Equitable and sustainable well-being over time: a functional approach

Il benessere equo-sostenibile nel tempo: un approccio funzionale

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Abstract In recent years, studies on well-being have risen to prominence and it has been widely accepted that well-being should be considered a multidimensional phenomenon, beyond its economic feature. In vein of multidimensionality, Italy launched the Equitable and Sustainable well-being (BES) as the official framework for measuring well-being. Exploiting the availability of BES indicators over different sequential years, our research is aimed at computing well-being efficiencies of the Italian province capital cities. The procedure employed in this paper integrates the Malmquist DEA scores with the diversity profile to rank the cities. This joint approach has the benefit to be recast into a functional framework.
Abstract *Negli ultimi anni sono saliti alla ribalta gli studi sul benessere ed è stato ampiamente accettato che il benessere dovrebbe essere considerato un fenomeno multidimensionale, al di là della sua caratteristica economica. All'insegna della multidimensionalità, l'Italia ha lanciato il benessere equo e sostenibile (BES) come quadro ufficiale per misurare il benessere. Sfruttando la disponibilità di indicatori BES su diversi anni consecutivi, la nostra ricerca è finalizzata al calcolo delle efficienze dei capoluoghi di provincia italiani nel promuovere il benessere. La procedura utilizzata in questo lavoro integra i punteggi degli indici di Malmquist con il profilo di diversità per classificare le città. Questo approccio congiunto ha il vantaggio di poter essere ricollocato all'interno dell'analisi funzionale.*

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1 Introduction

Over the recent decades, the interest in the well-being measurement is constantly increased worldwide. In reviewing the recent trends on this topic,we acknowledge a more elevated scientific standard and a more rigor of approaches proposed for national and international comparisons of well-being. One major theme in this, has been the recognition of the limitations of macroeconomic indicators, Gross Domestic Product (GDP) in particular, as proxies for describing and measuring the factors affecting people's lives [8]. Accordingly, there has been a shift toward a conceptualization of well-being as a multidimensional phenomenon, as emerged in various arenas including the United Nations, the OECD, and numerous national, regional, and local governments [2, 12, 15]. Focusing on the Italian scenario, we consider, in this paper, the Equitable and Sustainable Well-Being Index, whose Italian acronym, used hereafter, is BES, elaborated by a joint initiative of the National Committee for Economy and Labour (CNEL) and the ISTAT, with the final aim of developing a collective definition of progress in the Italian society and producing a shared set of indicators of the most relevant economic, social and environmental domains. The theoretical framework adopted by Istat within the BES project can be deemed as the adjusted version, for the Italian context, of the conceptual model published by [9]. Alongside with the national experience, Istat has implemented the BES framework at local level and launched UrBES project to measure well-being at an urban level. Exploiting the availability of UrBES indicators over different sequential years, the core idea of this research is to compare over time the efficiencies of Italian provinces capital cities in producing equitable and sustainable well-being, by employing entropy based Malmquist indices [11] recast into a Functional Data Analysis (FDA) approach [14]. The remainder of this paper is structured as follows. In Section 2, we provide details of the theoretical background of the Malmquist based entropy procedure and its reformulation into a functional framework while in Section 3 we describe the urban well-being data.

2 Methodological framework

The analysis considers different stages. Firstly, we rely on Mazziotta-Pareto's method of penalties [16] to summarize the statistical indicators into the domains individuated by the UrBes project. Subsequently, within the Data Envelopment Analysis framework, we coupled Malmquist index with Shannon's entropy to represent the overall change in the efficiency of Italian provinces capital provinces in promoting well-being. Finally, the ranking of Italian Provinces capital cities is facilitated through a reformulation of the entropy based Malmquist procedure where a family of diversity indices is considered. In what follows, we provide the basics of both procedures.

2.1 Entropy based Malmquist Productive Index (MPI)

Data Envelopment Analysis (DEA), firstly introduced by Charnes et al. [1], is a non parametric approach originally developed to measure the relative efficiency of Decision Making Units (DMUs) within production context characterized by multiple inputs and outputs. The literature review reveals that during the last decades the scope of DEA has broadened considerably, with successful applications to social indicators (see, among others, [3]). A proper extension of DEA to the field of composite indicators implies to consider a DEA model with only outputs. To avoid the inconsistencies that arise in a model without inputs, a single unitary input has to be included as in [10]. A very useful tool, in DEA, to calculate the relative performance of a DMU at different periods of time is the Malmquist Productivity Index (MPI). Suppose that there are n DMUs to be evaluated in lights of m input and s output. We denote with X_j and Y_j the input and output vector at DMU_j and θ it is a measure of technical efficiency of DMU_0. The MPI allows to calculate the relative performance at different periods of time and is a cross measure, which considers the production of a DMU as the efficient frontier built in the next or previous instant:

$$MPI_0 = \left[\frac{\theta^t x_0^t y_0^t}{\theta^t x_0^{t+1} y_0^{t+1}} \frac{\theta^{t+1} x_0^t y_0^t}{\theta^{t+1} x_0^{t+1} y_0^{t+1}} \right]^{\frac{1}{2}} \tag{1}$$

where $\theta^t x_0^t y_0^t$ and $\theta^t x_0^{t+1} y_0^{t+1}$, $\theta^{t+1} x_0^t y_0^t$ and $\theta^{t+1} x_0^{t+1} y_0^{t+1}$ are respectively the input oriented efficiency measures of DMU_0 at period t and t+1. MPI_0 measures the productivity change between periods t and t+1 [6]. The overall tendency in productivity changes of DMUs over time periods is traditionally obtained through the average of productivity indices of sequential times, implicitly assuming that all sectional indices equally affect the level of productivity. In this work, to eliminate the equal-weight effect, we take into account Fallahnejad's algorithm [7] who applies the Shannon's entropy to obtain more objective weights in aggregating the Malmquist productivity indices.

For the N DMU_s and the relative inputs and outputs for k+1 times $(t_0, t_1, ..., t_k)$, the Malmquist indices at two sequential times t and t+1 can be computed. They are denoted by $MPI_{jt}, \quad j = 1 \ldots n, \quad t = 1, \ldots k$ and summarised as in Table 1.

The weighted Malmquist productivity index for each DMUj $(WMPI_j)$ is obtained via some established steps, as detailed below.

In **Step 1**, the matrix in Table 1 is normalized dividing each element of the column by the sum of column:

$$p_{jt} = \frac{MPI_{jt}}{\sum_{j=1}^{n} MPI_{jt}} \tag{2}$$

Table 1 MPI matrix

	MPI1	MPI2	....	MPIk
DMU_1	MPI_{11}	MPI_{12}	....	MPI_{1k}
DMU_2	MPI_{21}	MPI_{22}	....	MPI_{2k}
DMU_3	MPI_{31}	MPI_{32}	....	MPI_{3k}
....	....	....	....	....
....	....	....	....	....
DMU_n	MPI_{n1}	MPI_{n2}	....	MPI_{nk}

The above normalization allows to eliminate anomalies due to different measurement units and scales.

In **Step 2**, the entropy h_t for all normalized MPI is calculated as:

$$h_t = -h_0 \sum_{j=1}^{n} p_{jt} ln p_{ij} \tag{3}$$

Step 3 involves the computation of the degree of diversification, defined as:

$$d_t = 1 - h_t, \quad t = 1, \dots k \tag{4}$$

If the values of the productivity of the DMUs are close, the weight of a given year can be considered weak in the aggregating process.

In **Step 4** the degree of importance of MPI at time t is obtained by setting

$$w_t = \frac{d_t}{\sum_{s=1}^{k} d_s}, \quad t = 1, \dots k \tag{5}$$

where $\sum_{t=1}^{k} w_t = 1$.
In **step 5**, the weighted MPI is calculated as:

$$WMPI_j = \sum_{t=1}^{k} w_t MPI_{jt}, \quad j = 1, \dots n. \tag{6}$$

2.2 Functional Malmquist Productive Index (FMPI)

As shown in Section 2, the weighted productivity index for each DMUs has been obtained by exploiting the Shannon's entropy method. In our context, the Shannon's entropy can be regarded as a diversity measure of DMUs. Actually, different indices could have been taken into account to delineate such diversity which could have lead to different rankings. To overcome this limitation, a possible solution is to a consider a family of diversity indices, dependent upon a single continuous variable, that depict graphically a diversity profile. Formally, to evaluate the DMUs diversity, we consider the β diversity profile proposed by Patil and Taille [13] in the environmental setting:

$$\Delta_\beta = \sum_{i=1}^{n} \frac{(1-p_i)^\beta}{\beta} p_i \quad \beta \geq -1 \tag{7}$$

In Eq.7, $\sum_{i=1}^{n} \frac{(1-p_i)^\beta}{\beta}$ can be interpreted as a measure of relevance of each DMUs for the change in productivity over time expressed by each MPI and p_i is defined as above. Note that Shannon's entropy is a particular case of β diversity profile, resulting when $\beta \to 0$. The benefit of using Δ_β instead of the single Shannon's entropy index, as suggested in the procedure reviewed in the previous section, is to rely on a larger spectrum of diversity measures, obtained varying β from -1 to 1. It follows that Δ_β is a convex curve that can be studied in a functional framework. In this work, our proposal is to rewrite the Steps 3-5 of the original Fallahnejad's procedure by replacing h_t with Δ_β. Specifically, in the Step 4, w_t is reformulated as $w_t(\beta) = \frac{d_t(\beta)}{\sum_{s=1}^{k} d_s(\beta)} \quad \forall \beta$, whereas in Step 5, $WMPI_j$ is consequently replaced by $WMPI_j(\beta) = \sum_{t=1}^{k} w_t(\beta) MPI_{jt} \quad \forall \beta$. Thus, for each DMUs we are able to obtain a functional weighted MPI. The comparative analysis of the weighted MPI curves and the consequently ranking of DMUs are achieved by means of functional tools (analysis of derivatives, radius of curvature and length of a curve), as suggested in [4].

3 Application to UrBes data

To meet the statistical information needs of local communities, Istat designed Bes at local level in cooperation with local authorities, investigating the specific information needs of Italian Municipalities, Provinces and Metropolitan Cities and tuning a shared theoretical framework [5]. Bes measures at local level maintain a high level of quality and consistency with the Bes indicators system and constantly follow the evolution of the Bes framework. The set of indicators, illustrating the 12 domains relevant for the measurement of well-being, is updated and illustrated annually in the Bes report. For the application of procedure illustrated above, we refer to the Ur-Bes framework which appraises well-being by a great deal of variables. In 2020, the set of indicators has been expanded to 152 (it was 130 in previous editions), with a deep revision that takes into account the transformations that have characterised Italian society in the last decade, including those linked to the spread of the COVID-19 pandemic. Owing to the data unavailability and missing values, our analysis is restricted to 103 Province capital cities and takes into account eight out of twelve domains of the original Ur-Bes dataset. In particular, we focus on the following domains: “Health”, “Education and Training”, “Work and Life Balance”, “Economic well-being”, “Social Relationships”, “Security”, “Landscape and Cultural Heritage”, “Environment”.

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