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Corresponding Author: Professor Carlo Lucheroni, PhD

Corresponding Author's Institution: University of Camerino

First Author: Carlo Lucheroni, PhD

Order of Authors: Carlo Lucheroni, PhD; Carlo Mari

Abstract: In this paper we review and extend the stochastic LCOE portfolio theory, a mean-risk analysis of electricity generation investment portfolios, focusing on the distinction between risk and deviation risk measures in terms of risk distribution shaping. Using standard and more advanced stochastic optimization risk measures, we derive optimal portfolios in the case of fossil fuels only, and in the case which includes the nuclear asset, interpreted as a risk free asset useful to hedge and reduce LCOE dispersion around its mean, in a US market case study. Four CO2 price volatility scenarios are used to illustrate how the theory handles the impact of indirect correlation among different fuel technologies induced by CO2 costs on the determination of optimal portfolios.

Dear Editors,

we're submitting the revised version of the paper "Risk Shaping of Optimal Electricity Portfolios in the Stochastic LCOE Theory".

We hope that in the revised text and in the response letter we satisfactorily addressed and answered all comments made by the kind Reviewers.

Best regards,

Carlo Lucheroni and Carlo Mari

*Detailed Response to Reviewers

Response to Reviewers

We wish to thank the SI Guest Editor and Reviewer #2 for their kind and encouraging appreciation words.

Detailed responses to individual reviews and indications where we modified the text and outlined each change made, point by point, follow.

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*SI GE:

>I suggest to discuss further, for instance in the conclusions section, nuclear LCOE with respect to the construction time uncertain, which is well document with a recent case occurring in Finland. Also, the use of the results conveyed in this paper to frame analysis policy would be welcome.

-We modified the Conclusions Section, and partially rewrote it.

We explicitly discussed the modeling possibilities coming from taking into account uncertainty in construction times. In this case a statistics is needed, and it can be constructed by collecting data on operating reactors that were built and connected to the grid in the last years, say from late nineties to today, for as many reactors as possible. If the kind Reviewer refers himself to the Olkiluoto 3 reactor in the Finnish Olkiluoto plant, that reactor hasn't been completed, it is about 10 years late in the schedule, and cannot be included in such collection. In any case the Referee comment is very interesting and appropriate, because it confirms that construction times variability is a large source of cost uncertainty.
Commenting on the relationship between stochastic LCOE theory and policy issues is a complicated but very interesting challenge, which probably would deserve a paper for itself. In the Conclusions we just mentioned its relevance, and a possible use of the stochastic LCOE theory in CO2 policy assessment.

Reviewer #1: No report was timely delivered so the reviewer was uninvited.

Reviewer #2:

>The only thing I find missing in the analysis of price correlations between gas and coal. It seems to me, by looking at Figure 2, that these two prices have significant positive correlations that should not be neglected. Given that the model is a Monte Carlo simulation, an easy assessment of the role of correlation only requires modelling multidimensional Wiener processes instead of considering independent Wiener processes. Either this correlation could be estimated using statistical analysis or as with the standard deviation of carbon prices different values could be studied such as {0, 0.25, 0.5, 0.75, 1}.

-Admittedly, by looking at the curves by eye, one could have the impression that the two processes are correlated. Before deciding to model the coal and gas price processes as independent on each other we actually computed the correlation between them, which resulted very low, equal to -0.088. In effects, the eye is mislead by a sort of similar trends in the two curves, but the volatility of the gas curve is so large that it actually destroys any kind of correlation between the two. We apologize for not having explicitly included this measure in Table 2, which we have modified in the revised version. We also modified the text including explicitly in the discussion this parameter. We also explicitly added a mention to the fact that in the literature Hogue (2012) adopts a pair of uncorrelated geometric Brownian motions to describe coal and gas market price series.

Highlights (for review)

- Stochastic LCOE theory is a powerful tool to study generation portfolios in their asset composition.
- Stochastic LCOE theory can be used for generation portfolio optimization under different risk measures.
- CVaR Deviation is a risk measure particularly appropriate for this problem to account for asymmetric tail risk.
- The three risk sources taken into account in the model are coal, natural gas and CO_2 market prices.
- In the model, under these three risk sources, a nuclear asset can be used as a risk free asset.

Risk Shaping of Optimal Electricity Portfolios in the Stochastic LCOE Theory

Carlo Lucheroni *　　Carlo Mari †

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*School of Science and Technologies, University of Camerino, Via M. delle Carceri 9, 62032 Camerino, Italy. Email: carlo.lucheroni@unicam.it

†Dept. of Economics, University "G. d'Annunzio" of Chieti-Pescara, Viale Pindaro 42, 65127 Pescara, Italy. Email: carlo.mari@unich.it

Abstract

In this paper we review and extend the stochastic LCOE portfolio theory, a mean-risk analysis of electricity generation investment portfolios, focusing on the distinction between risk and deviation risk measures in terms of risk distribution shaping. Using standard and more advanced stochastic optimization risk measures, we derive optimal portfolios in the case of fossil fuels only, and in the case which includes the nuclear asset, interpreted as a risk free asset useful to hedge and reduce LCOE dispersion around its mean, in a US market case study. Four CO_2 price volatility scenarios are used to illustrate how the theory handles the impact of indirect correlation among different fuel technologies induced by CO_2 costs on the determination of optimal portfolios.

1 Introduction

Mean-risk modern portfolio theory has been applied for the last ten years to the planning problem of the optimal choice of electricity generation investment portfolios. This line of research begun with a seminal paper by Awerbuch and Berger [1] in which a mean-variance approach was used for energy portfolios, i.e. portfolios invested in energy production technologies. In this way, that risk management and optimization approach was introduced in this research field. In a refinement of this work the optimal choice of electricity generation portfolios was again explored within a mean-variance approach but now using Net Present Value (NPV), a deterministic quantity that projects expected future costs and revenues back to the evaluation time. Specifically, in Roques et al. [2] and Roques et al. [3] NPV was turned stochastic by making costs and revenues stochastic. All distributions used there were gaussian, and Monte Carlo simulation techniques were used in order to determine the probability distributions of the stochastic NPV.

Another approach, in competition with NPV, is based on using LCOE (Levelized Cost Of Electricity), usually a deterministic quantity. The deterministic LCOE is affine to the deterministic NPV, because the LCOE is that generation cost projected back to the evaluation time that makes NPV equal to zero (see Mari [4] for a discussion, and references therein). Moreover, as it will be shown in this paper, in the deterministic case the portfolio which maximizes the NPV per MWh minimizes LCOE. Yet, a deterministic, typical LCOE theory doesn't include risk. In analogy to the stochastic NPV approach, the LCOE was turned stochastic in Mari [5], where a mean-variance LCOE theory for generation portfolios selection was developed. Differently from the NPV case, the approach developed by Mari [5] is based on risk management of generation costs, and does not include the risk of revenues, which is the risk associated with uncertain electricity market prices. The reason of this choice was that present generation costs and risks are expected to impact on future electricity prices through technology evolution and demand/supply interplay, so that a future dependence between electricity prices and generation cost risks is very difficult to assess and disentangle from current data in a model developed in the present. A risk analysis solely based on costs and not including revenues seems therefore a much more firmly grounded choice than that of using both revenues and costs.

Because it was pointed out that financial risk in the electric energy sector is mainly due to the high volatility of fossil fuels and CO_2 prices, which evolve in time in an unpredictable way (García-Martos et al. [6]), in Mari [5] and in this paper three sources of financial risks are taken into account, namely the stochastic dynamics of coal and gas market prices and the stochastic dynamics of CO2 prices. The main tenet of the stochastic LCOE theory is that the joint effect of fossil fuel prices volatility and the CO2 price volatility can induce rational electricity producers to diversify their baseload generation portfolios in order to minimize the impact of such factors on the cost risk of electricity production, and this is shown by numerically solving a portfolio optimization problem. It can be anticipated here that the risk-reducing diversification is not trivial because the portfolio two main components, i.e. the gas and coal technologies, are coupled through the CO2 price process. Moreover, nuclear technology, i.e. a CO2 free asset which is not risky from a CO2 point of view, can be introduced in the generation mix as a further, risk free asset, in order to even more mitigate costs risk, with the positive side-effect of reducing the fossil fuel component, i.e. the carbon emitting component.

The discussion in the paper will take advantage from the use of a recently introduced risk measure, easy to compute by linear stochastic optimization. In fact, in general, stochastic LCOE distributions are not gaussian, having asymmetric long thick tails. The standard Markowitz mean-variance analysis [7] can then be considered only as a starting approach for an accurate stochastic LCOE theory, which has hence to be developed with more suitable risk measures. VaR (Value at Risk) and CVaR (Conditional Value at Risk) could be chosen as the obvious candidates for this extension (Sarikalyn et al. [8]). Yet, it turns out that they are not the best choice for this problem, because they are not dispersion measures (unlike standard deviation), and are not suitable to play the same role as the role that standard deviation plays in a Markowitz approach. It will be shown that CVaR Deviation (CVaRD) (Rockafellar et al. [9]), a dispersion measure never used before in this context, and very seldom used in general, provides a much better alternative, and turns out perfectly suited for this application.

In this paper, 1) besides extending the Markowitz analysis of the stochastic LCOE theory proposed by Mari [5] by using VaR, CVaR and CVaRD as risk and deviation measures, 2) we also improve the underlying stochastic cost model to account for more suitable stochastic processes with respect to the geometric Brownian motion used in [5] to describe the evolution of fossil fuels market prices. This is mainly necessary for the gas price process used in the following case study of the US market, for which in the data there is evidence of mean reversion and jumps, an aspect which is not taken into account in geometric Brownian motion models.

More specifically, after this Introduction, in Sec.2 we review the stochastic LCOE theory. In Sec.3 we set up new and more realistic dynamic stochastic equations for the cost model of the theory. Specifically, we propose a more accurate description of fossil fuels market prices using a mean-reverting jump-diffusion process to describe the dynamics of gas prices and a geometric brownian motion to model coal prices. We estimate both processes for the case of the US market, for which abundant data are available. Capital costs and other many costs included in our analysis are referred to US market too. In order to make our case study more complete, we then analyze four different CO_2 price evolution scenarios in order to show CO_2 indirect effects on single-fuel LCOEs cross-correlation, and the implications of this correlation on the portfolio selection, the central mechanism of the stochastic LCOE theory. We will assume that CO_2 prices evolve in time according to a geometric Brownian motion and the four different scenarios refer to different volatility values of the CO_2 price process. In Sec.4 we discuss a choice of available risk and deviation measures. In Sec.5, using the estimated model, we study those gas and coal portfolios which are optimal under four risk and deviation measures, namely standard deviation, VaR, CVaR and CVaRD. We show how CVaRD improves, in regard to the theory proposed by Mari [5], the shape of the portfolios LCOE distribution when aversion to asymmetric tail risk is considered. In Sec.6 we include the nuclear asset as a risk free asset and hedging instrument, selecting deviation optimal portfolios using a deviation-expected LCOE plane, and discuss the important role of a risk free asset in the portfolio selection problem. In Sec.7 we conclude.

2 The Stochastic LCOE Theory

In this Section we briefly recapitulate the classical deterministic definition of LCOE, and show how to extend it to a stochastic setting.

Consider a project of an electricity generating plant, seen as a cash flow stream on a yearly timetable n ($n = -N, \dots, 0, \dots, M$) for which $n = -N < 0$ is the construction starting time, $n = 0$ is the end of construction time, the evaluation time and the operations starting time. $M \geq 1$ is the end of operations time (see Fig.1).

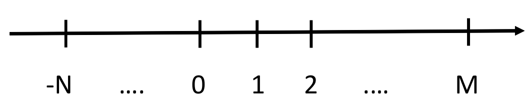

Figure 1: Project timeline.

Classically, the Levelized Cost of Electricity (LCOE, or "levelized cost" LC in short) is defined as that nonnegative price $P^{\mathrm{LC,x}}$ (assumed constant in time, and expressed in real money units) of the electricity produced by a specific generation technology x which makes the present value of expected revenues from electricity sales equal to the present value (PV) of all expected costs met during the plant life-cycle (investment costs, operating costs, incremental capital costs, decommissioning costs and carbon charges when due). The

LCOE represents the generating costs at the plant level (busbar costs) and does't include transmission and distribution costs and all possible network infrastructures adjustments [16]. To determine the LCOE, PVs are computed by using a discount rate that must provide equity investors the adequate return for the assumed risk. In general, this return is quantified by the Weighted Average Cost of Capital (WACC) which accounts for the possibility that a given project can be financed by a mix of equity and debt [11]. Assessing the LCOE through the WACC method allows one to include the level of risk perceived by investors (both equity holders and bondholders) through the debt fraction of the investment in the discount rate. The LCOE is then a break-even reference unitary cost, internal to the project, to be compared with the expected market electricity price. Given a set of technologies, $P^{\mathrm{LC,x}}$ is also useful to compare among each other the levelized costs of generation alternatives. Regarding the set of alternatives, this paper will focus only on the baseload production of electricity obtained from nuclear energy and two fossil sources, coal and natural gas.

The LCOE for a specific technology x is evaluated at $n = 0$ after equating present values of revenues and costs, by solving for $P^{\mathrm{LC,x}}$ from

$$\sum_{n=1}^{M}(P^{\mathrm{LC,x}}Q_n^x)(1+i)^{n-n_b}F_{0,n} = \sum_{n=1}^{M}(C_n^x + T_n^x)F_{0,n} + I_0^x. \tag{1}$$

In the l.h.s. of Eq.(1), Q_n^x denotes the amount of electricity produced during each period and it will be assumed constant (as $Q^x = NW^x \times 8760 \times CF^x$, where NW^x is the nominal capacity of the plant and CF^x the Capacity Factor of that plant), i is the expected yearly inflation rate (since the LCOE has to be expressed in real terms), n_b refers to the base year, and

$$F_{0,n} = \frac{1}{(1+WACC)^n} \tag{2}$$

is the discount factor in the WACC evaluation scheme, where $WACC$ is kept constant for the whole life of the project. The technology label x can take the values nu (nuclear), co (coal), ga (gas). In the r.h.s. of Eq.(1) C_n^x denotes expected nominal operating expenses which are incurred throughout the operational life of the plant. C_n^x includes (fixed and variable) operation and maintenance (O&M) costs, fuel costs (not included in the variable O&M costs), radioactive wastes management costs and set-aside decommissioning funds in the case of nuclear energy. Fixed and variable O&M costs and fuel costs are computed using real escalation rates. With regard to fossil fuels, costs have to include carbon market costs, or carbon taxes, or abatement expenses, to account for carbon emissions costs. Yearly nominal tax liabilities

$$T_n^x = T_c(R_n^x - C_n^x - dep_n^x), \tag{3}$$

are computed by subtracting cost C_n^x and asset depreciation dep_n^x from sales revenues R_n, being T_c the tax rate. I_0^x is the pre-operations nominal investment, starting at $n = -N$ and ending at $n = 0$, but computed as a lump sum. I_0^x is computed in the following way. Denoting by $\bar{O}_n^x$ the real amount of the overnight cost allocated to year n (again with reference to Fig. 1), the nominal amount O_n^x at year n can be expressed as,

$$O_n^x = (1+i)^{n-n_b}\bar{O}_n^x \qquad n = -N, \cdots, -1, 0. \tag{4}$$

Then, within the WACC approach,

$$I_0^x = O_{-N}^x(1+WACC)^N + \cdots + O_{-1}^x(1+WACC) + O_0^x. \tag{5}$$

Eq.(1) has a very interesting structure, which is found also in other Economics contexts[1]. Since revenues have the form

$$R_n^x = P^{\mathrm{LC,x}}\, Q^x(1+i)^{n-n_b}, \tag{6}$$

using Eq.(3) in Eq.(1), we get the LCOE valuation formula

$$P^{\mathrm{LC,x}} = \frac{\sum_{n=1}^M C_n^x F_{0,n}}{Q^x \sum_{n=1}^M (1+i)^{n-n_b} F_{0,n}} + \frac{I_0^x - T_c \sum_{n=1}^M dep_n^x F_{0,n}}{(1-T_c)Q^x \sum_{n=1}^M (1+i)^{n-n_b} F_{0,n}}. \tag{7}$$

Eq.(7) is valid for a single-technology (labeled by x) project. For a multi-technology project, i.e. a portfolio of technologies, the total price P^{LC} is the sum over technology indices

$$P^{\mathrm{LC,w}} = \sum_x P^{\mathrm{LC,x}} \frac{Q^x}{Q^{\mathrm{TOT}}} = \sum_x w^x P^{\mathrm{LC,x}}, \tag{8}$$

[1] $P^{\mathrm{LC,x}}$ inside the l.h.s. can be interpreted as the current time equivalent of the r.h.s., the overall present value of costs, once the equation is solved for $P^{\mathrm{LC,x}}$. This structure was for example discussed in the context of growth theory in Mirrlees and Stern [12], which in turn based themselves on Rothschild and Stiglitz [13], where for the first time a quantity analog to $P^{\mathrm{LC,x}}$ was interpreted as a Balanced Growth Equivalent (BGE), and the r.h.s. term as the welfare level.

where $Q^{\rm TOT} = \sum_x Q^x$, and

$$w^x = \frac{Q^x}{Q^{\rm TOT}} \tag{9}$$

is the weight of technology x in the portfolio. When Q^x is expressed in MWh and prices are in dollars, $P^{\rm LC,w}$ is expressed in real dollars per MWh and C^x_n, I^x_0, dep^x_n in nominal dollars.

Although the LCOE approach has been used as an alternative to more traditional evaluation techniques based on NPV, it should be noticed that the maximization of the NPV per unit of output (e.g., one MWh of electricity)

$$\text{NPV}_Q = \frac{\text{NPV}}{Q^{\rm TOT}}$$

as a choice criterion for selecting in the deterministic frame optimal portfolios, is anyway equivalent to the minimization of the LCOE [2]. To see this, let us recall that for a portfolio of technologies the NPV is defined as the PV of expected revenues from electricity sales at market prices minus the PV of expected costs, and it can be expressed as

$$\text{NPV} = (1-T_c)\left[Q^{\rm TOT}\sum_{n=1}^{M} P^e_n(1+i)^{n-n_b}F_{0,n} - \sum_{n=1}^{M}\sum_x \left(C^x_n F_{0,n} + \frac{I^x_0 - T_c dep^x_n F_{0,n}}{1-T_c}\right)\right], \tag{10}$$

where P^e_n is the expected market real price of electricity in the year n, or equivalently

$$\text{NPV} = (1-T_c)Q^{\rm TOT}\left[\sum_{n=1}^{M} P^e_n(1+i)^{n-n_b}F_{0,n} - \sum_{n=1}^{M}(1+i)^{n-n_b}F_{0,n}\sum_x \frac{Q^x}{Q^{\rm TOT}}P^{\rm LC,x}\right], \tag{11}$$

in which Eq.(7) and Eq.(8) have been used. Then, NPV_Q can be cast in the following useful two-terms form

$$\text{NPV}_Q = V_0 - V_1 P^{\rm LC,w}, \tag{12}$$

where $V_0 = (1-T_c)\sum_{n=1}^{M} P^e_n(1+i)^{n-n_b}F_{0,n}$ is independent (for baseload generation) from the portfolio composition and $V_1 = (1-T_c)\sum_{n=1}^{M}(1+i)^{n-n_b}F_{0,n}$ is a constant. Thus, the generation portfolio that maximizes NPV_Q is the portfolio that minimizes LCOE.

Tab.1 details all technical data and costs included in our analysis, for nuclear and fossil fuel technologies, denominated in US dollars referred to the base year 2012, i.e. in real dollars. Data shown in Tab.1 are collected from the "Annual Energy Outlook 2013"

[2] Notice that the relevant quantity for generation portfolio optimization is not the NPV itself, because doubling the size of a plant would double the NPV, which is not what is sought by portfolio optimization.

(AEO 2013) [14] as reported in "Updated Capital Cost Estimates for Utility Scale Electricity Generating Plants" [15] provided by the U.S. Energy Information Administration, integrated with data from "The Future of Nuclear Power" by the Massachusetts Institute of Technology [16] and its last update "Update of the MIT 2003 - Future of Nuclear Power" [?, 17]. In accordance to the Annual Energy Outlook 2013, we assume an expected inflation rate $i = 1.75\%$ per annum, and a tax rate $T_c = 37\%$. As a reference case, we adopt a nominal WACC rate of 8.5%, in agreement with the assumption of a real weighted average cost of capital of 6.6% adopted in "Levelized cost of new generation resources in the Annual Energy Outlook 2013" [18].

Inserting a deterministic operation costs sequence C_n^x in Eq.(7), assessed for example as a succession of expected values, generates a deterministic $P^{\mathrm{LC,x}}$. Promoting the sequence C_n^x to a stochastic process, due to a set of risky sources stochastic paths ω, makes a (time-independent) stochastic variable

$$P^{\mathrm{LC,x}}(\omega) \tag{13}$$

of the LC, with a distribution $p(P^{\mathrm{LC,x}})$, an expected value $\mu^{\mathrm{LC,x}}$ and a variance $(\sigma^{\mathrm{LC,x}})^2$. In what follows, we will refer to $P^{\mathrm{LC,x}}$ of Eq.(13) as "stochastic LCOE", or more simply LCOE or LC. As specified in Introduction, three sources of financial risks are taken into account, namely the stochastic dynamics of coal and gas market prices and the the stochastic dynamics of CO2 prices.

More specifically, stochastic dynamic processes for nominal fossil fuel prices, $X_n^x(\omega)$ ($x =$ 'co','ga'), and CO_2 price, $Z_n(\omega)$ (per ton), from 0 to M can be inserted in C_n^x, turning it into a stochastic process $C_n^x(\omega)$. In this case it is more convenient to rewrite Eq. (7) for $x =$ 'co','ga' as a linear combination of fuel and carbon contributions, to get

$$P^{\mathrm{LC,x}}(\omega) = A^x \sum_{n=1}^{M} X_n^x(\omega) F_{0,n} + B^x \sum_{n=1}^{M} Z_n(\omega) F_{0,n} + K^x, \tag{14}$$

where K^x is the deterministic component of the LC accounting for all residual terms in Eq.(7), different from fuel and CO_2 costs. Moreover, in Eq.(14)

$$A^x = \frac{H^x}{1000 \sum_{n=1}^{M} (1+i)^{n-n_b} F_{0,n}}, \tag{15}$$

	Units	Nuclear	Coal	Gas
Technology symbol		nu	co	ga
Capacity factor		90%	85%	87%
Heat rate	Btu/kWh	10452	8800	7050
Overnight cost	$/kW	5530	2934	917
Fixed O&M costs	$/kW/year	93.28	31.18	13.17
Variable O&M costs	mills/kWh	2.14	4.47	3.60
Fuel costs	$/mmBtu	0.74	stoch	stoch
CO_2 intensity	Kg-C/mmBtu	0	25.8	14.5
Waste fee	$/kWh	0.001	–	–
Decommissioning cost	$ million	750	–	–
O&M real escalation rate		1.0%	1.0%	1.0%
Fuel real escalation rate		0.5%	1.0%	2.0%
Construction period	# of years	6	4	3
Operations start		2018	2018	2018
Plant life	# of years	40	40	40
Depreciation scheme		MACRS,15	MACRS,20	MACRS,15

Table 1: Technical assumptions. All dollar amounts are in year 2012 dollars. Overnight costs are assumed to be uniformly distributed on the construction period. O&M stands for operation and maintenance. Mill stands for 1/1000 of a dollar. mmBTU stands for one million BTUs. Depreciation is developed according to the MACRS (Modified Accelerated Cost Recovery System) scheme as reproduced in Appendix 1. 'stoch' stands for stochastic.

$$B^x = \frac{S^x}{\sum_{n=1}^{M}(1+i)^{n-n_b}F_{0,n}}, \tag{16}$$

where H^x is the fuel heat rate and S_x is the CO_2 intensity (expressed in tCO_2/MWh). Declaring K^x deterministic in Eq. (14) implies that we assumed that other costs have a negligible variance w.r.t. fuel costs volatility. To be noticed that these other costs not only have a negligible variance, but in a gas fired plant about 75% of the generation cost depends on the cost of natural gas, and even if the volatility of coal prices is lower than the volatility of gas prices, in a coal fired plant coal costs are responsible for more than 35% of the generating costs [4]. A further aspect of Eq.(14) has to be highlighted. When $x =$ 'co','ga', notice that the second term of both $P^{\mathrm{LC,co}}(\omega)$ and $P^{\mathrm{LC,ga}}(\omega)$ is not zero and in both cases contains the same process $Z_n(\omega)$, making the levelized costs of coal and gas correlated. This correlation will show up in the variance of a coal and gas portfolio.

Under our three sources of risk hypothesis, we assume that $P^{\mathrm{LC,nu}}(\omega)$ follows a deterministic price path because electricity production from nuclear energy doesn't release CO_2. A nuclear plant can be seen, therefore, as a risk-free asset in an otherwise risky portfolio. In the following, this feature will be used to hedge the volatility of the LC due to fossil fuels and carbon prices volatility.

For a portfolio of technologies, each with weight w^x, the LC will be the sum

$$P^{\mathrm{LC,w}}(\omega) = \sum_x w^x P^{\mathrm{LC,x}}(\omega), \tag{17}$$

parametrically dependent on w. We will call $P^{\mathrm{LC,w}}(\omega)$ "portfolio LCOE". Its expectation is

$$\mu^{\mathrm{LC,w}} = E[P^{\mathrm{LC,w}}(\omega)] = \sum_x w^x \mu^{\mathrm{LC,x}} \tag{18}$$

where $\mu^{\mathrm{LC,x}} = E[P^{\mathrm{LC,x}}(\omega)]$. Clearly, its variance

$$(\sigma^{\mathrm{LC,w}})^2 = E[(P^{\mathrm{LC,w}}(\omega) - \mu^{\mathrm{LC,w}})^2] \tag{19}$$

will not in general be equal to the weighted sum of the component variances $(\sigma^{\mathrm{LC,x}})^2$. Eq. (17) is then useful because it clearly takes into account the fact that the levelized unit cost of generating electricity is very sensitive not only to fuel and CO_2 prices volatilities, but also to the way these volatilities interact.

In this stochastic frame Eq.(12) is still valid, just considering that the V_0 term on the r.h.s. will contain an additional source of risk, namely the market electricity price P^e risk, even though not dependent on portfolio composition. A possible co-dependence of stochastic P^e with stochastic $P^{\mathrm{LC,w}}$ could be taken into consideration, but since current generation costs and risks will impact on future electricity prices through technology evolution and

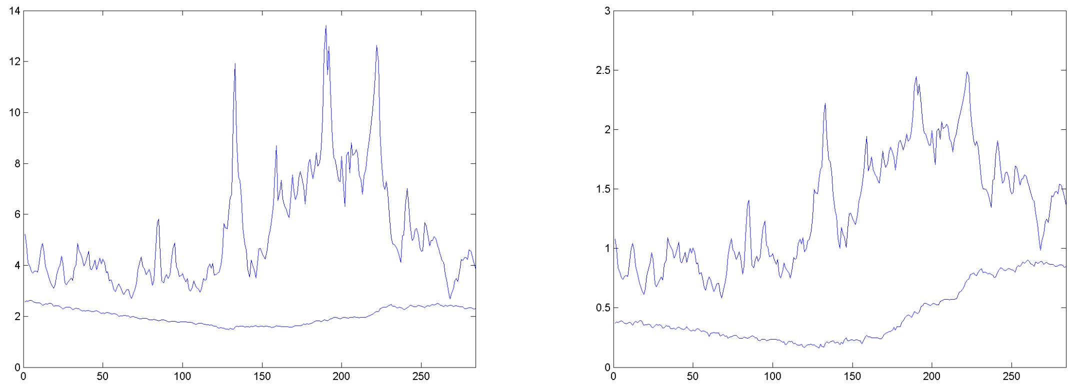

Figure 2: Historical behavior of fuel real prices (left, gas upper curve, coal lower curve) and log-prices (right, gas upper curve, coal lower curve) since January, 1990 until August, 2013. Prices are deflated and expressed in dollars (2012) per Million Btu.

demand/supply interplay, a future dependence of P^e on $P^{\mathrm{LC,w}}$ would be very difficult to assess and disentangle from generation cost risks, so that we don't include it in our model.

3 Accurately Modelling Fossil Fuels Market Prices

In this Section we propose a stochastic dynamical model to describe the time evolution of fossil fuels nominal market prices $X^x(\omega)$ to be inserted in $P^{\mathrm{LC,w}}(\omega)$, which improves on the cost model of Mari [5], in which only geometric Brownian processes were considered. Fig.2 shows the historical behavior since January 1990 until August 2013 of real US coal and gas market prices that we are going to use to model nominal prices. Both series are taken at a monthly frequency, deflated, and expressed in 2012 dollars per million Btu (mmBtu). Data were downloaded from the U.S. Energy Information Administration at site `www.eia.doe.gov/totalenergy`. Tab.2 provides the descriptive statistics of the monthly changes in the natural logarithm of market prices.

To account for the observed features of the historical price paths, we propose a stochastic model in continuous time t, in which the time evolution of the coal price is described by a geometric Brownian motion, and the dynamics of the natural gas price is described by a

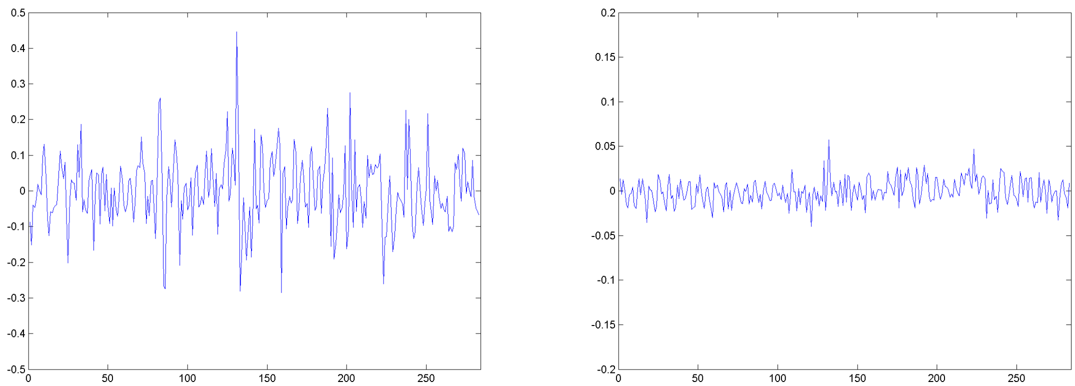

Figure 3: Historical behavior of monthly changes of fuel prices in natural logarithms of market prices (gas left, coal right) (L.H.R. scale amplified).

	Coal	Gas
Start	Jan 1990	Jan 1990
End	Aug 2013	Aug 2013
# of points	284	284
Mean	-0.0004	-0.0011
Std. dev.	0.0139	0.0982
Skew	0.2714	0.2572
Kurt	3.6915	4.6945
ρ_f	-0.088	

Table 2: Descriptive statistics of monthly percent changes of fuels price processes. Their correlation coefficient is indicated as ρ_f.

mean-reverting jump-diffusion process. Although geometric Brownian motion is often used to model fossil fuels prices dynamics [19], it should be noted that such a stochastic process may not capture completely the observed dynamics of market prices. Some evidence exists for more complicated behavior showing mean reversion around some long run value, jumps and stochastic volatility [6]. Calling $X^{\mathrm{co}}(\omega)$ the coal nominal price process and $X^{\mathrm{ga}}(\omega)$ the gas nominal price process, the $X^{\mathrm{co}}(\omega)$ dynamics is described by

$$\frac{dX^{\mathrm{co}}}{X^{\mathrm{co}}} = (\pi^{\mathrm{co}} + \pi)dt + \sigma^{\mathrm{co}} dW^{\mathrm{co}}, \tag{20}$$

where in $\pi^{\mathrm{co}} = \ln(1 + \gamma^{\mathrm{co}})$ the quantity γ^{co} is the real escalation rate of the coal price, chosen as reported in Tab.3, and in $\pi = \ln(1 + i)$ the quantity i is the expected inflation rate, which we choose as $i = 0.0175$ (taken from AEO 2013). σ^{co} is the volatility of coal prices, which is the only parameter to be estimated on time-series data for this dynamics, and $W^{\mathrm{co}}(\omega)$ is a standard Brownian motion.

Coal	Gas
$\gamma^{\mathrm{co}} = 0.01$	$\gamma^{\mathrm{ga}} = 0.02$
$\pi^{\mathrm{co}} = \ln(1.01)$	$\pi^{\mathrm{ga}} = \ln(1.02)$

Table 3: Real escalation rates for fossil fuels prices, taken from AEO 2013.

To determine the nominal dynamics of $X^{\mathrm{ga}}(\omega)$ we pose

$$X^{\mathrm{ga}}(\omega) = e^{(\pi^{\mathrm{ga}}+\pi)(t-t_b)} \tilde{X}^{\mathrm{ga}}(\omega), \tag{21}$$

where $\tilde{X}^{\mathrm{ga}}(\omega)$ is the real gas price process, and the exponential factor accounts for both inflation and real escalation rate of the fuel. In $\pi^{\mathrm{ga}} = \ln(1 + \gamma^{\mathrm{ga}})$ the quantity γ^{ga} is the real escalation rate of the gas price (see again Tab.3). After defining

$$\tilde{\Xi}^{\mathrm{ga}}(\omega) = \log \tilde{X}^{\mathrm{ga}}(\omega), \tag{22}$$

the dynamics of $\tilde{\Xi}^{\mathrm{ga}}(\omega)$ is chosen according to the following stochastic differential equation:

$$d\tilde{\Xi}^{\mathrm{ga}} = (\theta^{\mathrm{ga}} - \alpha^{\mathrm{ga}} \tilde{\Xi}^{\mathrm{ga}})dt + \sigma^{\mathrm{ga}} dW^{\mathrm{ga}} + JdN, \tag{23}$$

	Coal		Gas	
θ^{ga}			0.0432	(0.0256)
α^{ga}			0.0292	(0.0158)
λ^{J}			0.2542	(0.1501)
σ^{J}			0.1258	(0.0256)
σ^{x}	0.0139	(0.0006)	0.0737	(0.0086)
LL^{x}	807.94		264.58	

Table 4: Estimation results. Standard errors are between parentheses. LL^{x} stands for loglikelihood of fuel x.

where θ^{ga} and α^{ga} are mean reversion parameters, σ^{ga} is the gas price volatility and $W^{\mathrm{ga}}(\omega)$ is a standard Brownian motion. $N(\omega)$ is a Poisson process with constant intensity λ^{J}, and the jump amplitude J is distributed as a normal random variable with zero mean and standard deviation σ^{J}. We assume that $N(\omega)$, $W^{\mathrm{co}}(\omega)$ and $W^{\mathrm{ga}}(\omega)$ are mutually independent processes. Although in Tab.2 the estimated correlation coefficient between the coal and gas prices monthly percent changes $\rho_f = -0.088$ is not exactly zero, in agreement with Hogue [19] we assume it equal to zero. This is going to be consistent also with the fact that the quantitatively important correlation between coal and gas LCOEs results from the CO2 coupling due to CO2 volatility.

The dynamical parameters, namely σ^{co} for coal prices and the five parameters θ^{ga}, α^{ga}, σ^{ga}, λ^{J}, and σ^{J} for gas prices, were estimated on our data set by maximum likelihood, the results being summarized in Tab.4. Tab.5 displays the first four moments of the model distribution of monthly log-returns, obtained averaging over 5000 simulated paths randomly generated using estimated parameters. The statistical analysis of simulated trajectories shows a very interesting agreement with empirical data: the values of the first four moments of the simulated distributions of power prices and prices are very close to the empirical ones. The dynamics of nominal carbon prices is assessed according to a geometric Brownian motion

	Coal	Gas
Mean	-0.0001 (0.0008)	-0.0005 (0.0014)
Std.dev.	0.0139 (0.0006)	0.0983 (0.0056)
Skewness	0.0035 (0.1387)	0.0038 (0.2851)
Kurtosis	2.9740 (0.2738)	4.4475 (0.7952)

Table 5: Simulated moments. Standard errors are between parentheses.

of the type,

$$\frac{dZ}{Z} = \pi dt + \sigma^{\mathrm{ca}} dW^{\mathrm{ca}}, \tag{24}$$

where σ^{ca} is the carbon volatility and $W^{\mathrm{ca}}(\omega)$ is a standard Brownian motion which is assumed to be independent of $W^{\mathrm{co}}(\omega)$, $W^{\mathrm{ga}}(\omega)$ and $N(\omega)$. In this case, volatility won't be estimated but it will be taken as a parameter. The initial market price of CO_2 emissions has been fixed in 25 \$/ton. As discussed before, the dynamics of Z affects the time evolution of both LCOEs, introducing positive correlation between fossil fuel LCOEs. The quantification of such a correlation, as well as the values of the volatility of LCOE for both coal and natural gas generating technologies, can be obtained by using Monte Carlo techniques. Simulations have been performed in four different scenarios, namely assuming a carbon price volatility equal to 10%, 15%, 20% and 25% respectively. These assumptions try to depict low, medium, high and very high carbon prices volatility scenarios [20] in order to capture the relevance of hedging effects. The distributions of a sample of stochastic levelized costs obtained using this procedure, for coal and gas, are shown in Fig.4 when $\sigma^{\mathrm{ca}} = 0.20$, computed using of 100000 Monte Carlo randomly generated trajectories ω. For each trajectory, computed using the antithetic variable method, an evolution path for fossil fuel prices and carbon prices was obtained and, along such paths, LCOE values were computed according to single-technology Eq.(7). The first four LCOE moments are shown in Tab.6 for all four σ^{ca} scenarios. In Fig.4, since the higher is the LCOE (which is nonnegative) the higher is the risk not to cover the costs, on the x axis of the graph the LCOE is shown reversed (i.e. with a minus sign), so that the left tail of the distributions represents a risk of loss. This convention will be useful

σ^{ca}	fuel	$\mu^{LC,x}$	$\sigma^{LC,x}$	$P^{LC,x}$ skew.	$P^{LC,x}$ kurt.	ρ
0.10	coal	97.4	7.1	0.7	4.1	0.25
	gas	77.2	8.6	0.5	3.5	
0.15	coal	97.4	10.0	1.5	9.0	0.41
	gas	77.2	9.1	0.6	3.7	
0.20	coal	97.4	13.6	2.8	21.5	0.56
	gas	77.2	10.0	1.0	6.1	
0.25	coal	97.4	18.2	5.6	94.2	0.68
	gas	77.2	11.3	2.3	27.1	

Table 6: First four central moments of the $P^{LC,x}$ (LCOE) distribution.

in what follows. Notice that the two distributions in Fig.4 are strongly asymmetric with long tails, and have clearly different shapes and a nonnegative support. In order to show how long are their left tails, a dot marks the minimum −LCOE value which the computations find. Long tails mean that low probability events do exist such that breakeven is largely missed. The deterministic approach to LCOE doesn't take this risk into account.

4 LCOE Risk Analysis

In this Section we discuss a selection of risk measures under which to choose an optimal combination of gas and coal assets, nuclear for the moment excluded, in terms of an optimal choice of the portfolio weights w_1 and $w_2 = 1 - w_1$ ($1 \geq w_1 \geq 0$, no short selling allowed) introduced in Eq. (17). We will show that a measure called CVaR Deviation, not very often used in literature, is yet particularly suited to our stochastic LCOE case, and improves on usual Markowitz variance. Before starting, just notice that LCOEs are costs, not returns, with a strictly non-negative support. Hereafter, all Monte Carlo simulations use 10000 LCOE values only (instead of 100000), in order to limit stochastic optimization computational times, especially in the case of portfolios which include nuclear.

A Markowitz portfolio analysis was used to show how to manage the stochastic LCOE

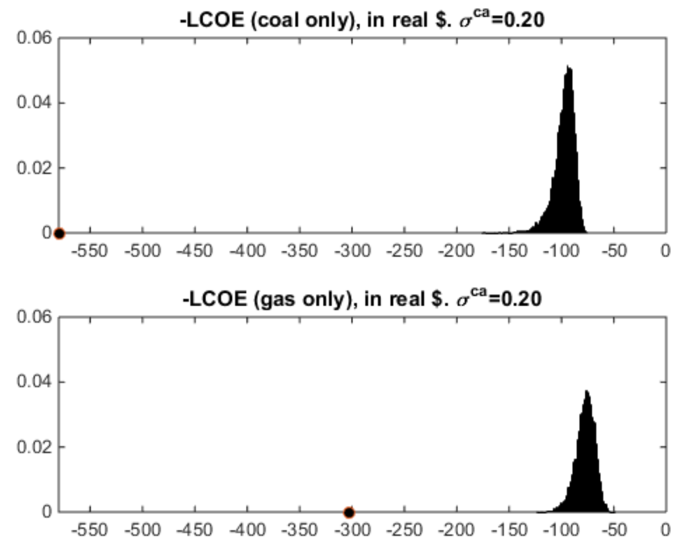

Figure 4: Single fuel LCOEs, for $\sigma^{\mathrm{ca}} = 0.20$. UPPER: coal; LOWER: gas. The horizontal axis is reversed. The dot on the abscissa marks the lowest $-$LCOE values found in each simulation. 100000 runs where used. The two sample distributions are clearly different.

risk [5], for example by computing the variance $(\sigma^{\mathrm{LC,w}})^2$ for a sequence of admissible $\mu^{\mathrm{LC,w}}$, and looking at the generation mix that minimizes the portfolio variance. For a pair of risky assets like coal and gas, $(\sigma^{\mathrm{LC,w}})^2$ vs. $\mu^{\mathrm{LC,w}}$ is a parabola with a minimum at $\mu^{\mathrm{LC}}_{\mathrm{mvp}}$, the minimum variance portfolio, at optimal w^*. In this way, optimization can be introduced in the stochastic LCOE problem, proposing portfolios of assets with minimum 'dispersion' about μ^{LC,w^*}. Investment risk is controlled choosing assets in such a way that uncertainty around expected LCOEs is minimal. The rationale is that, since LCOE estimates always include uncertainty because of fluctuating fuel prices, instead of getting rid of this uncertainty by hiding it under the carpet, it is wiser to arrange the investment in such a way to control it in the best possible way.

Yet, variance is the typical Markowitz risk measure, sensitive to correlation but not to asymmetries or long tails of distributions, whereas our LCOE distributions are long-tailed and skewed. A possible alternative risk measure, more sensitive to tail risk, is Value at Risk (VaR). In general, given 1) a vector of random variables $y(\omega)$ representing asset prices or returns, with joint probability density $p(y)$, 2) a vector of choice variables w representing portfolio weights, 3) a loss function $f(\omega) = f(w, y(\omega))$ representing portfolio losses (if the variables $y(\omega)$ represented returns it could be for example that $f(\omega) = -w'\, y(\omega)$, which is a random variable with values in percent), 4) a threshold h for the losses $f(\omega)$, 5) a probability α, the VaR for a given portfolio of components w is defined as a quantile

$$\mathrm{VaR}_{\alpha}^{w}(f(\omega)) = \min_{h}\{\int_{f(w,y(\omega))\leq h} p(y)\, dy(\omega) \geq \alpha\}, \tag{25}$$

i.e. as the minimum threshold $h^{*} = \mathrm{VaR}_{\alpha}^{w}(f(\omega))$ above which the probability of losses is equal or greater than α. For example, in the standard Markowitz case when losses are expressed as returns and in percent, $\mathrm{VaR}_{95\%}^{w} = 10\%$ represents a portfolio which doesn't loose more than 10% in 95% of the cases. Yet, in general, VaR is not very suitable to optimization uses when the underlying distribution is not gaussian (Pachamanova and Fabozzi [21]) and doesn't give information about what happens in the adverse cases. A better alternative to VaR is Conditional Value at Risk (CVaR), sensitive to tail risk and asymmetry, and with good minimization properties, expecially from a numerical point of view. CVaR for a portfolio of risks parametrized by w was introduced by Uryasev [22] and by Rockafellar and Uryasev [23] (see also Krokhml et al. [24]) in terms of VaR, as

$$\mathrm{CVaR}_{\alpha}^{w}(f(\omega)) = \frac{1}{1-\alpha}\int_{f(w,y(\omega))\geq \mathrm{VaR}_{w}^{\alpha}} f(w, y(\omega))\, p(y)\, dy(\omega) \tag{26}$$

Eq.(26) shows that CVaR can be interpreted as the expected value of losses greater in value than VaR (in the example above, $\mathrm{CVaR}_{95\%}^{w}$ would give the percent expected to be lost in the worst 5% cases). A first consequence of this is that $\mathrm{CVaR}_{\alpha}^{w} \geq \mathrm{VaR}_{\alpha}^{w}$. A second consequence is that, as an expected value, a CVaR can be estimated on sample data, i.e. in many circumstances the CVaR of a portfolio can be numerically obtained by linear (i.e. convex) stochastic optimization on sampled portfolio values with respect to an internal variable that at its optimal value turns out to be the VaR itself (Sarykalin et al. [8] and references therein). Estimates of minimum CVaR portfolios can be obtained as a joint linear stochastic minimization in both this variable and the portfolio coefficients. Some details of these results are sketched in Appendix 2.

There is more about CVaR that is useful to our purpose. CVaR belongs to a class of *risk measures in a more technical sense* introduced by Artzner et al. [25], thereafter called ADEH risk measures. CVaR is also a "strictly expectation bounded risk measure" (Rockafellar et al. [26]), a subclass $\mathfrak{R}(f(\omega))$ of ADEH risk measures. Measures in this class have useful common properties, like the fact that, for a constant c,

$$\text{CVaR}_\alpha^w(f(\omega) - c) = \text{CVaR}_\alpha^w(f(\omega)) + c. \tag{27}$$

In constrast, standard deviation, the root of the variance $(\sigma^{\text{LC}})^2 = E[(P^{\text{LC}}(\omega) - \mu^{\text{LC}})^2]$, is a *deviation measure*, belonging to an alternative class $\mathfrak{D}(f(\omega))$ connected one-to-one to $\mathfrak{R}(f(\omega))$ [26]. Generally speaking, each risk measure in $\mathfrak{R}(f(\omega))$ evaluates outcomes in an absolute way, whereas its partner deviation in $\mathfrak{D}(f(\omega))$ evaluates distribution widths. This one-to-one correspondence between risk measures $R \in \mathfrak{R}(f(\omega))$ and their partner deviation measures $D \in \mathfrak{D}(f(\omega))$ is estabilished as

$$D(f(\omega)) = R(f(\omega) - E[f(\omega)]) = R(f(\omega)) + E[f(\omega)] \tag{28}$$

where Eq.(27) was used for the second equality (for an updated and extended discussion of this correspondence see also [8, 9]). Measures in $\mathfrak{D}(f(\omega))$ have properties similar to those of the standard deviation (for example they are nonnegative and vanish only if $f = E[f]$), but in general, $D(f(\omega)) \neq D(-f(\omega))$, i.e. they can be asymmetric. More specifically, defined CVaR as in Eq.(26), the CVaR Deviation (CVaRD) for a portfolio can be defined, according to Eq.(28), as

$$\text{CVaRD}_\alpha^w(f(\omega)) = \text{CVaR}_\alpha^w(f(\omega) - E[f(\omega)]), \tag{29}$$

so that, in a lighter notation where $E^w = E[f(\omega)]$,

$$\text{CVaRD}_\alpha^w = \text{CVaR}_\alpha^w + E^w. \tag{30}$$

Besides being an important theoretical result, Eq.(30) has a practical advantage: knowledge of CVaR_α^w is sufficient to compute CVaRD_α^w. Because of the structure of the $\mathfrak{R}(f(\omega))/\mathfrak{D}(f(\omega))$ theory, it is only in their CVaRD form that CVaR optimization results can be safely compared to Markowitz standard deviation optimization results. It will be now shown in which sense minimum CVaRD LCOE portfolios improve on minimum variance LCOE portfolios.

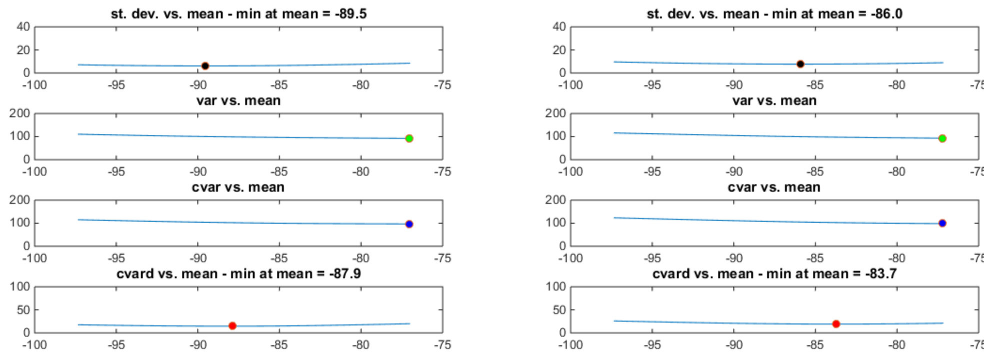

Figure 5: Risk measures (from top to bottom: standard deviation, VaR, CVaR, CVaRD) vs. mean. LHS: $\sigma^{ca} = 0.10$; RHS: $\sigma^{ca} = 0.15$. Color keys: CVaR (blue), VaR (green), CVaRD (red), standard deviation (black).

5 Coal and Gas Portfolios

In the Markowitz analysis of the optimal coal and gas portfolio (with w_1 coal and $w_2 = 1-w_1$ gas weights), covariance measures the degree of dependency between the two coal-only and gas-only LCOEs. In our model, this mutual dependency is controlled by σ^{ca}. Fig.5 and Fig.6 show a sequence of four CO_2 scenarios, for $\sigma^{\text{ca}} = 0.10, 0.15, 0.20, 0.25$. Markowitz theory requires that for low dependency (e.g. no dependency at all, then zero correlation) a well diversified mix of the two assests is preferred to single asset portfolios. In Fig.5, left panel, the case for low correlation $\sigma^{\text{ca}} = 0.10$ scenario is shown. In the top graph the portfolio standard deviation $\sigma^{\text{LC,w}}$ as a function of $\mu^{\text{LC,w}}$ is shown, as $w_1 \in [0,1]$ and $w_2 = 1 - w_1$ vary. The dot indicates the position of the minimum risk portfolio according to the Markowitz criterion. The Markowitz inefficient frontier stands on the l.h.s. of the dot, where the expected LCOE is very negative (a bad thing for investors). This minimum variance portfolio mixes coal and gas assets. In the second and third graphs $\text{VaR}^{w}_{95\%}$ and $\text{CVaR}^{w}_{95\%}$ as a function of $\mu^{\text{LC,w}}$ are shown. VaR or CVaR are chosen as risk measures to take into account the long asymmetric tails of Fig.4, by agents adverse to left tail risk that don't want to end up with a too high LCOE w.r.t. an expected electricity market price, i.e. a precise level. In this case, choosing a portfolio according to the minimum VaR or

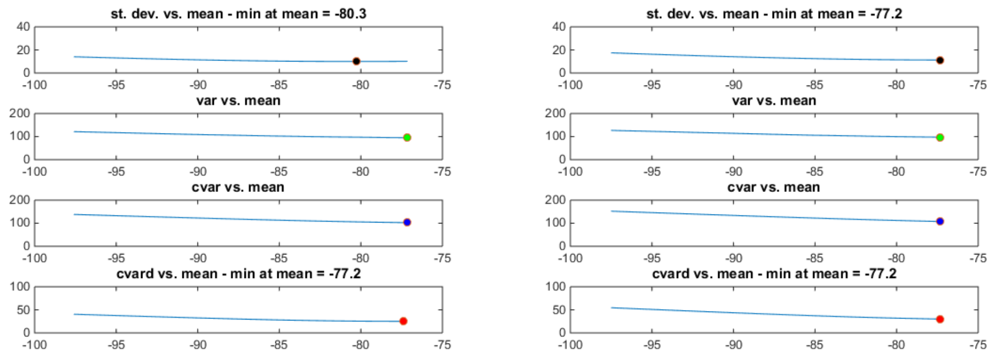

Figure 6: Risk measures (from top to bottom: standard deviation, VaR, CVaR, CVaRD) vs. mean. LHS: $\sigma^{ca} = 0.20$; RHS: $\sigma^{ca} = 0.25$. Color keys: CVaR (blue), VaR (green), CVaRD (red), standard deviation (black).

CVaR criterium would favour only-gas generation (the extremum is on the r.h.s.). This is consequential with the fact that, in current market data, gas and coal expected LCOEs are so distant one from the other that CVaR diversification doesn't work - gas is always the best choice. But an agent *selectively* adverse to uncertainty on the left tail of the LCOE, i.e. adverse to the width of its distribution (not with respect to any reference point as in the CVaR case) and not to LCOEs being close to zero (a good thing for investors), would minimize $\mathrm{CVaRD}^{\mathbf{w}}_{95\%}$, as in the fourth graph. In this case, the portfolio obtained would be mixed, even though different from the Markowitz portfolio in expected value and components. It is also evident from the Figure that CVaRD is not symmetric, differently from the variance case. In Fig.5, right panel, the same measures are shown for $\sigma^{\mathrm{ca}} = 0.15$. More correlation displaces the minimum towards the r.h.s. edge, with a difference between standard deviation and CVaRD, since CVaRD is more adverse to tail risk. In the case of standard deviation, increasing σ^{ca} makes coal riskier than gas ($\sigma^{LC,co} > \sigma^{LC,ga}$, cfr. Tab. 6, fourth column) so that an only-gas portfolio is more and more preferred. VaR and CVaR portfolios stick to the only-gas choice. In general, variances and CVaRD minima move to the r.h.s. w.r.t. the left panel. In Fig.6, left panel, the case $\sigma^{\mathrm{ca}} = 0.20$ is shown. Whereas the Markowitz criterion still suggests a mixed portfolio, VaR, CVaR and CVaRD now agree

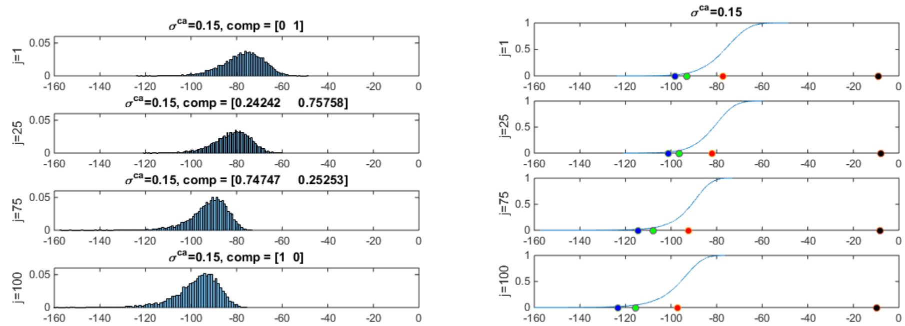

Figure 7: Sequence of 4 out of 100 portfolios for $\sigma^{ca} = 0.15$, as component w_1 varies from 0 (index $j = 1$, gas-only) to 1 (index $j = 100$, coal-only). LHS: distribution densities, RHS: corresponding cumulative distributions and their (minus) CVaR (blue), VaR (green), CVaRD (red), standard deviation (black). Compare with RHS of Fig. 5 (same color keys).

on a gas-only portfolio. Correlation is very high and tail averse agents prefer gas, which has a shorter tail. In Fig.6, right panel, the case $\sigma^{\mathrm{ca}} = 0.25$ is shown. Maximum correlation, all four measures agree on gas-only portfolios. Notice that in all four cases $\mathrm{CVaR}_\alpha^{\mathbf{w}} \geq \mathrm{VaR}_\alpha^{\mathbf{w}}$. It is then interesting to look at the shape of the portfolio distribution density, as w_1 increases from 0 (gas-only portfolio) to 1 (coal-only portfolio), for a given σ^{ca}.

L.h.s. panel of Fig.7, from top to bottom, shows such a scan for $\sigma^{ca} = 0.15$, for the four combinations for which $w_1 = 0, 0.2424, 0.7474, 1$, in our reversed-axis convention. On the r.h.s. panel the corresponding cumulative distributions are plotted, in order to help one's eye to locate the 95% VaR position (which is defined in terms of a fixed 0.05 level for the cumulative distribution, not shown in the graphs, see Eq.(25)). On their abscissas, the four positions of the risk measures is marked as a set of four points, one set for each w_1 value (compare also with Fig.5, which uses the same color code). Notice that quantities like CVaR (blue), VaR (green), CVaRD (red) and standard deviation (black) are positive definite, so that, in order to show them on the plot, their sign was flipped. As expected, $\mathrm{CVaR}_\alpha^{\mathbf{w}} \geq \mathrm{VaR}_\alpha^{\mathbf{w}}$. Notice that CVaRD is always larger in absolute value than variance. Since this scan doesn't cover all values of w_1, it is obviously not possible to see here that the minimum variance and minimum CVaRD distributions lay somewhere between the first

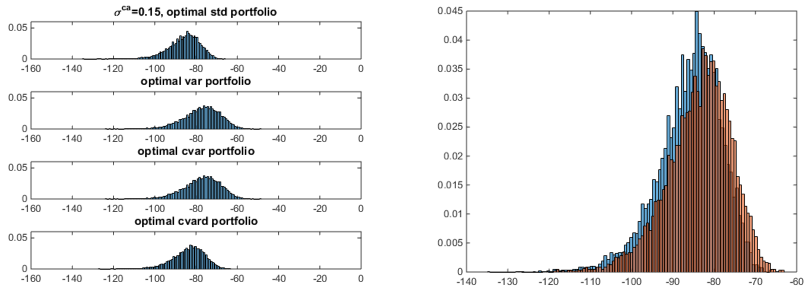

Figure 8: Portfolio distributions, for $\sigma^{ca} = 0.15$, which are optimal under the four different risk measures. LHS: list of all four distributions, i.e. optimal under variance, VaR, CVaR, CVaRD. RHS: direct comparison between variance (blue) and CVaRD (red) distributions, i.e. between deviation measures only - optimal variance first weight is $w_1 = 0.4343$, optimal CVaRD first weight is $w_1 = 0.3232$.

(i.e. top) and the second graph. L.h.s. panel of Fig.8 shows the $\sigma^{ca} = 0.15$ optimal distributions under (top to bottom) standard deviation, VaR, CVaR and CVaRD measures. VaR and CVaR are very conservative in terms of large LCOEs and coincide with the gas-only distribution. Standard deviation and CVaRD accept more tail risk and diversify their components. R.h.s. panel of Fig.8 directly compares the standard deviation (blue) and the CVaRD (red) optimal distributions. Being more left-tail risk adverse, CVaRD optimum selects an asset mix with $w_1 = 0.3232$ (less coal) in contrast to a $w_1 = 0.4343$ (more coal) of the variance optimum, so that in CVaRD mix there is a smaller probability of a too large LCOE (bad thing) than in the case of the variance optimum, whereas low LCOE probability (good thing) is enhanced in comparison to the variance case. This is the reason why, in the case of the stochastic LCOE, we propose CVaRD as a better alternative to usual Markowitz variance.

6 Including the Nuclear Asset

When the zero-variance nuclear asset is included in the problem, interesting results appear in analogy with the mean - standard deviation Markowitz plane [5], where the portfolios

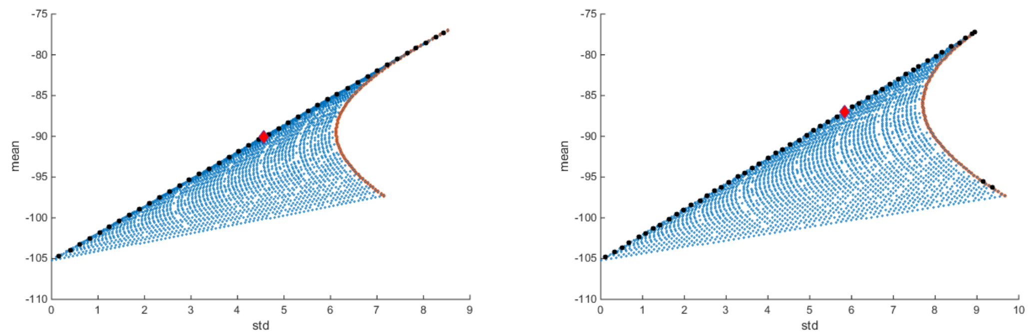

Figure 9: Mean - standard deviation plane. LCOE of nuclear is 105.2. Red line: efficient frontier for zero nuclear. Black dotted line: efficient frontier. Red dot: see text. L.H.S. $\sigma^{ca} = 0.10$. R.H.S. $\sigma^{ca} = 0.15$.

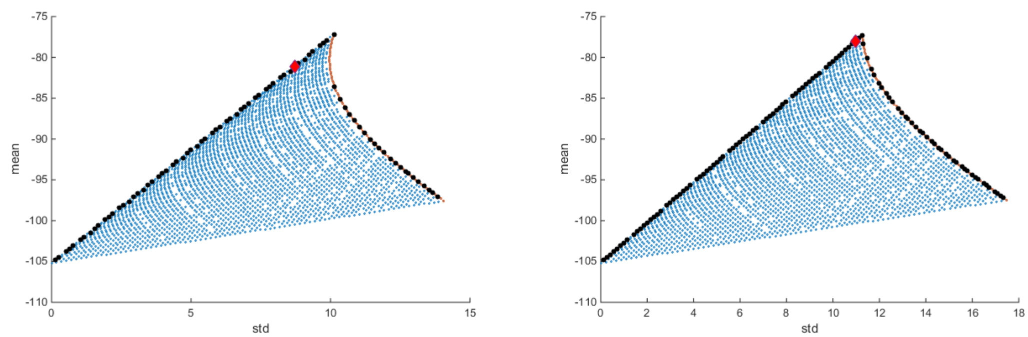

Figure 10: Mean - standard deviation plane. LCOE of nuclear is 105.2. Red line: efficient frontier for zero nuclear. Black dotted line: efficient frontier. Red dot: see text.L.H.S. $\sigma^{ca} = 0.20$. R.H.S. $\sigma^{ca} = 0.25$.

efficient frontier can be drawn and the inclusion of a risk-free asset like a bond induces the presence of a "capital market line" (suitably reinterpreted), tangent to the efficient frontier in the tangent portfolio.

Using then our reversed-axis convention, Fig.9 and Fig.10 show on a usual Markowitz plane all possible portfolios that can be obtained when nuclear is included, for the same sequence of CO_2 volatility values used above. The LCOE of the nuclear asset is computed (again, using data from Tab.1) as 105.2. As expected, the inclusion of a nuclear asset allows one to identify a portfolio (OP, marked with a red dot in Fig.9 and Fig.10) with the same

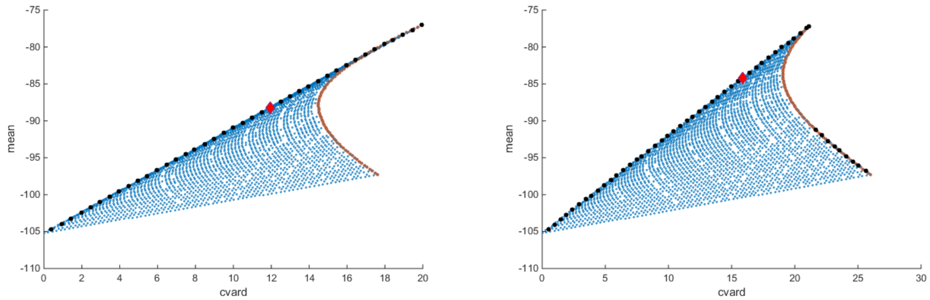

Figure 11: Mean - CVaRD plane. LCOE of nuclear is 105.2. Red line: efficient frontier for zero nuclear. Black dotted line: efficient frontier. Red dot: see text. L.H.S. $\sigma^{ca} = 0.10$. R.H.S. $\sigma^{ca} = 0.15$.

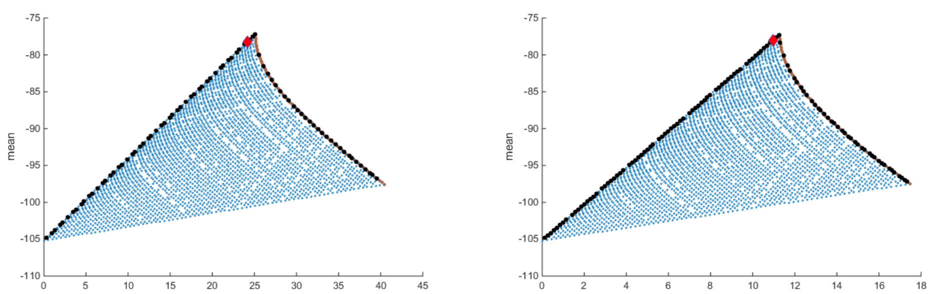

Figure 12: Mean - CVaRD plane. LCOE of nuclear is 105.2. Red line: efficient frontier for zero nuclear. Black dotted line: efficient frontier. Red dot: see text. L.H.S. $\sigma^{ca} = 0.20$. R.H.S. $\sigma^{ca} = 0.25$.

expected LCOE of the minimum variance portfolio but with a reduced standard deviation risk. In a less obvious way, a CVaRD - expected LCOE plane which includes nuclear displays patterns similar to the standard deviation case. This can be seen in Fig.11 and Fig.12, where all possible three-component portfolios are plotted. Each point is obtained as a separate stochastic optimization. The fact that a CVaRD - expected values plane has to show the same features of the standard Markowitz plane is discussed for example by Rockafellar et al. [27]. In general, the presence of the nuclear asset reduces the portfolio risk under both risk deviation measures. Looking at the two sets of Figures, as σ^{ca} increases from $\sigma^{ca} = 0.10$ onward, OP gets closer to the gas-only portfolio (the prong in the upper right hand corner of the portfolio universe), i.e. the gas weight increases. The stochastic LCOE theory provides a quantitative estimate of the component weights. More specifically, under the CVaRD measure, as σ^{ca} goes from 0.10 to 0.25, the gas weight goes from 58% to 97%. Correspondingly, the other two components get reduced from 8% to 0% in the case of coal and from 34% to 3% in the case of nuclear. Notice that an analysis that includes nuclear is less restrictive on the selection of optimal portfolios among which risk adverse investors can choose. Without nuclear, the minimum deviation portfolio is always the best one. With nuclear, the investor can tune the risk of its portfolio from zero (only nuclear) to the OP (red dot) risk value, which in any case is a portfolio less risky than the minimum deviation portfolio that can be obtained excluding nuclear. On the contrary, an investor in any case adverse to high LCOE values always chooses a gas only portfolio.

7 Concluding Remarks

In this paper we discussed the stochastic LCOE theory when its underlying price model uses a realistic very volatile fossil fuels dynamics coupled to a CO_2 pivotal dynamics. In particular, we showed how to optimally shape portfolio LCOE distributions when selective asymmetric tail-risk aversion is considered, using a CVaRD approach. We wish to remark that ours is a rare financial case in which CVaR is not appropriate, and in any case not very telling. This is consequential with the fact that, in current market data, gas and coal expected LCOEs are so distant one from the other that CVaR diversification doesn't work - gas is always the best choice. We also discussed the role of the nuclear asset in this theory. We considered portfolios which include gas, coal and nuclear, assuming that the nuclear component plays the role of a risk free asset in terms of its LCOE.

Yet, we are aware that the nuclear LCOE cannot be considered risk free when the construction time-span becomes uncertain, since this variable has a large impact on the LCOE (Kessides [28]). As it is well documented in literature, the nuclear fuel cost itself has a very low impact on LCOE (about 8%, see Mari [4]). In contrast, and as a consequence of this, the uncertain duration of the construction period is the main source of financial cost risk for the nuclear source. This would make the nuclear source an asset which LCOE is risky but still uncorrelated with the other fossil fuel LCOEs. Even if this asset is taken as risky, it can be anyway used in a well diversified baseload energy portfolio for the purpose of hedging, with maybe a lesser effect, that can be yet quantitatively assessed only using our stochastic LCOE theory. The uncertainty about construction time can be assessed and modeled by collecting data on operating reactors that were built and connected to the grid in the last years.

Our stochastic LCOE approach is interesting also because it can be used to understand, from the point of view of a policy maker which manages CO2 prices, how rational investors would react to a given policy scheme in a free market context. Taking into consideration this rational behavior, the policy maker could refine the design of CO2 pricing market mechanisms.

We leave all this to future investigation.

Appendix 1: Further technical assumptions

Asset depreciation dep_n^x of Eq.(3) is technology dependent and it is computed using the MACRS system (Modified Accelerated Cost Recovery System) displayed in Tab. 7.

	MACRS,15	MACRS,20
Year 1	5.00%	3.750%
Year 2	9.50%	7.219%
Year 3	8.55%	6.677%
Year 4	7.70%	6.177%
Year 5	6.93%	5.713%
Year 6	6.23%	5.285%
Year 7	5.90%	4.888%
Year 8	5.90%	4.522%
Year 9	5.91%	4.462%
Year 10	5.90%	4.461%
Year 11	5.91%	4.462%
Year 12	5.90%	4.461%
Year 13	5.91%	4.462%
Year 14	5.90%	4.461%
Year 15	5.91%	4.462%
Year 16	2.95%	4.461%
Year 17		4.462%
Year 18		4.461%
Year 19		4.462%
Year 20		4.461%
Year 21		2.231%

Table 7: Depreciation Schedule.

Appendix 2: Numerical CVaR computation

Recall that, for a stochastic variable L of support y (like $f(w, y(\omega))$ in Eq.(26)),

$$\mathrm{Prob}(L \geq y) = -\frac{d}{dy} E[\max(L - y, 0)].$$

Consider the stochastic unconstrained program objective, parametric in α (the probability α in Eq.(26), i.e. a confidence level),

$$g_\alpha(y) = y + \frac{1}{1-\alpha} E[\max(L - y, 0)].$$

The necessary condition to find the argmin of the program

$$\min_y \quad g_\alpha(y)$$

is obtained by setting the derivative of the objective to zero, to give

$$1 - \alpha - \mathrm{Prob}(L \geq y) = 0,$$

which returns the condition for optimal y^*

$$\mathrm{Prob}(L \geq y^*) = 1 - \alpha.$$

Thus, $y^* = \mathrm{argmin}_y\, g_\alpha(y) = \mathrm{VaR}_\alpha$, from the definition of VaR in Eq.(25). The value of the objective at optimality, i.e. the optimal expected value of the problem, is

$$\begin{aligned} & y^* + \left(\frac{1}{1-\alpha}\right) E[\max(L - y^*, 0)] = \\ & \left(\frac{1}{1-\alpha}\right) \left((1-\alpha) y^* + \int_{y^*}^{\infty} (y - y^*)\, p(y)\, dy\right) = \\ & \left(\frac{1}{1-\alpha}\right) \int_{y^*}^{\infty} y\, p(y)\, dy = E[L | L \geq y^*], \end{aligned}$$

because $\int_{y^*}^{\infty} p(y) dy = (1 - \alpha)$. This value can be written as the conditional expected value

$$E[L | L \geq \mathrm{VaR}_\alpha]$$

which is the CVaR_α of the definition in Eq.(26). Furthermore, if a set of values y_i ($i = 1, \ldots, N$) sampled from L is available, the objective $g_\alpha(y)$ can be approximated by

$$g_\alpha(y) \approx \hat{g}_\alpha(y) = y + \frac{1}{N(1-\alpha)} \sum_{i=1}^{N} \max(y_i - y, 0).$$

Since the (nonlinear) max function in this expression can be rewritten as a linear combination of accessory nonnegative variables, an estimate of y^*, i.e. the CVaR of L, is obtained as the solution of a convex stochastic optimization problem [23].

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