

Article

Forecasting the Power Generation Mix in Italy Based on Grey Markov Models

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Abstract: This study considers an application of the first-order Grey Markov Model to foresee the values of Italian power generation in relation to the available energy sources. The model is used to fit data from the Italian energy system from 2000 to 2022. The integration of Markovian error introduces a random element to the model, which is able now to capture inherent uncertainties and misalignments between the Grey Model predictions and the real data. This application provides valuable insights for strategic planning in the energy sector and future developments. The results show good accuracy of the predictions, which could provide powerful information for the effective implementation of energy policies concerning the evolution of energy demand in the country. Results show an improvement in the performance of more than 50% in terms of Root Mean Squared Error (RMSE) when the Markov chain is integrated in the analysis. Despite advancements, Italy's 2032 energy mix will still significantly rely on fossil fuels, emphasizing the need for sustained efforts beyond 2032 to enhance sustainability.

Keywords: Markov chain; grey model; energy mix; electricity



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1. Introduction

Grey systems theory (GST) was initiated by Julong Deng in 1982 [1] and further developed in [2]. It represents an approach aimed at forecasting systems characterized by small sample sizes and poor information. This methodology excels in managing systems where only fragments of information are known by identifying, unearthing, and refining the valuable data present. Grey systems theory has proven to be an effective strategy for handling uncertainty in information. In applied sciences, it is common to encounter systems that are not fully understood and are supported by minimal datasets and vague information, highlighting the need to apply this theory.

Since its inception, GST has witnessed consistent expansion in terms of cross-disciplinary applications and in the diversity of its methodologies. In particular, the proposal of novel approaches and techniques for addressing data that are unknown, incomplete, or inadequate is at the core of recent developments. GST has been employed across a broad spectrum of fields, for example, in predictive scientometrics [3], energy and environmental emissions [4,5], materials science [6], engineering disciplines [7], the development of electric vehicles [8], environmental studies [9], the management of water leaks [10], and the fields of economics and social sciences [11], and additional examples are illustrated in [12,13].

Generally, the Grey Model $GM(M, N)$ comprises M -order partial differential equations and N variables. The principle of Grey modeling is founded on the assumption that a process's underlying mechanisms can be abstracted by a specific, standard partial differential equation, which mirrors the method's degree of complexity. The estimation

of model parameters is conducted through the analysis of accident data. This modeling framework is distinguished by its ability to use just four pieces of data to project future events [14]. In the literature, two single-variable Grey Models are predominantly utilized: the first-order $GM(1, 1)$ and the second-order $GM(2, 1)$.

In practice, observed data are frequently subject to various random influences, leading to data series that exhibit unpredictability and volatility. The $GM(1, 1)$ forecasting model is an effective tool for data sequences with minimal data, but it struggles with sequences that exhibit significant fluctuations [15]. To overcome this issue and to increase forecast precision, the Grey Markov Chain Model $MCGM(1, 1)$ was created to correct the residual errors produced by $GM(1, 1)$. This model merges the first-order single-variable $GM(1, 1)$ Grey forecasting model with a Markov-chain (MC) model that adjusts for residuals [16,17]. This integration of models has been widely applied to datasets that are small in size but characterized by randomness and fluctuation [18–20]. The core concept behind the $MCGM(1, 1)$ model involves identifying discrete states for the residual errors from $GM(1, 1)$ and computing Markov probability transition matrices to grasp the statistical properties of these random residuals. Subsequently, adjustments are made based on these Markov matrices to refine the forecasting accuracy of the original $GM(1, 1)$ model.

In this article, one of the main contributions is the implementation and full retraction of the model described above for forecasting the mix of primary energy consumption in mature industrial economies, using Italy as an ideal representative case.

The importance of this topic lies in the strategic planning and policymaking to meet environmental goals and energy security. Forecasting the energy mix enables the assessment of future energy scenarios, including the adoption of renewable energy sources, and helps in evaluating their impact on carbon emissions reduction and energy security. A recent application study to the Australian framework is discussed in [21].

For Italy, as part of the European Union (EU), forecasting the energy mix is crucial for aligning with the ambitious targets of the EU for reducing greenhouse gas emissions and increasing the share of renewable energy in total energy consumption by 2030 and beyond. Moreover, the relevance of Italy as a case study is due to its status as a mature industrial economy. Italy's unique geographical position makes it particularly sensitive to shifts in renewable energy sources and the implications of having no nuclear power facilities. These aspects are crucial, as they influence both the national energy strategy and the broader European energy landscape. Understanding the future energy mix aids in planning for the necessary infrastructure developments, such as grid upgrades and energy storage solutions, to accommodate an increased share of variable renewable energy sources like wind and solar power; see [22]. This prediction is also important, as it directly informs strategies for carbon sequestration, an essential component in mitigating CO_2 emissions from the depletion of fossil sources. Understanding the scale of carbon sequestration required enables us to align our strategies with international efforts to reduce the environmental impact of energy production, thereby contributing to the global sustainability objectives [23].

Moreover, forecasting the energy mix is essential for identifying potential challenges and opportunities within the energy transition. For instance, it can highlight the need for Italy to diversify its energy sources to reduce dependence on imported fossil fuels, thus enhancing energy security. It also allows for the identification of economic opportunities related to the energy transition, including job creation in the renewable energy sector and potential for technological innovation [24].

Furthermore, an accurate forecast of the energy mix is vital for ensuring the reliability and stability of the energy supply. As Italy moves towards a more sustainable energy system, understanding the balance between supply and demand, the integration of renewable energy and the role of energy efficiency becomes increasingly important. This insight is critical for maintaining a stable energy supply while meeting environmental targets.

In conclusion, forecasting the energy mix is of paramount importance for Italy as it navigates its energy transition. It supports strategic planning, policy development, and in-

frastructure investment decisions aimed at achieving a sustainable, secure, and competitive energy system.

Given the absence of prior applications of this model for this specific purpose in the literature, we have endeavored to outline its implementation steps towards the applied results.

This study provides an insightful look into Italy's future energy composition, using a forecasting model that integrates the Grey system theory with Markov chains to address the uncertainties in energy prediction. The obtained results reveal a significant trend towards the adoption of renewable energy sources while indicating that fossil fuels will still play a dominant role in Italy's energy mix by 2030. This dual dependence underscores the complexity of transitioning to a sustainable energy framework and highlights the critical need for Italy to enhance its efforts in renewable energy development. The results serve as a call to action for policymakers, suggesting that while strides are being made towards a greener energy future, more aggressive policies and investments are required to reduce fossil fuel dependency significantly. This study not only sheds light on the potential energy landscape of Italy in the coming years but also emphasizes the importance of innovative forecasting techniques in planning and policymaking for energy transition.

The paper is organized as follows. In Section 2, we present the model. Specifically, in Section 2.1, we illustrate the steps of the Grey Model $GM(1,1)$, and in Section 2.2, we explain how to apply the Markov chain for error adjustment. Subsequently, in Section 3, we introduce our case study focused on forecasting the consumption of primary energy from each energy source for Italy until 2032 and we demonstrate the accuracy of our forecasts and the robustness of our models through a sensitivity analysis as well. In Section 4, the conclusions of our work are given.

2. The Model

The $GM(1,1)$ model employs a first-order differential equation to describe the forecasting target, and it stands as a primary tool in Grey system forecasting. Additionally, the enhanced $MCGM(1,1)$ approach, merging the principles of the $GM(1,1)$ model with those of the Markov chain, delivers improved statistical assessments for datasets that demonstrate significant fluctuations. The Grey Markov Model does not require restrictive assumptions concerning the relationships between variables and data stationarity [25]. However, for reasonable outcomes, its deterministic component should be able to explain the data trend, while the Markov chain should enhance the deterministic predictions.

2.1. $GM(1,1)$ Model

The $GM(1,1)$ model is widely recognized for its ability to generate reliable short-term forecasts using a limited dataset of non-negative values. In this section, we will discuss the steps required to implement this model.

Let $X^{(0)}$ be the vector representing the original data sequence containing n observations:

$$\mathbf{X}^{(0)} = (x_1^{(0)}, x_2^{(0)}, \dots, x_n^{(0)})^T. \quad (1)$$

Firstly, we perform a one-time accumulated generating operation in the following way:

$$x_k^{(1)} = \sum_{i=1}^k x_i^{(0)}, k = 1, 2, \dots, n, \quad (2)$$

in order to create the $\mathbf{X}^{(1)}$ array

$$\mathbf{X}^{(1)} = (x_1^{(1)}, x_2^{(1)}, \dots, x_k^{(1)}, \dots, x_n^{(1)})^T. \quad (3)$$

It should be observed that the first element of the vector in Equation (1) corresponds to the first element of the vector in Equation (2), symbolically represented as $x_1^{(0)} = x_1^{(1)}$.

Then, set $Z^{(1)} = (z_2^{(1)}, z_3^{(1)}, \dots, z_n^{(1)})$ as the vector of the background values computed as follows:

$$z_k^{(1)} = \frac{x_k^{(1)} + x_{k-1}^{(1)}}{2}, \quad k = 2, \dots, n. \quad (4)$$

This is equivalent to stating that the general element $z_k^{(1)}$ is the arithmetic mean of the two neighboring data points.

The GM(1,1) equation is defined as follows:

$$x_k^{(0)} + az_k^{(1)} = b \quad (5)$$

where a is the developing coefficient and b is the Grey effect.

Next, the cumulative values $x_1^{(1)}, x_2^{(1)}, \dots, x_n^{(1)}$ can be approximated by the first-order Grey differential equation:

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = b, \quad (6)$$

Based on the principles of Least Square Estimation (LSE), the estimation of parameters \hat{a} and \hat{b} can proceed in the following way:

$$[\hat{a}, \hat{b}]^T = (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \mathbf{y}, \quad (7)$$

where

$$\mathbf{B} = \begin{bmatrix} -z_2^{(1)} & 1 \\ -z_3^{(1)} & 1 \\ \vdots & \vdots \\ -z_n^{(1)} & 1 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} x_2^{(0)} \\ x_3^{(0)} \\ \vdots \\ x_n^{(0)} \end{bmatrix}. \quad (8)$$

To calculate an estimate of the Standard Error (SE) associated with the parameters \hat{a} and \hat{b} , we compute

$$SE^2 = \widehat{Var} \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \left(\frac{\sum_{k=1}^n (x_k^{(0)} - \hat{x}_k^{(0)})^2}{n-2} \right) (\mathbf{B}^T \mathbf{B})^{-1}, \quad (9)$$

Finally, the solution of (6) leads us to the time response function

$$\hat{x}_k^{(1)} = \left(x_1^{(0)} - \frac{\hat{b}}{\hat{a}} \right) e^{-\hat{a}(k-1)} + \frac{\hat{b}}{\hat{a}}, \quad k = 1, 2, \dots \quad (10)$$

Therefore, the predicted values of $X^{(0)}$ are:

$$\hat{x}_k^{(0)} = \hat{x}_k^{(1)} - \hat{x}_{k-1}^{(1)} = (1 - e^{\hat{a}}) \left(x_1^{(0)} - \frac{\hat{b}}{\hat{a}} \right) e^{-\hat{a}(k-1)}, \quad k = 2, \dots, \quad (11)$$

given the assumption that the first value is known, that is $\hat{x}_1^{(0)} = x_1^{(0)}$.

2.2. Markov Chain Residual Modification

Observed data often fluctuate due to various random influences. Consequently, accurate forecasts with the GM(1,1) model are difficult to obtain. To overcome this problem, we integrate a Markov chain approach to refine the residual errors. In particular, the MCGM(1,1) model merges the Grey forecasting approach of GM(1,1) with a Markov chain mechanism to adjust for residual errors. The core concept behind the MCGM(1,1) involves first identifying discrete states for the residual errors from GM(1,1), then calculating Markov probability transition matrices to understand the statistical behavior of

these residuals, and finally applying corrections based on these matrices. This process enhances the original forecasting accuracy of the GM(1,1) model. The step-by-step residual modification is described in the current section.

First, we define which residuals to be used. There are examples in the literature that show the use of absolute errors [25] and others that show the use of relative errors [20,26]. In our investigation, we focus on relative errors represented in the form

$$r\varepsilon_k = \frac{\hat{x}_k^{(0)} - x_k^{(0)}}{x_k^{(0)}}, \quad k = 1, 2, \dots, n. \quad (12)$$

Let $\varepsilon_k = (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n)$ be the chosen series of residual values, ε_{min} be the minimum, and ε_{max} be the maximum. We split the range of residuals $[\varepsilon_{min}, \varepsilon_{max}]$ into q intervals of the same length. The generic interval is denoted by

$$r_w = \left[\varepsilon_{min} + (w-1) \frac{\varepsilon_{max} - \varepsilon_{min}}{q}; \varepsilon_{min} + w \frac{\varepsilon_{max} - \varepsilon_{min}}{q} \right], \quad w = 1, \dots, q. \quad (13)$$

Each of them is assumed to be a state. In particular, state 1 is the interval with the lower bound equal to ε_{min} , and state q is the one with ε_{max} as the upper bound.

This means that the sequence of residual values is converted into a sequence of states assuming values in the set $E = \{r_1, r_2, \dots, r_q\}$. The state of residual ε_k depends on which interval it belongs to. The specific number of state divisions is not rigidly defined; rather, it is determined based on comprehensive factors such as the sample size and the error range of the fit [27]. Classifying them into three to five states is typically deemed suitable [28].

Now, take c_w as the notation for the representative value of each state r_w , $w = 1, 2, \dots, q$ whose lower bound is l_w and upper bound is u_w . In some circumstances, simple choices are considered. For example, the formula proposed in [25] is a weighted average of the lower and upper bounds of the intervals:

$$c_w = \alpha_w l_w + (1 - \alpha_w) u_w, \quad w = 1, 2, \dots, q, \quad (14)$$

where α_w are the weights that can be assigned to the extremes of each interval. In general, various choices are possible by producing statistical estimations of the representative values of the states of the Markov process. This is commonly performed in diverse application settings; see, for example, [29,30]. Accordingly, we consider the following estimation formula:

$$c_w = \frac{\sum_{k=1}^n \mathbb{1}_{\{\varepsilon_k \in r_w\}} \cdot (\varepsilon_k)}{\sum_{k=1}^n \mathbb{1}_{\{\varepsilon_k \in r_w\}}}. \quad (15)$$

The next assumption is to consider this series of states as generated by a Markov chain $\{R_k\}_{k \in \mathbb{N}}$. Hence, for every time $k \in \mathbb{N}$, it results that

$$\mathbb{P}[R_{k+1} = c_j | R_k = c_i, R_{k-1} = c_h, \dots] = \mathbb{P}[R_{k+1} = c_j | R_k = c_i] =: p_{ij} \equiv p_{ij}^{(1)}. \quad (16)$$

The above probabilities define the so-called one-step transition probability matrix, $\mathbf{P} = (p_{ij})_{i,j \in E}$.

Now, we introduce the transition probability matrix $\mathbf{P}^{(m)}$ as an $q \times q$ matrix with elements $p_{ij}^{(m)}$, where i and j denote the rows and columns respectively, and m counts the number of the steps. Indeed, $\mathbf{P}^{(m)}$ is called the m -steps transition probability matrix and represents the m -th power of the matrix \mathbf{P} :

$$P^{(m)} = \begin{bmatrix} p_{11}^{(m)} & p_{12}^{(m)} & \cdots & p_{1q}^{(m)} \\ p_{21}^{(m)} & p_{22}^{(m)} & \cdots & p_{2q}^{(m)} \\ \vdots & \vdots & \ddots & \vdots \\ p_{q1}^{(m)} & p_{q2}^{(m)} & \cdots & p_{qq}^{(m)} \end{bmatrix} \quad (17)$$

According to the Markov chain assumption, it results that

$$p_{ij}^{(m)} = \mathbb{P}[R_{k+m} = c_j | R_k = c_i]. \quad (18)$$

Each element $p_{ij}^{(m)}$ represents the probability of $R_{k+m} = c_j$ given that $R_k = c_i$, i.e., the residual process transits from state c_i to state c_j in m steps. The one-step transition probability matrix is estimated as follows:

$$p_{ij}^{(1)} = \frac{t_{ij}^{(1)}}{t_i}, i, j = 1, 2, \dots, q. \quad (19)$$

The term $t_{ij}^{(1)}$ represents the total number of transitions from the i -th to the j -th state after $m = 1$ step, and t_i is the count of occurrences of state i within the residual sequence. To obtain the m -step transition matrix, the one-step matrix is raised to the m -th power.

Each row in $P^{(m)}$ has elements that sum to one. If the sum of a row's elements for state i is nil because we never observed a residual belonging to the i -th interval r_i , one choice could be to directly assign $p_{ii}^{(1)} = 1$. This choice implies that state r_i functions as an absorbing state.

Due to limited data, another problem may arise in the application of this model. It may be possible that a state of the process is only visited once and at the last observation of the time series of the residuals. In this eventuality, we are unable to estimate the transition probabilities over the row of the transition matrix which corresponds to the considered state. Here, the choice to characterize that state as an absorbing one poses a major risk when forecasting over a time span exceeding the observation time. In such a scenario, rendering state r_i to be absorbing exclusively dictates the predictions, thereby introducing distortions. This is because it is unrealistic to assume that the process remains in the residual state r_i until the end of the predictive interval.

Therefore, our recommendation is to assign to each element over the row an equal probability. This equates to the random initialization of errors within the system, subsequently allowing them to develop following the observed frequencies via the Markovian framework.

For example, this is equivalent to saying that for a number of states equal to $r = 4$, each row element would be equal to 0.25. In this way, when the chain reaches this particular state, it will not be absorbed by the starting state and it will be equally likely to reach in the next visit any other state.

The Markov chain method is employed to adjust the initial values obtained from the $GM(1, 1)$ model predictions, revising it by adding the effect of a predicted residual. We can now describe this procedure as follows.

Let us assume that at the current time k , the residual process is in the state i , that is, $\varepsilon_k \in r_i = [l_i, u_i]$. The $GM(1, 1)$ model produces forecasting $\hat{x}_{k+m}^{(0)}$ of the variable $x^{(0)}$ for future times $k + m$, $m = 0, 1, 2, \dots$. These forecasts are adjusted according to the formula

$$\hat{x}_{k+m}^{(0)} = \hat{x}_{k+m}^{(0)} (1 - \hat{\varepsilon}_k^{(m)}), \quad k = 1, 2, \dots, n. \quad (20)$$

The prediction $\hat{\varepsilon}_k^{(m)}$ can be computed conditionally on the state of the residual process R_k . Thus, if the residual at time k is in state r_i , the prediction is given by:

$$\hat{\varepsilon}_k^{(m)} = p_{i1}^{(m)} c_1 + p_{i2}^{(m)} c_2 + \dots + p_{ir}^{(m)} c_r, \quad k = 1, 2, \dots, n. \quad (21)$$

The prediction formula (20) expresses m -steps forecasting.

Thus, for $m = 0$, we obtain a contemporary adjustment according to the state r_i given by the relation

$$\hat{x}_k^{(0)} = \hat{x}_k^{(0)} (1 - c_i), \quad k = 1, 2, \dots, n. \quad (22)$$

which is obtained setting $m = 0$ into (20) and (21). Hence, the Markov chain corrects the initial prediction according to the representative value of the residual state at time k .

For $m = 1$ and for the residual at time k being at state r_i , we obtain

$$\hat{x}_{k+1}^{(0)} = \hat{x}_{k+1}^{(0)} (1 - \hat{\varepsilon}_k^{(1)}) = \hat{x}_{k+1}^{(0)} \left(1 - \sum_{j=1}^q p_{ij} c_j\right), \quad k = 1, 2, \dots, n. \quad (23)$$

In this case, the $GM(1, 1)$ forecast $\hat{x}_{k+1}^{(0)}$ is corrected according to the factor $1 - \sum_{j=1}^q p_{ij} c_j$ which contains the expected value of the residual process at the next time, conditional to the occupancy of the state r_i at current time k .

In general, as m varies, the model gives predictions for any future time $k + m$.

3. Case Study

The transition towards a sustainable and diversified energy mix is a critical challenge and opportunity for countries worldwide. Italy, with its unique geographical, economic, and social landscape, presents an intriguing case for examining how future energy consumption patterns might evolve. This case study delves into forecasting Italy's primary energy consumption mix-up to the year 2032, employing the innovative Grey Markov Model $MCGM(1, 1)$.

The data for our study were collected from the comprehensive database of "Our World in Data" (<https://ourworldindata.org/>), renowned for its extensive global energy statistics. This rich dataset provided us with annual figures of primary energy consumption across various sources in Italy, measured in terawatt-hours (TWh), spanning from the year 2000 to 2022. The energy sources detailed in this dataset include Biofuel, Coal, Gas, Hydro, Nuclear, Oil, Solar, and Wind.

Table 1 presents data on primary energy consumption across various sources measured in terawatt-hours for the years 2000 through 2022. Each row represents a year, showing how much energy was consumed from each source during that year. The dataset shows no missing values for any observation year, reflecting a complete data collection process. Starting from 2000, when reliance on traditional sources like Coal, Gas, and Oil was predominant, there has been a noticeable diversification and increase in renewable energy sources over the years. For instance, Solar and Wind energy consumption started from nearly negligible amounts in 2000 but show significant growth, indicating a shift towards more sustainable energy sources. Notably, Nuclear energy consumption remains at zero. Following the Chernobyl disaster in 1986, Italy held a referendum in 1987, which resulted in the decision to discontinue the use of nuclear power for energy production. The table is a comprehensive depiction of the evolving energy landscape, highlighting the gradual transition from fossil fuels to renewable energy sources.

Table 1. Primary energy consumption in terawatt-hours by source.

Year	Biofuel	Coal	Gas	Hydro	Nuclear	Oil	Solar	Wind
2000	0	145.580	678.785	130.699	0	1124.703	0.053	1.665
2001	0	154.697	680.646	137.521	0	1102.603	0.056	3.464
2002	0	159.678	676.029	115.349	0	1106.428	0.061	4.098
2003	0	172.299	745.332	106.348	0	1095.877	0.070	4.228
2004	2.754	193.013	773.418	122.003	0	1072.420	0.084	5.322
2005	1.933	191.534	827.695	103.275	0	1039.467	0.089	6.709

Table 1. Cont.

Year	Biofuel	Coal	Gas	Hydro	Nuclear	Oil	Solar	Wind
2006	2.178	193.911	810.588	105.263	0	1038.177	0.100	8.454
2007	1.977	189.848	814.561	92.788	0	1005.712	0.110	11.407
2008	7.983	183.701	808.515	116.963	0	946.612	0.542	13.660
2009	12.885	143.823	743.176	137.228	0	875.992	1.891	18.273
2010	15.864	159.028	791.499	141.879	0	853.391	5.289	25.330
2011	15.672	178.270	742.161	126.410	0	828.881	29.782	27.190
2012	17.494	182.733	713.566	114.819	0	772.214	51.718	36.761
2013	14.687	157.403	667.404	143.831	0	706.480	58.838	40.601
2014	12.546	153.447	589.716	158.604	0	671.279	60.430	41.119
2015	15.486	143.395	643.154	122.631	0	710.444	61.783	39.974
2016	15.401	127.732	675.458	113.592	0	704.790	59.174	47.353
2017	15.610	111.904	715.788	96.335	0	713.695	64.876	47.216
2018	15.979	102.567	692.141	129.177	0	730.328	59.936	46.873
2019	9.359	77.534	707.906	122.194	0	709.052	62.443	53.252
2020	13.227	59.253	676.252	125.100	0	585.492	65.504	49.273
2021	13.822	65.182	723.737	118.766	0	652.068	65.519	54.760
2022	12.217	84.773	652.692	73.416	0	686.015	71.749	53.870

Primary energy is the energy that is produced when a source is burned in its raw form. It includes the energy required by the end user in the form of electricity, transportation and heating, as well as inefficiencies and energy lost during the conversion of raw materials into a usable form. Since renewable sources are reported in terms of their electricity output, they are corrected by the ‘substitution method’ for efficiency losses in fossil fuels [31]. It attempts to match non-fossil energy sources to the inputs that would be required if they were generated from fossil fuels. To this end, energy generation from non-fossil sources has been divided by a standard ‘thermal efficiency factor’ by Energy Institute [32]. In this way, it is reliable to compare inefficient fossil fuel inputs to renewable energy sources, which do not have this inefficiency.

In our analysis, we will not use the data referring to biofuel from 2000 to 2003 because they contain zero values which would affect the results unrealistically.

For illustrative purposes, let us revisit all the stages of the previously described model, employing data related to the coal energy consumption in terawatt-hours, and set them as the following vector

$$X^{(0)} = (145.580, 154.697, 159.678, 172.299, 193.013, 191.534, 193.911, 189.848, 183.701, 143.823, 159.028, 178.270, 182.733, 157.403, 153.447, 143.395, 127.732, 111.904, 102.567, 77.534, 59.253, 65.182, 84.773)^T. \quad (24)$$

The vector containing the cumulative values is

$$X^{(1)} = (145.580, 300.277, 459.955, 632.254, 825.267, 1016.801, 1210.712, 1400.56, 1584.261, 1728.084, 1887.112, 2065.382, 2248.115, 2405.518, 2558.965, 2702.36, 2830.092, 2941.996, 3044.563, 3122.097, 3181.35, 3246.532, 3331.305)^T, \quad (25)$$

and consequently, we compute

$$Z^{(1)} = (222.9285, 380.116, 546.1045, 728.7605, 921.034, 1113.7565, 1305.636, 1492.4105, 1656.1725, 1807.598, 1976.247, 2156.7485, 2326.8165, 2482.2415, 2630.6625, 2766.226, 2886.044, 2993.2795, 3083.33, 3151.7235, 3213.941, 3288.9185)^T. \quad (26)$$

To estimate the parameters \hat{a} and \hat{b} , we use the following matrices:

$$B = \begin{bmatrix} -222.9285 & 1 \\ -380.116 & 1 \\ \vdots & \vdots \\ -3288.9185 & 1 \end{bmatrix}, y = \begin{bmatrix} 154.697 \\ 159.678 \\ \vdots \\ 84.773 \end{bmatrix}. \quad (27)$$

Using Equation (7), we obtain the estimation of the developing coefficient $\hat{a} = 0.034$ and the Grey effect $\hat{b} = 210.598$. Meanwhile, we use Equation (9) to compute the Standard Errors, obtaining $SE(\hat{a}) = 0.006$ and $SE(\hat{b}) = 13.320$.

Wanting to obtain a forecast up to 2032, which is ten years ahead of the last available data, we use Equation (10) to obtain the cumulated values $\hat{x}_k^{(1)}$, and Equation (11) to obtain the predicted values $\hat{x}_k^{(0)}$. The results of the GM(1,1) model are shown in Table 2. Moreover, in Figure 1, coal primary and predicted energy consumption in terawatt-hours are plotted. It shows that the prediction starts with the initial available data, rises up, and follows an exponential behavior.

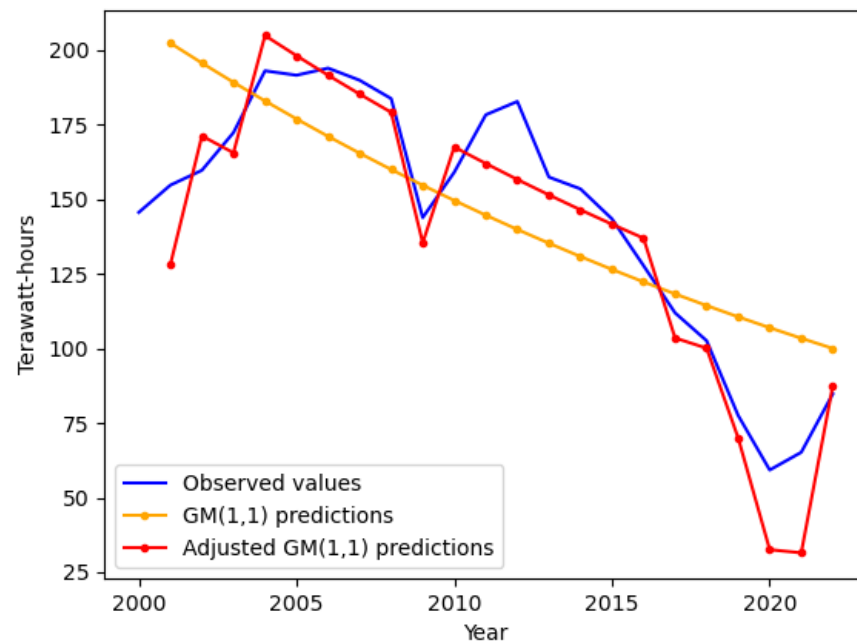


Figure 1. Coal primary energy consumption observed values and predictions in terawatt-hours.

Now, let us proceed with the correction of predictions through the Markov chain. Consider the initial data and compute the relative errors utilizing Equation (12). Results are recorded in Table 2. In this series, the minimum is $\varepsilon_{min} = -23.466$ and the maximum is $\varepsilon_{max} = 80.454$. Now, we split the range of residuals $[-23.466, 80.454]$ into $r = 4$ intervals and assume each one to be a state:

$$\begin{aligned} \text{State 1} &= [-23.466, 2.514], \\ \text{State 2} &= (2.514, 28.494], \\ \text{State 3} &= (28.494, 54.474], \\ \text{State 4} &= (54.474, 80.454]. \end{aligned} \quad (28)$$

In the last column of Table 2, states are assigned based on the interval each relative error falls into. From Equation (15), the centers of each state are $c_1 = -11.974$, $c_2 = 12.490$, $c_3 = 36.692$, and $c_4 = 69.54$.

Table 2. Error analysis, state assignment, and adjusted prediction for the primary energy coal consumption in terawatt-hours from 2000 to 2022.

Year <i>k</i>	Observed Value $x_k^{(0)}$	GM(1,1) Prediction $\hat{x}_k^{(0)}$	Absolute Residual $a\varepsilon_k$	Relative Error $r\varepsilon_k$	States <i>w</i>	Residual Adjustment $\hat{\varepsilon}_k^{(0)}$	Adjusted GM(1,1) Prediction $\hat{\hat{x}}_k^{(0)}$
2001	154.697	202.299	47.602	30.771	3	−36.692	128.071
2002	159.678	195.623	35.945	22.511	2	−12.490	171.190
2003	172.299	189.167	16.868	9.790	2	−12.490	165.540
2004	193.013	182.924	10.089	−5.227	1	11.974	204.828
2005	191.534	176.887	14.647	−7.647	1	11.974	198.068
2006	193.911	171.049	22.862	−11.79	1	11.974	191.531
2007	189.848	165.404	24.444	−12.875	1	11.974	185.210
2008	183.701	159.945	23.756	−12.932	1	11.974	179.098
2009	143.823	154.667	10.844	7.540	2	−12.490	135.349
2010	159.028	149.562	9.466	−5.952	1	11.974	167.472
2011	178.27	144.627	33.643	−18.872	1	11.974	161.945
2012	182.733	139.853	42.880	−23.466	1	11.974	156.600
2013	157.403	135.238	22.165	−14.082	1	11.974	151.432
2014	153.447	130.775	22.672	−14.775	1	11.974	146.434
2015	143.395	126.459	16.936	−11.811	1	11.974	141.602
2016	127.732	122.285	5.447	−4.264	1	11.974	136.928
2017	111.904	118.250	6.346	5.671	2	−12.490	103.481
2018	102.567	114.347	11.780	11.485	2	−12.490	100.065
2019	77.534	110.573	33.039	42.613	3	−36.692	70.002
2020	59.253	106.924	47.671	80.454	4	−69.540	32.569
2021	65.182	103.395	38.213	58.626	4	−69.540	31.495
2022	84.773	99.983	15.210	17.942	2	−12.490	87.495

Subsequently, the one-step transition probability matrix is computed through Equation (19):

$$P^{(1)} = \begin{bmatrix} 0.833 & 0.167 & 0 & 0 \\ 0.4 & 0.4 & 0.2 & 0 \\ 0 & 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0 & 0.5 \end{bmatrix}. \tag{29}$$

For example, the value 0.833 represents the probability of reaching state 1 given that the chain at the previous step was in state 1; 0.167 stands for the probability of reaching state 2 given that the chain was previously in state 1. Raising the matrix to the m -th power makes it possible to obtain the m -steps transition matrix. From the matrix in Equation (29), we obtain the following stationary distribution:

$$\pi = [0.632 \quad 0.263 \quad 0.053 \quad 0.052]. \tag{30}$$

Since we know that all the real values referred to the years 2000–2022, for these years, we use the one-step transition probability matrix, updating the state i in Equation (21). For the years 2023–2032, we use the state $i = 2$, which is the last known state, and the matrix is raised to the m -th power. Finally, once all the adjustments $\hat{\epsilon}_k^{(1)}$ are computed, Equation (20) is used to obtain the adjusted $GM(1, 1)$ predictions from 2001 to 2032. See Table 2 for the results referring to the years of the sample and Table 3 for the prediction from 2023 to 2032. Figure 2 summarizes the coal primary and predicted energy consumption in terawatt-hours through the $GM(1, 1)$ model before and after the Marvokian adjustment. Table 4 offers a clear comparison between the performance in terms of the Root Mean Squared Error (RMSE) of two models in predicting the consumption of various energy sources: the traditional $GM(1, 1)$ model and its adjusted version that integrates the Grey Model (GM) with a Markov chain. From the data analysis, it is evident that the adjusted model demonstrates significant improvements in its estimates. The dashed vertical line separates the years of our sample from the years related to the forecast. To its left, as we have previously explained, the one-step transition matrix was used, updating the forecast with the observed value each time, while on the right, the forecast was made starting from the last observed value, using the m -step transition matrix. Compared to the forecast made with the Grey Model, the Markovian correction allows for predictions that follow the trend of the observed data, enhancing the predictive capability.

Table 3. Adjusted prediction of coal primary energy consumption in terawatt-hours from 2023 to 2032.

Year k	$GM(1, 1)$ Prediction $\hat{x}_k^{(0)}$	Residual Adjustment $\hat{\epsilon}_k^{(m)}$	Adjusted $GM(1, 1)$ Prediction $\hat{\hat{x}}_k^{(0)}$
2023	96.683	−7.545	89.389
2024	93.493	−8.062	85.955
2025	90.407	−5.951	85.027
2026	87.424	−4.378	83.596
2027	84.538	−3.329	81.724
2028	81.748	−2.639	79.591
2029	79.050	−2.185	77.323
2030	76.442	−1.887	74.999
2031	73.919	−1.691	72.669
2032	71.479	−1.562	70.363

Table 4. Comparison of Root Mean Squared Errors (RMSEs) for various energy sources predicted by $GM(1, 1)$ and adjusted $GM(1, 1)$ models.

	Biofuel	Coal	Gas	Hydro	Oil	Solar	Wind
$GM(1,1)$	1.750	26.610	54.744	18.473	46.548	2.515	3.109
Adjusted $GM(1,1)$	0.727	14.096	19.875	10.149	16.471	0.363	1.401

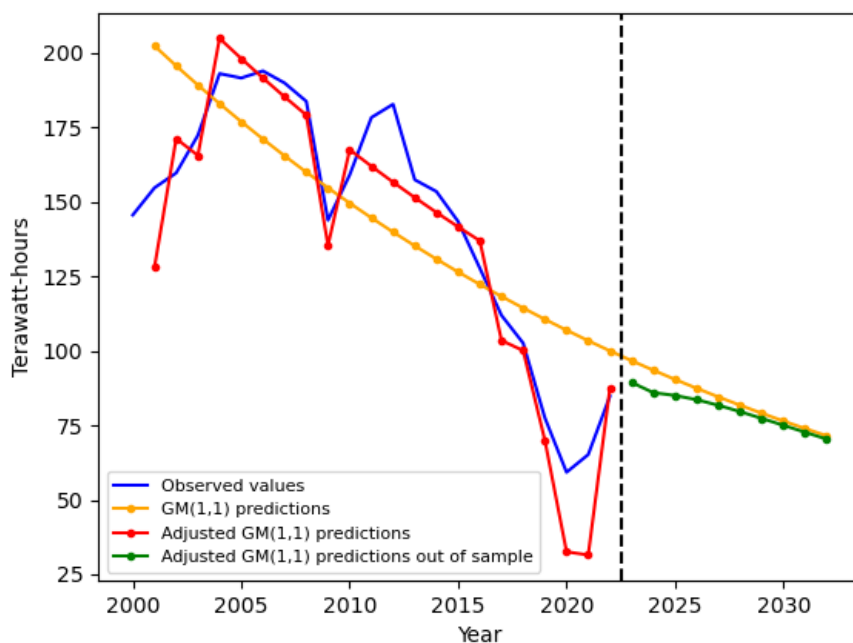


Figure 2. Coal primary and predicted energy consumption in terawatt-hours through GM(1,1) model before and after the adjustment.

All the previously described steps were repeated for each energy source, obtaining the predictions from 2023 to 2032. It should be noted that in the cases of Biofuel and Solar, it was not possible to calculate the representative values for all states because some states were never reached, as the relative errors pertained only to certain states. To overcome this problem, we employed Change Point analysis to identify the optimal point at which to truncate each data series [33]. The truncation point was determined to be in the year 2010 for Biofuel and in 2011 for Solar. After implementing these modifications, we were able to determine the representative values for each state and retrace all prior steps. In Table 5, all the GM(1,1) adjusted predictions are shown for all the energy sources, while in Table 6, the estimates together with the Standard Errors (provided in parenthesis) of the developing coefficient and the Grey effect for each energy source are provided.

Table 5. Primary energy consumption prediction in Italy in terawatt-hours from 2023 to 2032.

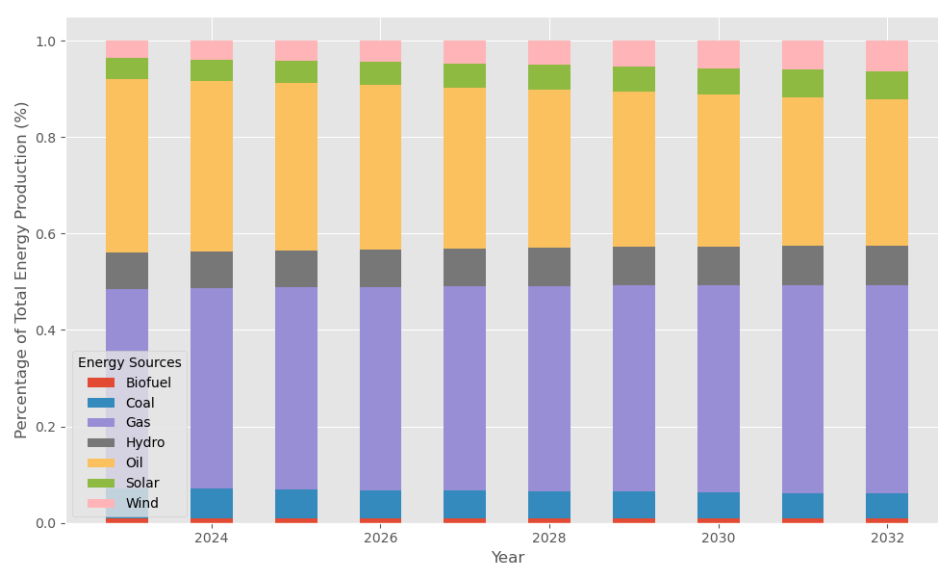
Year	Biofuel	Coal	Gas	Hydro	Oil	Solar	Wind
2023	11.461	89.389	694.271	94.266	573.561	70.008	76.235
2024	12.234	85.955	669.458	110.202	548.961	72.172	83.317
2025	11.370	85.027	675.409	111.143	537.014	73.477	91.057
2026	11.212	83.596	664.983	110.941	525.465	75.121	99.517
2027	10.991	81.724	663.762	110.773	512.196	76.688	108.762
2028	10.709	79.591	657.744	110.583	497.767	78.346	118.867
2029	10.471	77.323	654.454	110.387	482.989	80.010	129.910
2030	10.231	74.999	649.780	110.192	468.365	81.726	141.979
2031	9.995	72.669	645.926	109.997	454.118	83.469	155.169
2032	9.766	70.363	641.687	109.802	440.312	85.256	169.584

Table 6 contains the data relating to the estimate and the Standard Error of the developing coefficient and the Grey effect for each energy source.

Table 6. Estimate and the associated Standard Error of the developing coefficient and the Grey effect for each energy source.

Year	Biofuel	Coal	Gas	Hydro	Oil	Solar	Wind
\hat{a}	0.023 (0.008)	0.034 (0.006)	0.006 (0.003)	0.002 (0.005)	0.031 (0.001)	−0.021 (0.003)	−0.089 (0.010)
\hat{b}	16.745 (0.922)	210.598 (13.320)	777.177 (26.369)	121.821 (8.766)	1198.075 (24.775)	54.432 (1.170)	10.875 (2.765)

Figure 3 presents a forecasted breakdown of Italy's primary energy consumption mix from 2023 to 2032, measured in terawatt-hours (TWh). The stacked bar chart displays the proportionate contribution of each energy source to the total national energy production on a year-by-year basis.

**Figure 3.** Forecast of the Italian Primary Energy Consumption Mix (2023–2032).

The predictions highlight a steady increase in solar and wind energy, reflecting a global shift towards more sustainable and renewable energy sources. Biofuel predictions also show a consistent yet slight increase, indicating a modest contribution to Italy's energy mix. Coal consumption, on the other hand, is forecasted to decline, echoing the broader transition away from fossil fuels. The forecast for gas and oil consumption suggests a decrease, aligning with efforts to reduce carbon emissions and dependency on non-renewable energy sources. Hydroenergy consumption shows minor fluctuations, indicating stability in its contribution to the energy mix.

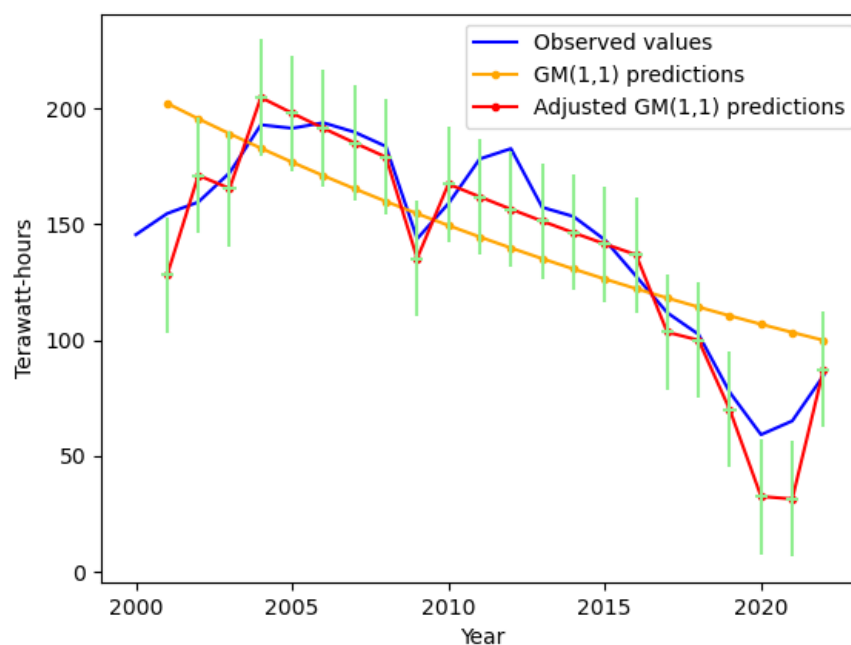
The figure may serve as a critical tool for policymakers, indicating potential trajectories for energy policy and investment. It reflects the ongoing transition in the Italian energy landscape, highlighting the gradual decline in fossil fuels and the ascent in renewables, which are essential for achieving long-term sustainability and energy security goals.

Table 7 presents the results of a sensitivity analysis on the $MCGM(1, 1)$ measured in terms of RMSE as parameters a and b are varied in the range of 2 times the Standard Error. The first row lists the values of parameter b , while the first column shows the values of parameter a . Their values were selected based on estimates obtained using the method of least squares, by adding and subtracting once and twice the Standard Errors, respectively. The trend of RMSE values generally increases from left to right across columns, suggesting the parameter's influence on the prediction error. The table provides a clear overview of how tuning parameters a and b can optimize model performance.

Table 7. Sensitivity Analysis of RMSE (Root Mean Squared Error) for varying parameters a and b in $MCGM(1,1)$ for coal primary energy consumption.

$a \backslash b$	172.329	186.165	210.598	213.835	227.671
0.021	18.079	21.884	33.015	34.852	43.640
0.027	14.313	15.027	20.952	22.087	27.755
0.034	13.820	12.086	14.096	14.783	18.587
0.040	16.327	13.306	11.891	12.100	13.944
0.046	19.672	15.721	11.798	11.638	12.043

Since the uncertainty of prediction is closely related to the SE of the two parameters involved, in Figure 4, we show the updated version of Figure 1 with error bars reflecting the expected uncertainty.

**Figure 4.** Coal primary energy consumption observed values and predictions in terawatt-hours with error bars.

4. Conclusions

In this comprehensive study, we introduced and applied the Grey Markov Model $GM(1,1)$ integrated with Markov Chain Residual Modification to forecast Italy's power generation mix-up to the year 2032. This approach allowed us to incorporate both the uncertainty inherent in historical data and their fluctuating nature, reducing the in-sample prediction error, affirming the integration of Grey systems and Markov chains and thus attempting to improve the forecast for Italy's primary energy consumption.

Our findings reveal a significant potential for renewable energy sources to increase their share in Italy's energy mix, indicating a gradual decrease in reliance on fossil fuels. This transition aligns with global efforts towards more sustainable and environmentally friendly energy production methods.

The insights garnered from this study underscore the necessity for Italy's energy policymakers to further encourage and invest in renewable energy sources. Strategic planning should incorporate the predicted shifts in energy production types, emphasizing sustainability and energy independence. Additionally, the model's predictions can serve

as a guide for adjusting current policies and devising new regulations to support the anticipated changes in the energy sector.

The evolving dynamics within the energy matrix suggest that shifts towards alternative energy sources may be approaching saturation. The potential for these sources to maintain the high growth rates observed in recent decades could be constrained by inherent limitations such as spatial requirements, intermittency of energy supply, and distribution challenges. These factors, rather than solely the governmental willingness to adopt these technologies, could significantly influence the future composition and sustainability of the energy matrix. These forecasts are based on current data, which should be updated with future observations, which is recommended via the repeated implementation of CPA.

While this study offers valuable forecasts and insights, it also opens several avenues for future research:

- Extending the model to other countries and regions to compare and contrast energy mix transitions on a global scale.
- Incorporating additional variables such as technological advancements, policy changes, and economic factors that could impact the power generation mix through a $GM(1, N)$ model.
- Exploring the potential impacts of increased renewable energy adoption on grid stability, energy prices, and environmental outcomes.

In conclusion, the application of Grey Markov Models to forecast Italy's power generation mix presents a promising tool for energy researchers and policymakers alike. By understanding future trends, Italy can better prepare for a sustainable energy future, reducing carbon emissions and fostering a resilient energy system. Further research in this area will undoubtedly refine these predictions and contribute to the global knowledge base on energy-forecasting methodologies.

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