needs. In particular, we study the relationship between the bank loan portfolio behavior and the policy interest rate level by analyzing: *i*) how banks identify the set of projects to be financed and *ii*) how the set of financed projects changes with the policy interest rate. The goal is to evaluate the dynamic effects of this behavior on a number of variables such as the bank liquidity and risk position.

We share with the recent literature, especially with the one on the monetary policy "risk-taking" channel cited above, the focus on the effects of low interest rates (Dell'Ariccia et al., 2014). However, the approach proposed by this papers offers some new insights.

First, we focus on the bank loan portfolio composition rather than on the composition of the assets of the bank balance sheet. As highlighted above, the latter approach "averages" out borrowers heterogeneity and tackle the issue of how much a bank should lend in total and how much should be allocated to other types of assets such as government bond or market shares.

Second, borrowers heterogeneity is accounted for by considering each project financed by the bank as a random process. The model has a mean-variance representation and we do the groundwork to frame the bank lending portfolio choice using Markowitz's portfolio theory.

Third, we argue that a bank dealing with low interest rates (Lombardi and Sgherri, 2007, for example, dwell on the interest rates level in recent years) operates in a "prickly" situation where the constrains which ensure its survival become binding and they "overshadow" the objective pursued by the bank managers in "normal times". In our framework the bank has to satisfy two constraints. Firstly, it has a liquidity constraint in the very short run and secondly, being mainly private enterprises, it has to match a profitability constraint. Our aim is to investigate how the bank behave to meet the profitability constraint when economic conditions change and how this affects the liquidity constraint.

Fourth, the computational approach we use allows to go beyond the one-period relationship between the bank and the borrower and let us gain a more *detailed knowledge* of the movements in the loan portfolio: we track the dynamics of the whole distribution that follows to a change in the environment faced by the bank.

The paper is organized as follows. In Section 2, we present our model of the banking activity and define the profitability and liquidity constraints. The important issue of how the lending activity is conducted to meet the profitability constraint is treated in section 2.2. In Section 3, we use computational tools to simulate a bank that behaves according to the rules obtained in the previous section. Among the results reported in Section 4, the evolution of the lending portfolio and the resulting impact on the bank liquidity play a key role. The analysis in sections 2-4 assumes the bank can solve asymmetric information by bearing screening and monitoring costs. This assumption is relaxed in section 5 which extends the model of the previous sections by considering a situation in which asymmetric information remains between the bank and borrowers despite screening an monitoring activities. In section 6 a number of issues which arise in our framework is discussed. We also discuss future extensions of the present analysis. Section 7 concludes.

2. Theory

We investigate a setting in which the bank is committed to a unique activity, i.e. lending to entrepreneurs that are willing to take over a production investment. It is important to highlight that we do not model the effects of competition among banks. It is the degree of substitution between credit and other source of financing that matter in the present work. Therefore, the market structure of the banking sector is not considered in our analysis and the reader can alternatively think of the whole banking system or a representative bank of a fully competitive credit market when the expression "the bank" is encountered.

In this section we present the theoretical aspects of our model. The bank balance sheet, profitability and liquidity are discussed in section 2.1. Section 2.2 presents the modeling of investment projects and the setting of interest rate for each project. Finally, in section 2.3 we first present the measures of risk and revenue of a single project and, through them, we obtain risk and revenue of the lending portfolio. The latter are particularly important because it is on them that the bank bases its decision.

2.1. Profitability and liquidity

The bank balance sheet in this model is as follows:

$$L + C = D + E \tag{1}$$

where L is liquidity, C credit, D deposits and E the bank equity. For the sake of simplicity, we assume that the interest rate on liquidity and on deposits is zero. Under these assumptions, the profit for a bank (Φ) can be expressed as

$$\Phi = \phi C - wC - eE$$

where ϕ is the average interest rate gained on loans, w the cost for each unit of credit and e are dividends.[2]

In the proposed model, the bank commits to keep a constant ratio between equity and lending[3]

$$E = \gamma C. \tag{2}$$

In this context a profitability constraint can be identified by imposing $\Phi \geq 0$. This bring us to

$$\phi \geq s \tag{3}$$

where $s = w + \gamma e$.

The interplay between influx and outflux of funds generates fluctuations in liquidity that must be faced by the bank. From a modeling point of view, it is usual to require $L \geq \hat{L} = kD$. To ease the exposition without loosing generality, we transform the inequality $L \geq \hat{L}$ in $L = \hat{l}\hat{L}$ with $\hat{l} \geq 1$. Substituting the just written equality and rule (2) into equation (1) we have

$$\hat{l}kD + C = D + \gamma C$$

that brings to

$$D = [(1-\gamma)/(1-\hat{l}k)]C. \tag{4}$$

Using this result, the liquidity constraint can be expressed as:

$$\hat{L} = lC$$

where $l = k(1-\gamma)/(1-\hat{l}k)$.

[2]Note that here w are "production costs" such as personnel and screening costs. Under our assumptions, these are the only costs for a bank because the interest rate on deposits is assumed to be equal to zero.

[3]The adoption of this commitment can be seen as an extreme simplification of the Basel accords contents (Basel Committee on Banking Supervision, April, 2003).

2.2. The lending activity

The bank finances investment projects. To be realized, each of them requires the same amount of funds (which we normalize to one). Projects are heterogeneous and each of them is characterized by its levels of revenue and risk. In this part of the paper, we analyze a situation in which the bank can solve asymmetric information problems by bearing screening costs, so that the bank knows the features of the project the entrepreneur is going to implement. This setting disciplines our investigation with regards to the projects to be considered and the level of the interest rate. These two aspects are discussed in sections 2.2.1 and 2.2.2 respectively.

2.2.1. Investment projects

A first consequence of having a bank which solves asymmetric information is that the bank can discard inefficient projects and considers only efficient ones to compose the loan portfolio.[4] Selecting efficient projects is a difficult task especially when the dynamic feature of investment projects (their outcome evolves over time) are taken into account. To deal with this more complicate framework, we adopt the simplification - often used by economic modelers - that projects have two possible outcomes.

We will first focus on single-period projects to better explain the role of efficient projects, then we extend the framework by considering dynamic aspects.

Single-period projects. In the single-period case, investment projects last only one period and have a minimum yield of ρ.[5] The bank lends at time t an amount of 1 for each project (below, the single project is identified with the lower script i). Project i outcome is realized after 1 period and is modeled as a random variable taking values $\{u''_{i,1} = 1 + \rho, s''_{i,1} = 1 + \rho + \pi_i\}$ with probabilities $\{1-p_i, p_i\}$. For future reference, we highlight that the revenue of a project is also a random variable taking values $\{u'_{i,1} = \rho, s'_{i,1} = \rho + \pi_i\}$ with the same probabilities given above. Note however that relevant information for a project are π_i and p_i, and, subtracting the common element $1 + \rho$, each project can be presented as a Bernoulli random variable (Π_i) which

[4]Using the terminology of portfolio theory, a project is efficient if does not exist any other project having a risk not higher and a revenue not lower than its own.

[5]In this paper ρ is treated as a parameter, but in future developments it could be modeled as depending on macroeconomic conditions.

takes values $\{u_{i,1} = 0, s_{i,1} = \pi_i\}$ with probabilities $\{1 - p_i, p_i\}$. Considering only efficient projects means that a project with a higher average revenue is associated with a higher level of risk. This requirement is satisfied if the derivatives of the mean and variance with respect to π_i are both positive. These derivatives depends on the functional form of p_i. In this paper we use the functional form $p_i = (1+\pi_i)^{-\alpha}$ which allows us to easily investigate both the situation where the bank can select only efficient projects and a setting where asymmetric information prevents this possibility. In Appendix A we show that using this functional form, we have a positive relationship between revenue and risk when $0 < \alpha \leq 1$.

Multi-period investments. The single period case will be used below to explain the basic implications of the model. However, through the paper we will work mainly with the multi-period case. We model a multi-period investment project as a set of Bernoulli trials (one random variable with two outcomes for each period of life of the investment). This framework allows us to give the following discrete representation of the bank-customer relationship. Borrower and lender meet periodically to evaluate the state of the project and they decide if the credit is prolonged or payed off.

More formally, a multi-period project is characterized by two sequences which give the outcome of the project if unsuccessful ($u''_{i,n}$) and successful ($s''_{i,n}$) in the nth trial.

As discussed above, if the bank solves asymmetric information, only the subset of efficient projects is considered in the analysis. A convenient way to consider efficient projects in a dynamic context builds on the single-period case by considering projects with the following features: $\{u''_{i,n} = (1+\rho)^n, s''_{i,n} = (1+\rho+\pi_i)^n\}$. The probability of a success in a trial (p_i) is kept constant over time and it decreases with π_i as in the single-period case.

2.2.2. The interest rate

A second consequence of having a bank which solves asymmetric information is that entrepreneurs are willing to pay an higher interest rate to the bank with respect to that observed in the financial market. In our framework, the interest rate is modeled as follows.

The Central Bank sets the policy interest rate r. According to portfolio theory, the market determines a risk premium β. These two elements determine the *capital market line*

$$r_i^m = r + \beta\sigma_i.$$

The *capital market line* gives the cost of funds an entrepreneur would bear using direct sources of financing.

The existence of asymmetric information makes it difficult for entrepreneurs to access financial markets. The bank offers a solution to this problem. In our model, we acknowledge the role of the bank assuming that entrepreneurs are willing to pay an interest rate which is at most equal to that identified by the capital market line (at a level of risk equal to that of their projects) plus a spread (δ). Consequently, the interest rate on loans is determined by:

$$r_i = \delta + r_i^m = \delta + r + \beta\sigma_i. \tag{5}$$

It is useful to recall that in our case, the standard deviation refers to the Bernoulli random variable Π_i: $\sigma_i = [\pi_i^2 p_i(1-p_i)]^{\frac{1}{2}}$.

Below, we will refer to equations (5) as the "augmented" capital market line.

2.3. The lending portfolio

In this section we analyze the multi-period case. We present the outcome of a single project and then we point the attention to a situation where a set of projects is considered. We will first get an intuition of the loan portfolio management implications by using single-period projects. The case of multi-period projects is more deeply investigated by numerical methods in sections 3 and 4.

2.3.1. A single project

We recall that in the multi-period case, the outcome of a project i is described by a set of Bernoulli trials ($\{u''_{i,n} = (1+\rho)^n, s''_{i,n} = (1+\rho+\pi_i)^n\}$).

In what follows, I denotes the interest accrued. Until a credit is not payed back, I evolves according to

$$I_{i,t+1} = I_{i,t}(1+r_i) + r_i.$$

Using the initial condition $I_0 = 0$ and iterating n times we obtain

$$I_{i,n,t} = \sum_{k=1}^{n} \binom{n}{k} r_i^k. \tag{6}$$

where $\binom{n}{k}$ denotes the binomial coefficient.[6]

[6]It is straightforward to verify that $(1+r_i)^n = 1 + I_{i,n}$.

This is the amount of interest a credit of type i prolonged $n-1$ times will give back if successful "enough". Considering that the possibility of the debt to be fulfilled depends on the $s''_{i,n}$ sequence, we say that a project is successful "enough" if $\rho+\pi_i \geq r_i$.[7] This implies that the bank will be always able to obtain the principal and interest by waiting for the project being successful. In the first part of the paper, we work in this favorable situation and we model the bank-entrepreneur relationship as follows. If the success has not been yet realized the credit is prolonged but its amount is increased to take into account the accrued interest. When the project is successful, the borrower pays off.

At a given point in time, the incoming interest flux from a credit that was prolonged $n-1$ times is a Bernoulli random variable $\mathcal{I}_{i,n,t}$ taking values $\{0, I_{i,n,t}\}$ with probabilities $\{1-p_i, p_i\}$ respectively. Because we are interested in analyzing the model in the portfolio theory framework, we determine the mean and variance of this variable:

$$\mu(\mathcal{I}_{i,n,t}) = I_{i,n,t} p_i,$$

$$\sigma^2(\mathcal{I}_{i,n,t}) = I^2_{i,n,t} p_i (1-p_i).$$

It it worth noting that, given δ and β, these equations can be expressed as functions of r and σ_i (see Appendix B for details):

$$\mu(\mathcal{I}_{i,n,t}) = \mu_{i,n,t}(r, \sigma_i)$$

$$\sigma^2(\mathcal{I}_{i,n,t}) = \sigma^2_{i,n,t}(r, \sigma_i).$$

2.3.2. The loan portfolio

The bank faces a large number of heterogeneous projects. According to our assumptions a project is fully characterized by π_i and p_i. We take account of the heterogeneity of the potential borrowers by introducing a probability density p_{π_i} which gives the fraction of all the available projects characterized by a given π_i. We also know that in our framework σ_i is an increasing function of π_i so that, the p_{π_i} can be transformed into the p_{σ_i} distribution which gives the fraction of projects having a given risk. This second representation is more convenient when the model is presented in terms of portfolio theory.

[7] One should take in mind that whenever $s''_{i,s}$ becomes smaller than $1+I_{i,n,t}$ the bank loose a part or the whole amount.

On the other hand, we know from the previous section that a type i credit that was prolonged $n-1$ times is successful (and the relative influx is realized) with probability p_i. In other terms, the evolution of the loan portfolio can be seen from a statistical point of view as an extremely complicated birth and death process.

Our aim is to gain insight into this complicate random process in order to choose the loan portfolio. In general terms, the bank has to choose a bounded and convex set belonging to the support of the p_{σ_i} random variables. To simplify, we fix the lower bound of the customer set to $\sigma = 0$ and we let the bank choose the upper bound.

For a bank financing a convex set $[0, \sigma]$ of projects, the total amount of interest received is

$$\mathcal{I}_t = \int_0^\sigma \sum_{n=1}^{\infty} C_{i,n,t} \mathcal{I}_{i,n,t} d\sigma_i \tag{7}$$

where $C_{i,n,t}$ is the credit which was allotted to type i borrowers in period $t-n$ and still not refunded (being the amount needed to implement a project equal to one, this variable also denotes the number of projects having characteristics i and n). The total amount of lending is defined in the following way:

$$C_t = \int_0^\sigma \sum_{n=1}^{\infty} C_{i,n,t} d\sigma_i.$$

$\mathcal{I}_t$ is a random variable being a sum of random variables. We are interested in the average of the projects interest influx:

$$\phi_t(\sigma, r) := \frac{\mu(\mathcal{I}_t)}{C_t} = \int_0^\sigma \sum_{i=1}^{\infty} p_{\sigma_i,n,t} \mu_{i,n,t}(\sigma_i, r) d\sigma_i, \tag{8}$$

where $p_{\sigma_i,n,t}$ is the proportion of financed projects having a risk equal to σ_i which have been financed n periods before.

Let us recall we focus on the behavior at low levels of the policy interest rates. In other words we work under the assumption of a binding profitability constraint. We draw attention on the fact that the profitability constraint must be satisfied in a relatively long period (say a year). Therefore, we set the bank problem of choosing the lending portfolio in terms of stationary quantities, so that the t lower script will be removed from equations. The average projects interest influx (equation 8) becomes

$$\phi(\sigma, r) := \frac{\mu(\mathcal{I})}{C} = \int_0^\sigma \sum_{i=1}^{\infty} p_{\sigma_i,n} \mu_{i,n}(\sigma_i, r) d\sigma_i. \tag{9}$$

By using equation (3) we identify in the value σ^s that satisfies the equation

$$\phi(\sigma^s, r) = s, \tag{10}$$

the upper bound of the loan portfolio. In other words, the bank has to finance projects belonging to the $[0, \sigma^s]$ set to satisfy the profitability constraint.

As anticipated above, a simple example of the σ^s computation can be obtained from the single-period case; in this case the revenue of each project can be seen as a Bernoulli random variable taking values $\{u'_{i,1} = \rho, s'_{i,1} = \rho + \pi_i\}$ with probabilities $\{1 - p_i, p_i\}$. Under this assumption, in each period the set of financed project can be viewed as a new random draw from the population of projects the bank faces. Equation (9) becomes:

$$\phi(\sigma, r) = \int_0^\sigma p_{\sigma_i,1} \mu_{i,1}(\sigma_i, r) d\sigma_i.$$

where, in this specific case,

$$\mu_{i,1}(\sigma_i, r) = \begin{cases} r_i & \text{if} \quad r_i \leq \rho \\ r_i p_i + \rho(1 - p_i) & \text{if} \quad r_i > \rho \end{cases}$$

and $p_{\sigma_i,1} = p_{\sigma_i} / \int_0^\sigma p_{\sigma_i} d\sigma_i$ is the probability of financing a project with a risk equal to σ_i given that only the projects with $\sigma_i \leq \sigma$ are financed. The upper bound of the set of customers for the single-period case is graphically identified in figure 1. To build the figure we have used the following settings: $s = \rho = 0.05$, $\delta = 0.01$ and $\beta = 0.02$. The "augmented" capital market line (equation 5) and the $\phi(\sigma, r)$ functions are represented for two values of the policy interest rate (2% and 1%; $\delta = 1\%$ implies intercepts at 3% and 2% respectively). The $\phi(\sigma, r)$ functions are obtained under a uniform distribution of π_is. σ^ss are determined by the intersection point of the ϕ functions with the s horizontal line. The figure shows how the bank responds by increasing the upper bound of the loan portfolio to a reduction of the policy interest rate. Under our assumption the bank lends to projects having a standard deviation belonging to the set $[0, \sigma_2^s]$ if the policy interest rate is 2% and to the wider set $[0, \sigma_1^s]$ if the policy interest rate is 1%.

More generally, the σ^s value depends directly on s and inversely on r and it is equal to zero as long as $r + \delta > s$.[8] Thus, a relevant consequence that

[8]The increase of σ^s is important in our model. Let us point out that, although we

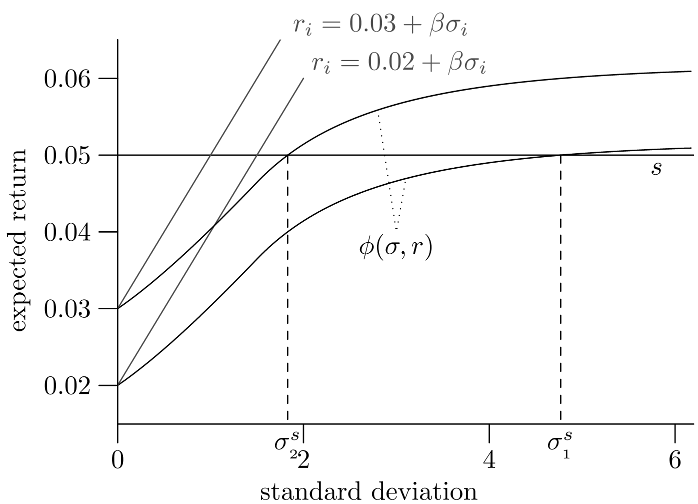

Figure 1: determination of the loan portfolio upper bound in the single-period projects case.

stems from this behavior is that lower policy interest rate pushes the bank to "board" additional customer having a higher level of risk to satisfy the profitability constraint.

Hereafter we analyze how the movements of the bank loan portfolio affect the dynamics of a number of relevant variables of the bank, in particular their liquidity. Because we will focus on the more complicate multi-period case, we will progress by numerical computations.

3. Calculations

We start the description of our calculations considering how the bank balance sheet changes with the policy interest rate. Our first consideration concerns deposits. It is widely accepted that people choose to be more liquid at lower interest rates so that a negative relationship between the bank deposits and the policy interest rate exists. Using the relationships presented in section 2.1 we can determine the others variables of the bank balance sheet

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analyze the effects of a reduction of the policy interest rate, our results could be also obtained by an increase of s. We have chosen to analyze the reduction of the policy interest rate because this implies that the bank management is forced to choose a higher σ^s to respect the profitability constraint. In contrast, an increase in s implies the adoption of a reckless behavior to obtain a higher profitability.

starting from deposits: r determines D, liquidity is $L = \hat{l}kD$, the level of credit can be established solving C in equation (4) and finally, the equity base is $E = \gamma C = L + C - D$.

Once, the total amount of credit is known, the lending activity goes on as follows. Projects are organized in a finite number of numbered cohorts. In a simulation time step, entrepreneurs running projects belonging to a cohort meet the bank. Each loan is payed off if the project is successful, otherwise it is prolonged. As mentioned above, in this part of the paper we analyze the very favorable situation in which, the successful outcome of the projects is alway enough to pay back the principal and the accrued interest. The bank obtains no influx of funds if credit is prolonged. At the end of the time step, the bank finances new investments if the total amount of outstanding credit is lower than that resulting from equation (4). New projects are financed until the total amount of loans given by equation (4) is reached by randomly drawing new π_is from a uniform distribution with boundaries 0 and π^s. These events repeat in the following time step with the next cohort, and when all cohorts are updated, a cohorts-cycle is concluded and a new cycle starts with the first cohort.[9]

The setting we use to investigate numerically the multi-period case are similar to those for the single-period case listed above. We set the profitability threshold (s) and the minimum revenue from projects (ρ) to 0.05, the risk premium parameter β to 0.02, the interest rate spread δ to 0.01 and $\alpha = 1$. Newly financed projects are assigned new π_is drawn from a uniform distribution. These settings imply that if the policy interest rate is 4% (and the "augmented" capital market line has intercept at 5%) the time series of cash influxes is constant at a level which satisfies the profitability constraint: with this level of the policy interest rate, the bank satisfies the profitability constraint without taking risk (this is because $s = \rho$). This level of the policy interest rate provides a benchmark and we consider to start our numerical simulations with it.

Concerning the balance sheet, we use a linear function for deposits:

$$D = c_1 - c_2 r.$$

For the sake of simplicity, we put $\gamma = \hat{l}k$ so that, according to equation (4)

[9] We could compare a simulation time step to a real world day. Since interest accrues everyday, we think a real world "actual day" fits the concept better than a "working day".

we have $C = D$. For future reference we can thus write

$$C = c_1 - c_2 r. \tag{11}$$

We set up the initial level of credit according to this equation. In our simulations, for example, we set $c_1 = 2000$ and $c_2 = 25000$ so that at the benchmark interest rate ($r = 0.04$), the bank finances 1000 projects having each one a null standard deviation. Projects are arranged in 30 cohorts.

4. Results

We evaluate the effects of moving the policy interest rate on some important bank variables first. Then, we will analyze in detail the evolution of the lending portfolio after the change in r.

As specified above, in our simulations the policy interest rate is initially set at 4%, it is moved at a lower level for a period and it is brought back to its initial level. Subsection 4.1 analyzes the effects of the r reduction, while subsection 4.2 focuses on the effects of bringing the policy interest rate to its original level.

4.1. A reduction in the policy interest rate

4.1.1. Bank variables

In our model, when the policy interest rate is moved downward, the bank lends to more risky projects in such a way that the profitability constraint is met. In other words, we increase the upper bound of the set of financed projects (π^s). Table 1 reports the value of π^s which ensures an average return of the lending portfolio equal to 0.05 at different levels of the policy interest rate. The table also reports the values of p^s and the mean (μ^s) and standard deviation (σ^s) of the Bernoulli random variable $\{u_i = 0, s_i = \pi^s\}$ with probabilities $\{1 - p^s, p^s\}$.

To explain our arguments without weighting down the exposition, in this section we present the results of lowering the policy interest rate to two different levels: 0% and 3%. They represent a mild and a deep policy interest rate reduction. What happens in between can be deduced from the outcome of these two levels. Data collected from our simulations on the total cash

r	$r+\delta$	π^s	$p^s = (1+\pi^s)^{-1}$	$\mu^s = \pi^s p^s$	$\sigma^s = [(\pi^s)^2 p^s (1-p^s)]^{\frac{1}{2}}$
0.04	0.05	0	1	0	0
0.03	0.04	1.42	0.413	0.587	0.699
0.02	0.03	2.8	0.263	0.737	1.233
0.01	0.02	4.35	0.187	0.813	1.696
0	0.01	6.15	0.14	0.860	2.133

Table 1: the value of π^s which ensures an average return of the lending portfolio equal to 0.05 at different levels of the policy interest rate. Using π^s, the variables p^s, μ^s and σ^s are computed and reported.

influx, the number of financed projects and the number of projects which refund are shown in figure 2.[10]

Looking at graph 2A, one can see how the decrease of the policy interest rate causes a sudden fall in the influx of funds, then, it gradually increases and converges to a value that is proportional to the number of financed projects. Due to the increase in deposits, the total amount of credit suddenly goes from 1000 to 1250 when r is lowered to 3% and to 2000 when $r = 0\%$ (see chart 2B). The number of successful projects decreases with the policy interest rate (chart 2C). Finally chart 2D reports the revenue obtained for each unit of lending which gives us a measure of the bank liquidity position. To evaluate the length of the transient state we add vertical lines to the charts. The dashed line signals the time of the interest rate change while a dashed line with dots is drawn every 12 cohorts-cycles after the change. Charts show that the convergence to the new stationary situation takes more time for $r = 0$ than for $r = 0.03$. However, the difference is far less evident in the number of successful projects than the variables which involves the amount of interest received.

Figure 2 suggests us to further investigate in two directions. The first one focuses on how the composition of the lending portfolio changes after a

[10] Charts report cohorts-cycles data. Let us make an example to further explain what a cohots-cycle is. Consider for example the 13th cohorts-cycle. It starts in simulation time 361 with cohort 1 and end in simulation time 390 with cohort 30. Bearing this in mind, the charts were build as follows. For each simulation run, we build a time series which associates at each time of a given cohort-cycle (in the our example interval 361-390) the sum taken on the cohorts-cycle of the observed variable. We perform 1000 simulation runs for each setting and we report the average of the 1000 time series we obtained in charts 2A, 2B and 2C. Chart 2D is the ratio of the time series reported in charts 2A and 2B.

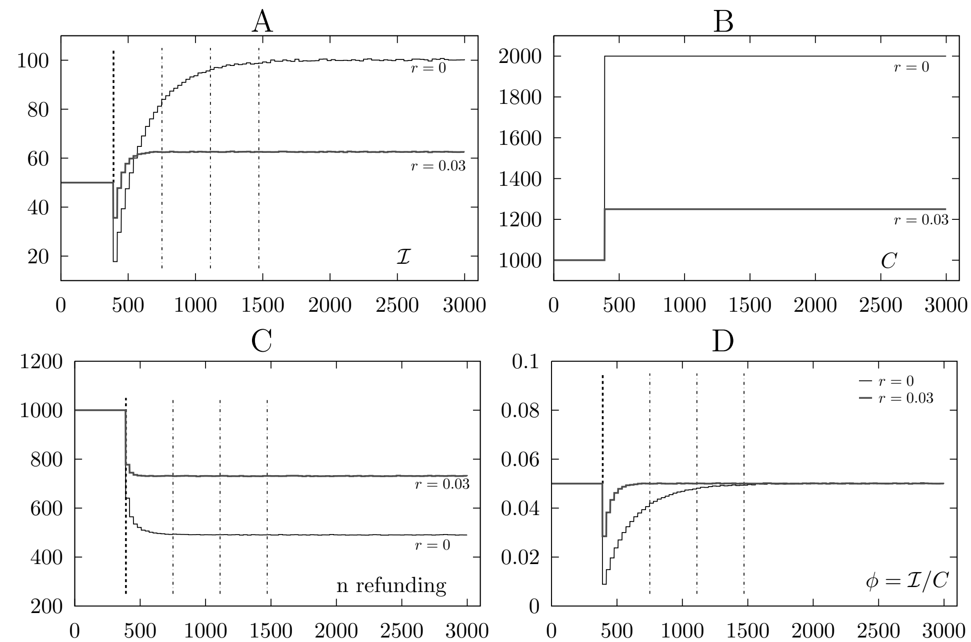

Figure 2: comparison of bank variables dynamics when r is lowered to 3% and 0% from an initial level of 4%. The change takes place in period 361 (the first cohort of 13th cohorts-cycle). The dashed line signals the time of the interest rate change. A dashed line with dots is drawn every 12 cohorts-cycles after the change.

downward movement of the policy interest rate and the second one concerns the statistical features of the bank liquidity in this new situation (sections 4.1.2 and 4.1.3 respectively).

4.1.2. Loan portfolio distribution dynamics

Beside the aggregate variables displayed in figure 2, in our simulations we have recorded the features of each project in each simulation round. It follows that for each simulation step, we have a collection of π_i that represents the bank loan portfolio. We use this data to build frequency distributions. The evolution of such distributions is of particular interest for this paper. Figure 3 allows a comparison of the dynamics of these distributions in the two cases we are reporting on. A kernel density estimation of the distribution obtained by pooling the data of each cohort in a given cohorts-cycle is computed and displayed in the figure every thirty simulation time steps. After the downward movement of the policy interest rate (time step 360 in the figure) the distribution is close to the uniform density as we expect from

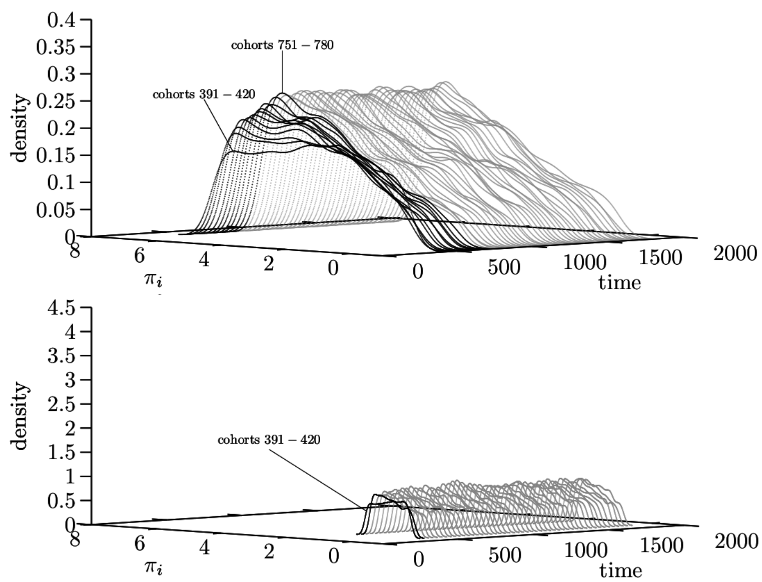

Figure 3: evolution of the loan portfolio after a decrease of the policy interest rate. Upper chart: transition from $r = 0.04$ to $r = 0$. Lower chart: transition from $r = 0.04$ to $r = 0.03$. Black highlights the transient period; gray is used when the shape of distributions does not change much over time.

r	π^s	C	$\sigma(\tilde{L})$	$\#\tilde{L}^-$ out of 10^5	$\bar{d}$
0.04	0	1000	0	0	0
0.03	1.42	1250	0.011	53199	2.134
0.02	2.8	1500	0.0157	54643	2.197
0.01	4.35	1750	0.0202	56734	2.315
0	6.15	2000	0.0258	58423	2.402

Table 2: a number of statistics from the excess liquidity time series at the stationary state

our assumption. Then it evolves towards a distribution whose density increases with π_i. The visual inspection of figure 3 shows that it takes about twelve cohorts-cycles to reach a stable distribution when the interest rate is lowered to 0 while the transition to a new distribution is achieved in a few cohorts-cycles when the policy interest rate is lowered to 0.03. The black is used in the figure to highlight the change of the distribution in the transient period, while the gray is used to draw distributions the shape of which does not change much over time. The z axes range is different in the two charts to allow for a more clear comparison with figure 6.

4.1.3. The statistical features of the new stationary state

In this section we focus on the properties of the bank excess liquidity when the steady state is reached. Let us first define what we mean for excess liquidity. From the discussion in the previous section we know that the total influx is a realization of the random variable defined in equation (7). We also know from section 2.1 that in each period the required liquidity is $\hat{L} = lC$. Therefore, we define the excess liquidity as follows:

$$\tilde{L} := L - \hat{L} = \mathcal{I} - lC.$$

To have a benchmark for our investigation we analyze the case where the average excess liquidity is zero. Definition (9) and condition (10) toghether ensure that $\mu(\mathcal{I}) = sC$ and consequently our benchmark case is obtained by setting $l = s$: in this case we have in fact $\mu(\tilde{L}) = 0$.

We have recorded data from simulations for the realization of $\mathcal{I}_t$ in a long period after the stationary distribution has been reached. A number of statistics computed from the time series obtained at different levels of the policy interest rate are reported in table 2. We note from the table that the volatility of $\tilde{L}$ is higher at lower levels of the policy interest rate. A second remark can be drawn from counting the number of liquidity shortages, that is counting how many times the $\tilde{L}$ is negative. The column $\#\tilde{L}^{-}$ of the table reports this information when simulations run for a long period (the recorded data points are 10^5). Data reported in this column show that the liquidity situation gradually worsens when the policy interest rate is lowered. A third comment comes from monitoring the persistence of liquidity shortages. We compute the length of a liquidity shortage as the number of consecutive periods in which a negative value of $\tilde{L}$ has been recorded. Let us denote with d the duration of liquidity shortages. The frequency distribution for this variable has been computed for the levels of the policy interest rate considered in table 2. The last column of the table reports the average length observed at each level of the considered policy interest rate. Figure 4 reports the semi-log plot of the whole duration distribution for the two values of r we are reporting on. The figure shows how the frequencies fall exponentially with the length of liquidity shortages. However, the frequency of long liquidity shortages is significantly higher at $r = 0$ than at $r = 0.03$.

The main point of our results is that the behavior of the bank, could limit the effectiveness of the policy interest rate movements; in our setting, lower policy interest rates may lead to greater difficulties in managing the bank liquidity, making it more difficult to resolve liquidity shortages.

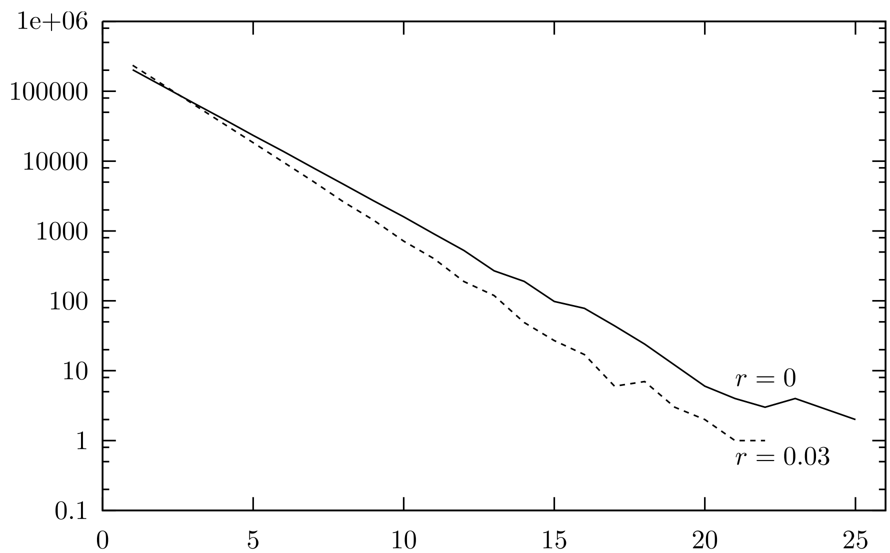

Figure 4: log of absolute frequencies of the length of liquidity shortage periods.

4.2. Moving to a higher policy interest rate

The evolution of the bank situation when the policy interest rate is brought back to the initial level is also an object of this study.

The situation is displayed in figure 5 which differs from figure 2 in that the policy interest rate is moved back to 4% in time step 1800. Again, dashed lines are used to signal the times of the interest rate changes and a dashed line with dots is drawn every 12 cohorts-cycle after the changes.

The total amount of interest influx follows an odd dynamic soon after the policy interest rate increase (see graph 5A). The dynamic is the result of two forces. The first one is the upper pressure applied by the increase of the policy interest rate. The second one operates through the evolution of the lending portfolio. The bank has not full control over the loan portfolio adjustment process soon after the interest rate increase. In our model, an increase of the interest rate causes a reduction of deposits and consequently the bank has to decrease its lending activity.

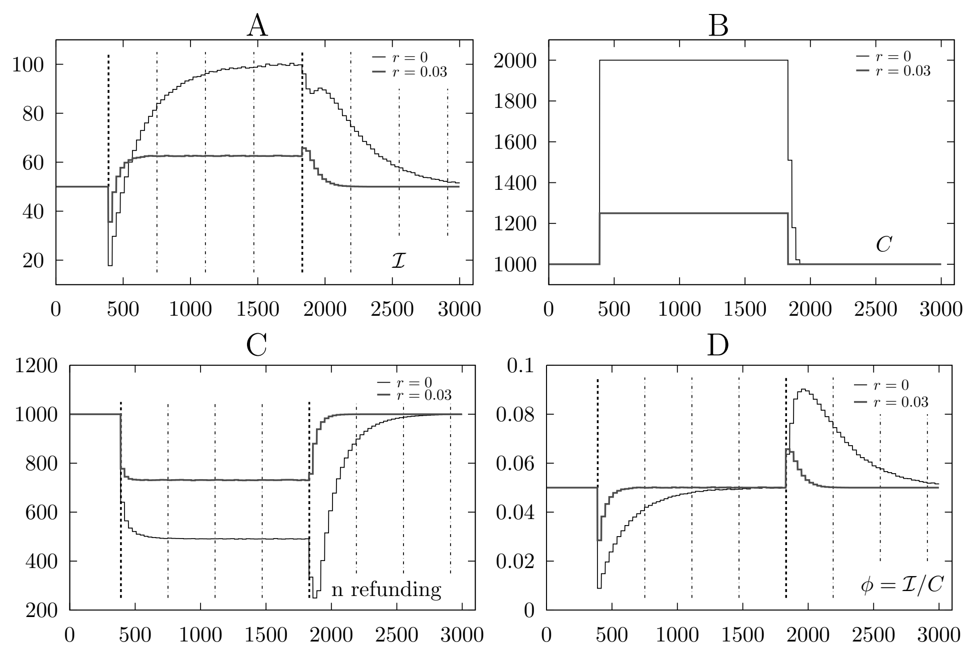

Figure 5: comparison of bank variables dynamics when r is lowered to 3% and 0% from an initial level of 4% and then brought back to the initial level. The interest rate reduction takes place in period 361 while the interest rate increase is in period 1801. The dashed lines signal the times of interest rate changes. A dashed line with dots is drawn every 12 cohorts-cycle after each change in r.

The bank can reduce the number of financed projects by avoiding lending to a new project when an old one pays back. When the reduction of credit is significant (as in the case the policy interest rate jumps from 0 to 4%), this "lending volume" effect outperforms the interest rate effect and the total interest rate influx decreases (see chart 5A).

The downward adjustment of the volume of lending takes place by the exit of borrowers characterized by low π_is because they have a high probability of paying off their debt. In other words, during the transition the lower bond of the bank portfolio is no more zero, but it starts growing for a while after an increase of the policy interest rate. The loan portfolio runs out of projects which pay off and, consequently, the number of refunders may fall temporarily after the policy interest rate change (see figure 5C). When the lower amount of lending implied by equation (11) is reached, projects with high probability to refund start entering again the lending portfolio. This happens because the bank substitutes successful projects with new ones whose π_i falls in the new interval $[0, \pi^s]$. However, while the projects which exited payed back a low interest amount, the new projects with a similar degree of risk pay higher interest rates because of the upward shift of the "augmented" capital market line. In this phase, the total amount of interest influx moves upward (see figure 5A).

The long run behavior can be understood by the following considerations. In our framework, projects with higher π_is are "locked" in the bank lending portfolio for longer time. Even if they were extended when the policy interest rate was low, they yield a high interest rate because of their high risk. Their substitution with lower risk projects (those with $\pi = 0$ because $\pi^s = 0$) causes a decrease of the total interest influx amount in the long run. As outlined above, chart 5D displays the liquidity available to the bank for each unit of credit. It shows that liquidity increases significantly in a few cohorts-cycles after the movement of the interest rate; then its trend inverts and gradually approach the steady state value.

Even after the interest increase the convergence to the steady situation slows down when $r = 0$ because of a higher π^s. A peculiarity can be noted for the number of projects which pay off: the convergence to the new steady situation is much slower after an increase in r than following a decrease (see chart 5C).

Building on figure 3, figure 6 highlights the dynamics of the loan portfolio after the policy interest rate increase. In line with our previous comments, the figure shows how the densities of high risk projects increase soon after

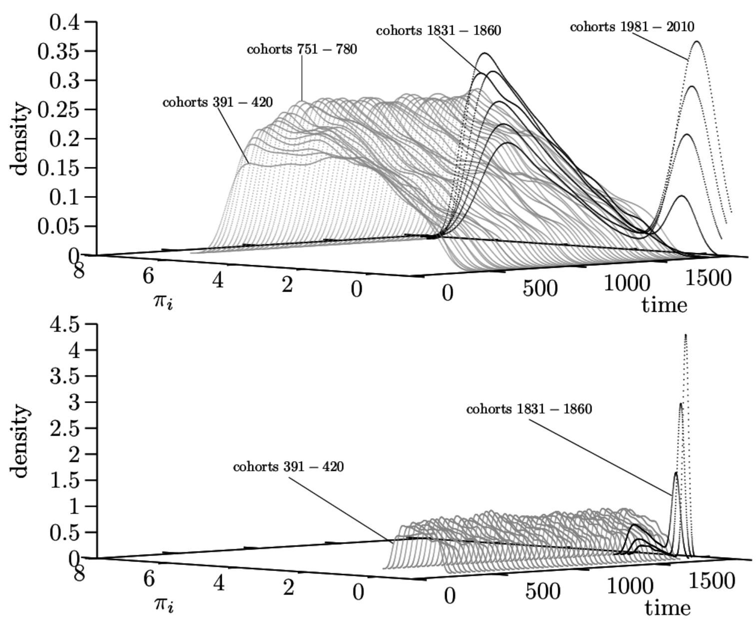

Figure 6: evolution of the loan portfolio after an increase of the policy interest rate. Upper chart: transition from $r = 0$ to $r = 0.04$. Lower chart: transition from $r = 0.03$ to $r = 0.04$. Black highlights the transient period; gray is used when the shape of distributions does not change much over time.

the policy interest change when $r = 0$. Then, the densities cumulate in $\pi_i = 0$ because for a policy interest rate equal to 4% only risk free projects are financed; the higher is the risk of a project, the longer it remains in the loan portfolio. The most relevant changes are highlighted in black as in figure 3.

Summing up, an increase of the policy interest rate causes a reduction of the risk of the bank loan portfolio in the long run, but this is achieved going first through an increase of the riskiness of the portfolio. At the aggregate level this mechanism is revealed by a trough in the number of refunding projects (visible in figure 5C).

5. Towards a real world situation

In previous sections we analyzed an "ideal" situation where the bank solves asymmetric information. This implies that only efficient projects are financed. In particular, in our settings the bank can always obtain the principal and interest when the project is successful. This makes the low value of the Bernoulli random variables irrelevant for the analysis. In fact, it is convenient for the bank to prolong the loan until the success is obtained.

In this section we integrate the theoretical analysis performed above with elements which characterize reality. Our final goal is to analyze the case in which the project bad result can be so small that the bank suffers a loss. A preliminary step to achieve this goal is to consider projects having a finite time horizon. Therefore, we evaluate the effect of financing projects with finite time horizon first and then those of losses. In both cases the low value of the Bernoulli random variables affects the model results.

5.1. Finite projects time horizon

In previous sections we considered two cases that, from the point of view of projects time horizon, represent two extremes: the single-period cases and the multi-period case. The latter can be thought of as the case in which the project time horizon is infinite. We now look at intermediate cases analyzing the dynamic of the system when projects - and thus loans - time horizon (hereafter denoted with $\bar{n}$) is gradually shortened. This can help shedding light on situations where economic agents mistrust the future because they are experiencing a gradual worsening of the overall economic conditions. In this section, the low value of the Bernoulli random variable is kept at 0 as in the analysis above, so that if the project is not successful after $\bar{n}-1$ renewals,

$\bar{n}$	π^s	$\phi\%$	$\sigma(\tilde{L})$	π^s	$\phi\%$	$\sigma(\tilde{L})$
25	6.15	4.884	2.142	6.33	5	2.22
20	6.15	4.798	1.965	6.48	5	2.085
15	6.15	4.654	1.729	6.85	5	1.908
10	6.15	4.418	1.405	7.7	5	1.633
5	6.15	4.031	0.929	11	5	1.174
4	6.15	3.924	0.801	13	5	1.039
3	6.15	3.8	0.652	16	5	0.859
2	6.15	3.649	0.464	24	5	0.634
1	6.15	3.455	0.148	55	5	0.154

Table 3: The effects of project duration on some relevant variables

the bank get the amount $(1+\rho)^{\bar{n}}$ at period $\bar{n}$. This feature will be removed below.

We perform several simulations setting $r = 0$ and gradually lowering $\bar{n}$. The simulations are carried out considering two different frameworks. In the first one, the boundaries of the lending portfolio are the same as those considered in the infinite time horizon case: $\pi_i \in [0, 6.15]$. In the second case, the upper bound (π^s) is adjusted to have an average revenue of the loan portfolio equal to 5%. The results of these exercises are reported in table 3. When π^s is kept constant at 6.15, both ϕ and σ decrease when the projects time horizon is reduced. The table also shows that to keep the average revenue to the target level, the portfolio upper bound must be significantly increased. This brings an increase in the volatility of the interest rate influx, $\sigma(\tilde{L})$, with respect to the $\pi^s = 6.15$ case at each level of $\bar{n}$.

It is worth reporting that high volatility clustering can be observed at low levels of the project time horizon. Figure 7 allows a comparison of the ϕ time series at $\bar{n} = 2$ and $\bar{n} = 5$. Although we know that the dynamics of events wich characterize our model affect the auto-correlation of the ϕ time series, the volatility clustering observed for example when $\bar{n} = 2$ is an unexpected phenomenon which is relevant for the bank liquidity. According to this result, the persistence of a high level of uncertainty about the future evolution of economic conditions - which reduces economic agents' willingness to subscribe long term contractual agreements - implies that periods of unstable funds influx may occasionally burst in periods of "tranquillity" complicating the bank liquidity management.

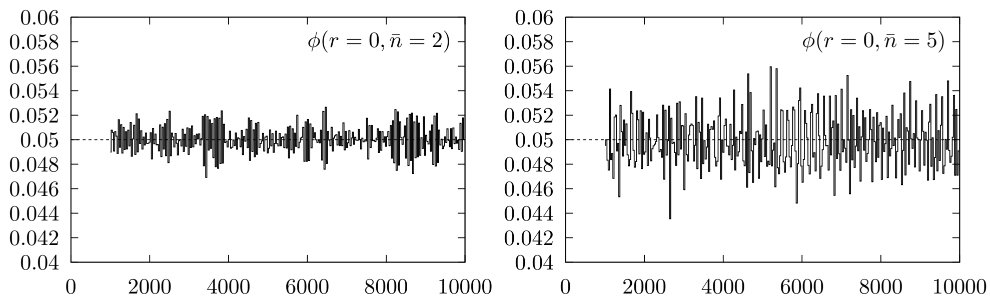

Figure 7: volatility clustering at short project duration

5.2. Losses

When the bank cannot completely remove asymmetric information it is no more able to select only efficient projects. To account for this possibility, the framework of the previous sections is modified as follows. First, we remove the favorable assumption that the lower bond of the Bernoulli random variable is zero allowing for negative values. Second, we remove the requirement that the average return of the project rises with the project risk.

The first innovation concerns Bernoulli random variables which take values $\{u_{i,1} = -b_i, s_{i,1} = \pi_i\}$ instead of $\{u_{i,1} = 0, s_{i,1} = \pi_i\}$ (probabilities are kept to $\{p_i, 1 - p_i\}$ as above for now). To set b_i, we first draw a random number from a beta distribution (say ξ_i). We them multiply it by π_i. Note that if $\xi_i = 1$, the Bernoulli random variable will be $\{u_{i,1} = -\pi_i, s_{i,1} = \pi_i\}$. We then perform a final check to exclude that the bank loses more than the principal: ξ_i that implies $1 + \rho - \xi_i \pi_i < 0$ is replaced by $\frac{1+\rho}{\pi_i}$. More formally we have

$$b_i = \min(1 + \rho, \xi_i \pi_i)$$

which implies $1 + \rho - b_i \geq 0$.[11]

We proceed in this way because, starting from the situation where the bank solves asymmetric information, we can produce a gradual worsening of the situation the bank faces by tuning the shape parameters of the beta distribution. If we denote the beta distribution with $\mathcal{B}(\mathcal{B}_1,\mathcal{B}_2)$, we can regulate

[11]In terms of the notation used in section 2.2.1 we have $\{u_{i,1} = -b_i, s_{i,1} = \pi_i\}$ for the Bernoulli random variable which characterize the project. From that we obtain the revenue random variable: $\{u'_{i,1} = \rho - b_i, s'_{i,1} = \rho + \pi_i\}$ and the whole amount random variable: $\{u''_{i,1} = 1 + \rho - b_i, s''_{i,1} = 1 + \rho + \pi_i\}$.

monitoring and screening effectiveness by tuning $\mathcal{B}_1$ and $\mathcal{B}_2$: asymmetric information problems are reduced (and thus the model of the previous section is approached) by lowering $\mathcal{B}_1$ and increasing $\mathcal{B}_2$. In what follows we model the worsening of the setting in which the bank operates by decreasing $\mathcal{B}_2$ while keeping $\mathcal{B}_1$ constant. In particular, we report in this section two cases characterized by $\mathcal{B}(1, 1000)$ and $\mathcal{B}(1, 500)$. The amount of losses suffered is lower in the first case, so that we will call it the "low losses" case and we will identify the variables with the l super script. The $\mathcal{B}(1, 500)$ case will be consequently called "high losses" and the h super script will be used.

An important issue in evaluating the effects of losses on the bank average return is how the interest rate on each loan is set. It is straightforward that the bank wants to compensate losses with an increase of the interest rate. However, this possibility depends on the highest interest rate the entrepreneurs are willing to pay. Note that for each level of π_i there are heterogeneous entrepreneurs characterized by different b_i. We call *prime* those entrepreneurs that, for each π_i, have the lowest b. It might be that *sub prime* entrepreneurs are willing to pay a higher interest rate, but, if they want to be indistinguishable from *prime* ones, they must declare to be willing to pay the same interest rate. We will consider two cases that differentiate for the difficulties entrepreneurs have in accessing financial markets. In the first case, prime entrepreneurs have no serious problems in accessing financial markets; therefore, the *capital market line* is the benchmark for setting the interest rate.

Figure 8 shows what happens to the bank average return and risk when the lending portfolio changes. Recall that in our model the bank finances projects in the convex sets $[0, \pi^s]$. Chart A in figure 8 displays the average return at three different $\bar{n}$ and, for each of them, the "low losses" and "high losses" cases. To ease the understanding of the figure, we specify the meaning of the labels that appear in the figure: $\bar{n}2^h$, for example, identifies a situation where the bank lend by contracts having a maximum length of 2 periods ($\bar{n}2$) to entrepreneurs whose bad result is obtained by using the "high losses" (super script ^{h}) case.

The chart shows that the average revenue of the lending portfolio has a maximizer at low $\bar{n}$. The maximizer increases with $\bar{n}$, and eventually disappears. Given $\bar{n}$, there is a gap in terms of average revenues between "high losses" and "low losses" cases which increases with π^s. This gap shrinks when $\bar{n}$ increases (the distance between the solid an dashed lines reduces when $\bar{n}$ increases). Figure 8B displays ϕ as a function of σ. It could be interpreted

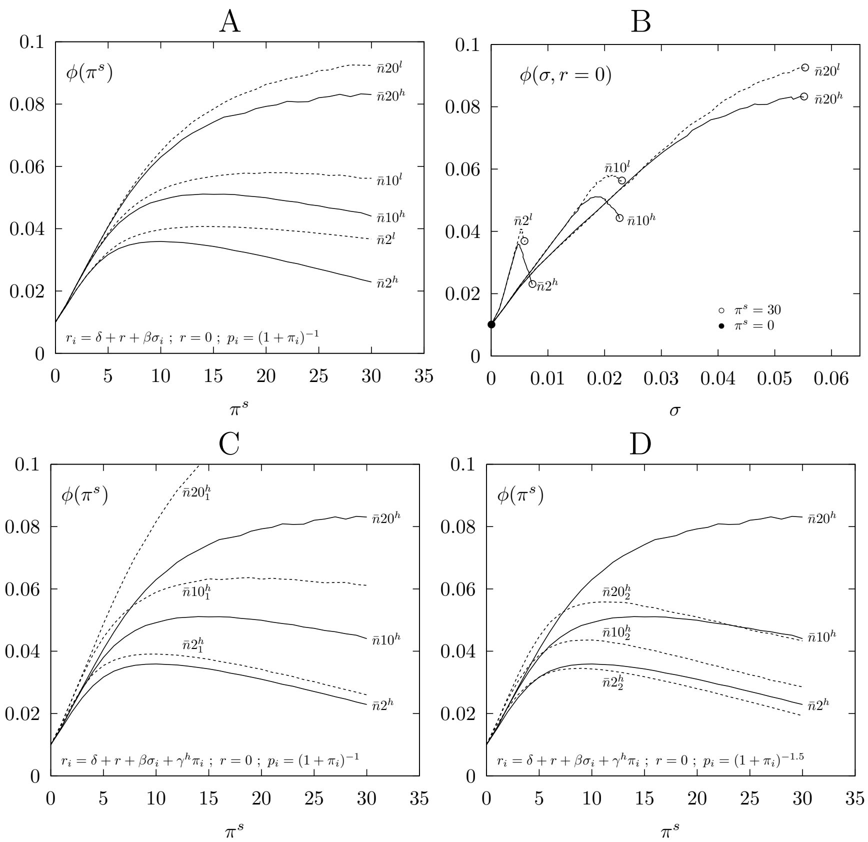

Figure 8: average revenue of the lending portfolio as a function of π^s (graphs A, C and D) and σ (graph B)

as the efficient frontier of portfolio theory.[12] π^s increases moving from left to right along the lines. These representation also shows that lending to entrepreneurs with too high π_i increases the loan portfolio risk while lowering its average return.

Recall that graphs 8A and 8B refer to a situation in which the financial market is an option for entrepreneurs, in fact we have used the "augmented" capital market line (equation 5) to obtain them. If entrepreneurs have serious difficulties in accessing financial markets, the bank can charge an additional interest rate spread to smooth out the effect of losses. Assuming the bank

[12]By using a bank objective function (Monti, 1972), this result could be used to reach the bank optimal solution. We leave this further investigation for future research.

can observe only π_i and that asymmetric information prevents a precise identification of b_i, we allow the bank to set the interest rate according to the following rule:

$$r_i = \delta + r + \beta\sigma_i + \gamma\pi_i. \tag{12}$$

Provided that asymmetric information does not prevent the bank to know the distribution of the b_is, we can set the last term in equation (12) ($\gamma\pi_i$) as the average loss suffered by the bank when π_i is observed. For the levels of $\mathcal{B}_1$ and $\mathcal{B}_2$ we consider, the average loss is very close to the average of the $\mathcal{B}(\mathcal{B}_1,\mathcal{B}_2)\pi_i$ random variable. We thus set $\gamma = \mathcal{B}_1/(\mathcal{B}_1+\mathcal{B}_2)$.

Figure 8C shows how the average revenue of the lending portfolio changes when the bank charges an additional interest rate spread. This new pricing strategy is most effective if $\bar{n}$ is high while it slightly increases the average revenue if $\bar{n}$ is low.

As hinted above, if the bank solves asymmetric information, the probability of projects being successful cannot decrease very fast with π_i. We show in Appendix A that, if the functional form $p_i = (1+\pi_i)^{-\alpha}$ is used, the condition $0 < \alpha \leq 1$ must hold to make both revenue and risk increase with π_i (conditions that characterize a system where asymmetric information is eliminated by the bank). Figure 8D shows the adverse consequences of facing $\alpha > 1$ due to asymmetric information. The chart highlights that an increase of α affects heavily the shape of the average return: the attainment of high target levels of revenue is now precluded even at high levels of $\bar{n}$. Lending to projects with an higher π_i loses its effectiveness as a device to increase the average return of the lending portfolio even if the bank asks for an additional interest rate spread (the dashed line labeled $\bar{n}20_2^h$ has a maximum).

6. Discussion

In this section we discuss a number of issues arising from our investigations by pointing out some critical points, possible future extensions and the relationship with existing empirical literature.

Policy interest rate and bank profits. The identification of determinants of banks profit is a topic which is drawing attention (Lee and Chih, 2013; Lee et al., 2014). In our model, lowering the policy interest rate implies a decrease of the bank profit. This is straightforward because we have assumed a null interest rate on deposits. One could argue that this profit reduction does not happen because in the real world the bank sets the interest rates on loans and deposits by practicing respectively a mark-up and a mark-down on the policy rate. In this way active and passive interest rates shift by the same amount and no effect should be observed on the bank profit. However this reasoning needs a careful assessment at low levels of the policy interest rate because the interest rate on deposits cannot be negative. Figure 9 can be

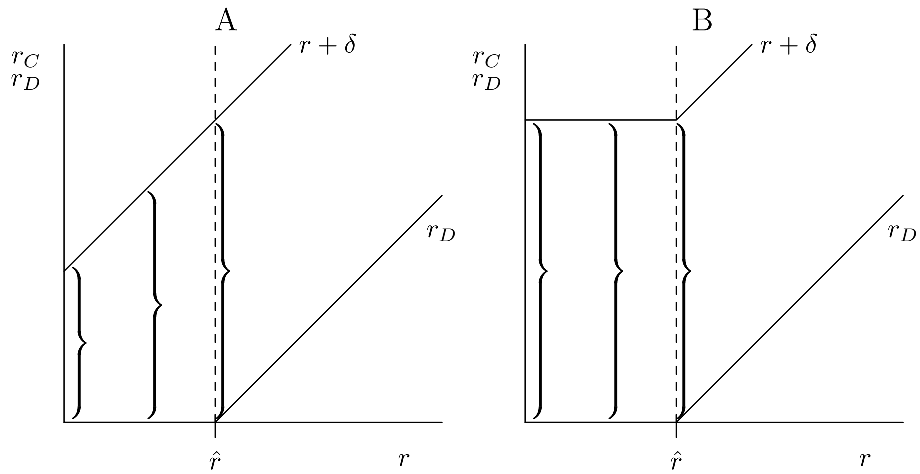

Figure 9: interest rates mark up and bank profit. $r_C = r + \delta$ is the interest rate on riskless borrowers; r_D interest rate on deposits. The size of the braces is proportional to the bank profit.

used to discuss this point. Note that in the figure, the size of the braces is proportional to the bank profit. If the bank keeps the mark up on loans constant like in figure 9A, there exists a positive interest rate below which its profit decreases ($\hat{r}$ in the figure). This is particularly important in this framework because we deal explicitly with low interest rate levels. A way to avoid profit reduction is to keep the interest rate on loans constant when the policy interest rate is below $\hat{r}$ like in figure 9B. If one believes in the validity of the latter solution, s/he has also to admit that monetary policy is ineffective through the interest rate channel because the reduction of the policy interest rate does not translate in a reduction of the interest rates for borrowers. On the contrary, if one believes the downward movement of the policy interest rate positively affects the situation of firms by lowering the rate at which they borrow, s/he should endorse the idea that a decrease of the policy interest rate below a given level causes a reduction of the bank profit. This paper adopts the latter argument.

Extending the framework. A more general discussion could be based on the observation that analyzing the bank liquidity is a "thorny" task, for example because it is affected by a number of other variables than the interest rate influx. In the model presented above we analyze a "neutral" benchmark case. For example, we assumed that deposits and loans have the same dynamics, so that they do not significantly affect bank liquidity. The extension to different situation is straightforward.

Our setting is also neutral in that the profitability threshold s, the liquidity threshold l and the minimum projects revenue ρ are kept constant in time and equal to each other. The analysis can be easily performed for different values of these parameters. A possible extension is to let s and l be state dependent and to make them evolving over time by calibrating their correlation with the business cycle. Concerning this point it would be interesting to analyze the role of capital adequacy standards already mentioned in the introduction. We showed that a decrease of the policy interest rate (other things being equal) increases σ^s, so that the average riskiness of bank activity increases. However, we have to take into account that capital adequacy rules require an increasing relationship between banks capital and the riskiness of their assets. More formally, we could let the γ parameter (which is used in equation 2) depend on a measure of the loan portfolio risk, $\gamma(\sigma^s)$ with $d\gamma/d\sigma^s > 0$. As a consequence, being $s = w + \gamma(\sigma^s)e$, a decrease in the policy interest rate causes an increase in σ^s, that in turn brings to an increase in s (this effect can be found in a recent model by Hitoshi, 2010).

Financial innovation. A largely debated question concerns the role of financial innovations for the stability of the financial system. This aspect can be analyzed in an extension of our model. Indeed financial innovation can be thought of as a new opportunity for the bank to allocate funds in an additional asset. In our model, all the opportunities available to the bank are described by the probability distribution p_{π_i}. There is a type of financial innovations that have the potentialities to worsen the overall banking system liquidity situation by making it possible for a bank to take additional risk. They are those that widen the support of p_{π_i}, allowing higher levels of π_i that were unaccessible before.[13] Collateralized Debt Obligations (CDOs),

[13] A financial innovation in our framework can be thought of as modifying the "efficient frontier" of the banks opportunities. The type of innovations that are more dangerous from our point of view are those which "stretch" the frontier to north-est.

for example, could be viewed as a kind of these innovations; Haensel and Krahnen (2007) provide evidence that CDOs "tends to raise the systematic risk of the issuing bank".

Relationship with empirical evidence. The investigation carried out in this paper is strictly theoretical. However, our findings are supported by the existing empirical evidence. Lang and Nakamura (1995) provide evidence from the federal Reserve's Survey on Terms of Bank Lending that a "flight to quality" (an increase of the percentage of safe borrowers) is observed after a tightening of the monetary policy. Studying the Japanese case, Watanabe finds that "a large loss of bank capital caused by the regulator's tougher policy towards banks in F[iscal]Y[ear] 1997 not only induced the contraction of the bank lending supply but, more importantly, caused the banks' reallocation of their lending supply to unhealthy industries with a higher concentration of non-performing loans (*evergreening*)" (Watanabe, 2010, p. 135). By using data from Italian banks, Albertazzi and Marchetti (2010) find "[...] evidence of a contraction of credit supply, associated to low bank capitalization and scarce liquidity, over the 6-month period following Lehman' bankruptcy". They also find "[...]that larger less-capitalized banks have reallocated their credit away from riskier firms. Quite strikingly, this 'flight to quality' has not been observed for smaller less-capitalized banks". The last statement is compatible with *evergreening* practices in larger less-capitalized banks in the period before the financial turmoil.

The cited empirical investigations could be interpreted as providing evidence for the existence of a positive relationship between banks health (in terms of profit an capitalization) and the quality of their loan portfolio. More generally, this strengthen the recent evidence that, in order to correctly evaluate the effects of monetary policy through the lending channel, bank risk conditions should be considered beside traditional indicators (Altunbas et al., 2010).

7. Conclusions

In this paper we take up the claim for the need of a more detailed knowledge of the bank behavior when the interest rate is low.

By relying on a computational approach, the paper focuses on the lending activity of commercial banks. The dynamics generated by the turnover of heterogeneous loans in the bank portfolio are analyzed considering the effects of the policy interest rate movements. In our model, a decrease of the policy interest rate implies a reduction of interest rate charged on each loan which in turn shrinks the bank average rate of return. If the bank can solve asymmetric information, the fall in the average return can be balanced out by changing the composition of the lending portfolio financing more risky entrepreneurs who ensure a higher return. However this comes at the cost of partially loosing control on the composition of the lending portfolio as well as sluggishness in approaching the new steady situation. When the bank cannot solve asymmetric information, our model confirms a result already obtained in the credit rationing literature: the average return from the lending activity has a maximum. In our model we can identify how the average return and risk obtained from the lending activity change with bank variables such as the interest rate charged on each loan, the project life length and the composition of the loan portfolio.

The proposed model give the possibility to identify the set of projects a bank finances under various economic conditions. Furthermore, it allows to evaluate the dynamic of some important variables implied by these choices. This paper also focuses on the dynamics of the bank liquidity; its analysis allows a precise understanding of the events following a change in the conditions faced by the bank and reveals unexpected features such as the increase of volatility clustering when the loan duration is shortened.

Beyond the cited microeconomic aspects, the model highlights mechanisms that operates in the transmission of macroeconomic policies impulses such as the movement of the reference interest rate decided by the central bank. In this respect, our work could be used as a component of a more wide computational model aiming at modeling the macroeconomy using a bottom up approach. We strongly believe this approach allows for a *detailed knowledge* of the state of the economic system and is thus very important for assessing pros and cons of policy actions.

Appendix A. Positive relationship between average and standard deviation

We recall that a Bernoulli random variable taking values $\{u, s\}$ with probability $\{1 - p_i, p_i\}$, where u is the result in case the success is not realized and s when it is, has a mean equal to

$$\mu_i = sp_i + u(1 - p_i)$$

and the variance is

$$\sigma_i^2 = (s - \mu_i)^2 p_i + (u - \mu_i)^2 (1 - p_i)$$

that, after some algebra, reduces to

$$\sigma_i^2 = (s - u)^2 p_i (1 - p_i)$$

If Π_i is a Bernoulli random variable taking values $\{0, \pi_i\}$ with probability $\{1 - p_i, p_i\}$ where $p_i = (1 + \pi_i)^{-\alpha}$, the average is

$$\mu_i = \pi_i (1 + \pi_i)^{-\alpha}.$$

The derivative is

$$\frac{d\mu_i}{d\pi_i} = (1 + \pi_i)^{-\alpha} - \alpha \pi_i (1 + \pi_i)^{-\alpha - 1} = (1 + \pi_i)^{-\alpha} [1 - \alpha \pi_i (1 + \pi_i)^{-1}]$$

that is positive for all non negative π_i as long as $\alpha \leq 1$.

The variance is

$$\sigma_i^2 = \pi_i^2 (1 + \pi_i)^{-\alpha} [1 - (1 + \pi_i)^{-\alpha}] = \pi_i^2 (1 + \pi_i)^{-\alpha} - \pi_i^2 (1 + \pi_i)^{-2\alpha} =$$

$$\pi_i^2 (1 + \pi_i)^{-2\alpha} [(1 + \pi_i)^{\alpha} - 1] = \mu_i^2 [(1 + \pi_i)^{\alpha} - 1].$$

The derivative is

$$\frac{d\sigma_i^2}{d\pi_i} = 2\mu_i \frac{d\mu_i}{d\pi_i} [(1 + \pi_i)^{\alpha} - 1] + \mu_i^2 \alpha (1 + \pi_i)^{\alpha - 1}$$

that is positive for $\alpha > 0$.

Appendix B. Mean and variance of a single project in the multi-period case

In the text we have

$$\mu(\mathcal{I}_{i,n,t}) = I_{i,n,t} p_i,$$
$$\sigma^2(\mathcal{I}_{i,n,t}) = I_{i,n,t}^2 p_i (1 - p_i).$$

substituting equation (6) they become

$$\mu(\mathcal{I}_{i,n,t}) = \sum_{k=1}^{n} \binom{n}{k} r_i^k p_i$$

$$\sigma^2(\mathcal{I}_{i,n,t}) = \left[\sum_{k=1}^{n} \binom{n}{k} r_i^k\right]^2 p_i(1-p_i).$$

Form the previous appendix we know that σ_i is a function of π_i and consequently π_i can be expressed as a function of σ_i which implies that the probability $p_i = (1-\pi_i)^{-1}$ is also a function of σ_i.

So we can write

$$\mu(\mathcal{I}_{i,n,t}) = \mu_{i,n,t}(r_i, \sigma_i)$$
$$\sigma^2(\mathcal{I}_{i,n,t}) = \sigma^2_{i,n,t}(r_i, \sigma_i).$$

Finally, r_i is determined by using equation (5) so that, given δ and β, it depends on the policy interest rate r and the standard deviation of the single project σ_i. Consequently we can write

$$\mu(\mathcal{I}_{i,n,t}) = \mu_{i,n,t}(r, \sigma_i)$$
$$\sigma^2(\mathcal{I}_{i,n,t}) = \sigma^2_{i,n,t}(r, \sigma_i).$$

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